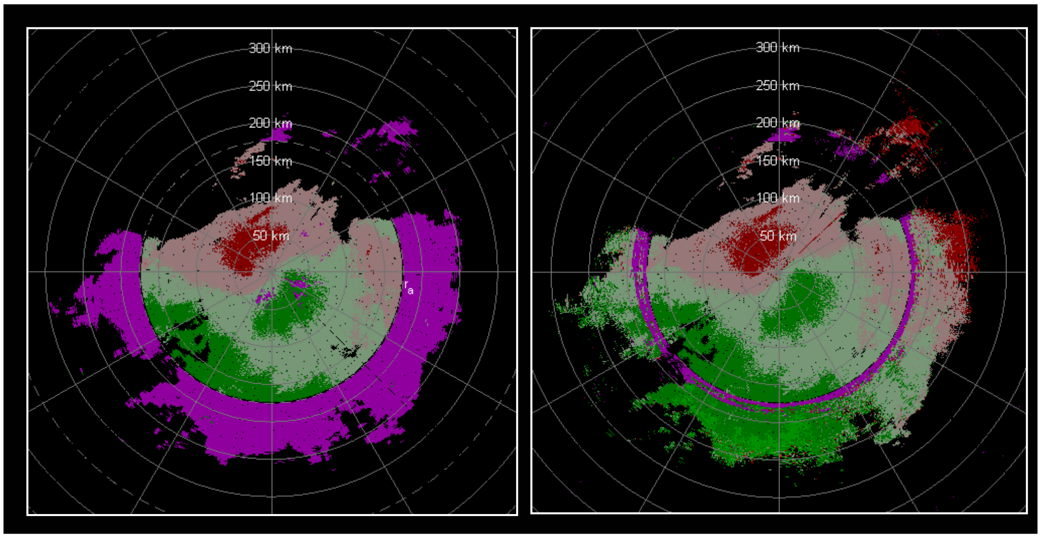
FY2004 NSSL-NCAR Interim Report

NEXRAD Range-Velocity Ambiguity Mitigation
SZ-2 Algorithm Recommendation



Range-velocity ambiguities on the current WSR-88D (left) and
using the recommended SZ-2 algorithm (right)
(KOUN, 10/08/02 1511 GMT)

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1. Introduction

This report describes the improvements to the SZ-2 algorithm as reported in the FY2003 NCAR-NSSL Interim Report, "NEXRAD Range-Velocity Ambiguity Mitigation SZ(8/64) Phase Coding Algorithm Recommendations", 15 August, 2003. The SZ-2 algorithm has been updated especially with respect to censoring and clutter filtering.

The herein recommended SZ-2 implementation is by-in-large an extension of the aforementioned Interim Report. However, the following important changes have been made: 1) ground clutter is no longer assumed to be only in the first trip, 2) a spectral based ground clutter filter "GMAP" by SIGMET is now used, and 3) censoring logic and thresholds have been updated.

To facilitate the programming of these changes, the recommended SZ-2 code builds on the existing prototype implementation by the ROC.

When implemented on the NEXRAD ORDA the herein recommended SZ-2 algorithm will significantly outperform the legacy range-velocity mitigation algorithm. However, the SZ-2 algorithm is still in its infancy and needs to be tested on much more experimental data. Further refinements can and should be made to obtain the best data quality and to minimize the amount of censored data.

2. SZ-2 Algorithm Description

The SZ-2 algorithm was first introduced by Sachidananda et al. (1998) in a study of range-velocity ambiguity mitigation using phase coding. Unlike the stand-alone SZ-1 algorithm, SZ-2 relies on power and spectrum width estimates obtained using a long pulse repetition time (PRT). The SZ-2 algorithm is computationally simpler than its stand-alone counterpart as it only tries to recover the Doppler velocities associated with a strong and weak trip signals and the spectrum widths associated with the strong trip signal. Analogous to the legacy "split cut", the volume coverage pattern (VCP) is designed such that a non-phase-coded scan using a long PRT (~ 3.1 ms) is immediately followed by a scan with phase-coded signals using a short PRT (~ 780 μs) at the same elevation angle. Hence, determination of the number and location of overlaid trips can be done by examining the overlay-free long-PRT powers.

The following is a functional description of the SZ-2 algorithm tailored for insertion into the signal processing pipeline of the RVP-8. The description is divided into two parts: long PRT processing and short PRT processing with emphasis given to the latter.

2.1. Long PRT Processing

2.1.1. Assumptions

1) There is no phase modulation of the transmitted pulses.
2) There are no overlaid echoes.
3) The number of pulses transmitted in the dwell time is $M_L^d < 32$. However, $M_L = 32$ pulses worth of data (centered relative to the dwell time) are supplied to the algorithm.
4) The number of range cells is $N_L = T_{s,L}/\Delta t$, where $T_{s,L}$ is the pulse repetition time (long PRT) and Δt is the range-time sampling period (e.g., in the legacy WSR-88D $\Delta t = 1.57$ μs).
5) The algorithm operates on one range cell of time-series data at a time (M_L samples).

2.1.2. Inputs

1) Time series data for range cell n: $V_L(m) = I_L(m) + jQ_L(m)$, for $0 \le m < M_L$, where m indexes the samples (or pulses).

2.1.3. Internal Outputs

These outputs are saved internally for later use in the short-PRT scan:

1) Clutter filtered power: P_L
2) GMAP removed power: C_L
3) Spectrum width: w_L

2.1.4. External Output

This output is sent to the ORPG:

1) Reflectivity: Z_L

2.1.5. Algorithm

SZ-2 processing in the long-PRT scan is an extension of the processing performed in any of the operational surveillance scans. Time-series data are clutter filtered using the GMAP clutter filter only in those locations where the bypass map indicates the presence of ground clutter. Clutter-filtered time-series data are used to compute total power and lag-one correlation (R_L) estimates. The signal power (P_L) is obtained after subtracting the noise power from the total power, and spectrum width (w_L) is estimated from the P_L/R_L ratio. P_L, w_L, and the powers removed by GMAP (C_L) are saved internally to be used later in the short-PRT processing. A reflectivity estimate, Z_L, is obtained from P_L as usual.

2.2. Short-PRT Processing

2.2.1. Assumptions

1) The phases of the transmitted pulses are modulated with the SZ(8/64) switching code.
2) The number of pulses transmitted in the dwell time is $M^d \le 64$. However, $M = 64$ pulses worth of data (centered relative to the dwell time) are supplied to the SZ-2 algorithm.
3) The number of range cells is $N = T_s/\Delta t$, where T_s is the pulse repetition time (short PRT) and Δt is the range-time sampling period (e.g., in the legacy WSR-88D $\Delta t = 1.57$ µs).
4) Range cells in the short PRT scan are perfectly aligned with range cells in the long PRT scan. This is important for the determination of short-PRT trips within the long-PRT data. Note: Misalignments may occur, for example, due to T_s/Δ_t not being an integer number or due to one or more samples being dropped during the transmit time.
5) The algorithm operates on one range cell (M samples) of time-series data at a time.

2.2.2. Inputs

1) Phase-coded time series data cohered to the 1st trip: $V(m) = I(m) + jQ(m)$, for $0 \le m < M$, where m indexes the samples (or pulses).
2) Ground-clutter-filtered powers and spectrum widths from the long-PRT scan: P_L and w_L. These vectors correspond to the long-PRT scan radial that is the closest in azimuth to the phase-coded radial in (1).
3) GMAP removed powers: C_L. This vector corresponds to the long-PRT scan radial that is the closest in azimuth to the phase-coded radial in (1).
4) Range-dependent ground clutter filter bypass map corresponding to the radial in (1): B. B can be either *FILTER* or *BYPASS*, indicating the presence or absence of clutter, respectively.
5) Measured SZ(8/64) switching code: $\psi(m)$, for $-3 \le m < M$.
6) Censoring thresholds:
   K_{SNR}: signal-to-noise (SNR) threshold for determination of significant returns,
   K_1: power ratio threshold for the determination of significant clutter in the non-overlaid case,
   K_2: power ratio threshold for the determination of significant clutter in the overlaid case,
   K_s: signal-to-noise ratio (SNR) threshold for determination of recovery of strong trip,
   K_w: signal-to-noise ratio (SNR) threshold for determination of recovery of weak trip,
   $K_r(w_{n1}, w_{n2})$: maximum strong-to-weak power ratios (p_1/p_2) for recovery of the weaker trip for different values of strong and weak trip normalized spectrum widths ($w_{n1} = w_1/2v_a$ and $w_{n2} = w_2/2v_{a,L}$, where v_a and $v_{a,L}$ are the maximum unambiguous velocities corresponding to the short and long PRT, respectively). The value of K_r is determined using the spectrum-width-dependent constants C_T (threshold), C_S (slope), and C_I (intercept) as indicated in step 21 of the algorithm.
   K_{CSR1}: clutter-to-strong-signal ratio (CSR) threshold for determination of recovery of all trips,
   K_{CSR2}: clutter-to-weak-signal ratio (CSR) threshold for determination of recovery of the weak trip ($K_{CSR2} \le K_{CSR1}$),

$w_{n,max}$: maximum valid normalized spectrum width estimated from the long-PRT data.

The table below shows the recommended values for the censoring thresholds in the SZ-2 algorithm. These are expected to be refined during the testing and validation stages of the SZ-2 algorithm implementation.

<table>
<tr><th>Censoring threshold</th><th colspan="3">Recommended value</th><th>Notes</th></tr>
<tr><td>K_{SNR}</td><td colspan="3">-</td><td>Actual value should be obtained from VCP definition</td></tr>
<tr><td>K_1</td><td colspan="3">100</td><td>20 dB</td></tr>
<tr><td>K_2</td><td colspan="3">10</td><td>10 dB</td></tr>
<tr><td>K_s</td><td colspan="3">1</td><td>0 dB</td></tr>
<tr><td>K_w</td><td colspan="3">3.16228</td><td>5 dB</td></tr>
<tr><td rowspan="4">K_r</td><td></td><td>$w_{n2} < 0.243$</td><td>$w_{n2} \geq 0.243$</td><td rowspan="4">Step 21 describes the computation of K_r based on C_T, C_S, and C_I</td></tr>
<tr><td>C_T</td><td>40 dB</td><td>35 dB</td></tr>
<tr><td>C_S</td><td>-20/3 dB</td><td>-20/3 dB</td></tr>
<tr><td>C_I</td><td>0.0699</td><td>0.0544</td></tr>
<tr><td>K_{CSR1}</td><td colspan="3">31622.8</td><td>45 dB</td></tr>
<tr><td>K_{CSR2}</td><td colspan="3">31622.8</td><td>45 dB</td></tr>
<tr><td>$w_{n,max}$</td><td colspan="3">0.25</td><td>This is equivalent to about 4.5 m s^{-1} for PRT #1 in the legacy WSR-88D</td></tr>
</table>

2.2.3. Outputs

1) Doppler velocities for 4 trips: $v = [v(0) \quad v(1) \quad v(2) \quad v(3)]$
2) Spectrum widths for 4 trips: $w = [w(0) \quad w(1) \quad w(2) \quad w(3)]$
3) Return types for Doppler velocity and spectrum width for 4 trips:

$$type_v = [type_v(0) \quad type_v(1) \quad type_v(2) \quad type_v(3)]$$
$$type_w = [type_w(0) \quad type_w(1) \quad type_w(2) \quad type_w(3)]$$

As in the legacy WSR-88D, *type* can take the values *NOISE_LIKE*, *SIGNAL_LIKE*, or *OVERLAID_LIKE*. These are used to qualify the base data moments sent to the RPG as being non-significant returns, significant returns, or unrecoverable overlaid echoes, respectively.

2.2.4. Algorithm

In what follows, assume that n is the current range cell number.

1) Overlaid trip determination (Inputs: P_L, C_L. Outputs: t_A, t_B, r, P, Q)

The signal powers (after noise and clutter have been removed) from trips 1 to 4, i.e., $P_L(n)$, $P_L(n + N)$, $P_L(n + 2N)$, and $P_L(n + 3N)$, are used to determine t_A and t_B, the recoverable trips, according to the following algorithm (note that this assumes perfect alignment of range cells between the long and short PRTs):

(Collect long-PRT filtered and unfiltered powers for 4 trips)
For $0 \le l < 4$
    If $n + lN < N_L$
        (Within the long-PRT range)
        (Filtered power)
        $P(l) = P_L(n + lN)$
        (Unfiltered or total power)
        $Q(l) = P(l) + C_L(n + lN)$
    Else
        (Outside of the long-PRT range)
        $P(l) = 0$
        $Q(l) = 0$
    End
    (Trip number)
    $t(l) = l$
End

(Rank long-PRT filtered powers)
Sort vectors P, Q, and t so that powers $P(0)$, $P(1)$, $P(2)$, and $P(3)$ are in descending order with their corresponding total powers as $Q(0)$, $Q(1)$, $Q(2)$, and $Q(3)$ and trip numbers as $t(0)$, $t(1)$, $t(2)$, and $t(3)$. Note that trip numbers are 0, 1, 2, or 3. In what follows, a −1 will be used to indicate an invalid trip number.

(Determine trip-to-rank mapping)
For $0 \le l < 4$
    $r[t(l)] = l$
End

Note: $t(rank)$ will be used to get the trip number for a given rank and $r(trip)$ to get the rank of a given trip.

(Determine potentially recoverable trips based on long-PRT filtered powers)
If $P(0) > NOISE.K_{SNR}$
  (The strongest trip signal is a significant return; therefore, it is recoverable)
  $t_A = t(0)$
  If $P(1) > NOISE.K_{SNR}$
    (The second strongest trip signal is a significant return; therefore, it is recoverable)
    $t_B = t(1)$
  Else
    (The second strongest trip signal is not a significant return; therefore, it is not recoverable)
    $t_B = -1$
  End
Else
  (The strongest trip signal is not a significant return; therefore, none of the trips are recoverable)
  $t_A = -1$
  $t_B = -1$
End

In the above algorithm, K_{SNR} is the SNR threshold to determine significant returns. This should be obtained from the VCP definition as in the legacy WSR-88D.

If $t_B = -1$, only one trip is recoverable.

If $t_A = -1$ and $t_B = -1$, none of the trips are recoverable and the algorithm continues at step 6.

2) Ground clutter location determination (Inputs: B, P, Q, t_A, t_B. Output: t_A, t_B, t_C, n_C)

Location of ground clutter is determined according to the ground clutter filter bypass map. Three cases are considered: (1) no clutter in any of the trips, (2) clutter in only one of the trips, and (3) clutter in more than one of the trips (this will be referred to as overlaid clutter).

(Determine trips with clutter)
$n_C = 0$
For $0 \leq l < 4$
  If $n + lN < N_L$
    (Within the long-PRT range)
    If $B(n + lN) = FILTER$
      (There is clutter in the l-th trip; therefore, store clutter trip number and increment clutter trip count)
      $clutter\_trips(n_C) = l$
      $n_C = n_C + 1$
    End
  End
End

(Handle clutter)
If $n_C = 0$
    (No clutter anywhere; therefore, clutter can be ignored)
    $t_C = -1$
ElseIf $n_C = 1$
    (Non-overlaid clutter)
    $t_C = clutter\_trips(0)$
    If $t_C \neq t_A$ and $t_C \neq t_B$
        (Clutter is not with any of the two strongest trips)
        If $t_B = -1$ and $Q[r(t_C)] < Q(0)\, K_1$
            (There is only one trip to recover and clutter is K_1-times weaker than it; therefore, clutter can be ignored)
            $t_C = -1$
        End
        If $t_B \neq -1$ and $Q[r(t_C)] < Q(1)\, K_1$
            (There are two trips to recover and clutter is K_1-times weaker than the second strongest trip; therefore, clutter can be ignored)
            $t_C = -1$
        End
    End
Else
    (Overlaid clutter)
    (Second strongest trip is unrecoverable)
    $t_B = -1$
    (Determine whether strongest trip is recoverable)
    If $P(0) > [Q(1) + Q(2) + Q(3) + NOISE]K_2$
        (The strongest power is K_2-times stronger than the out-of-trip total powers; therefore, strongest trip may be recoverable and only clutter in trip $t(0)$ may be considered)
        (Determine if there is clutter in trip $t(0)$; initially assume that clutter will be ignored)
        $t_C = -1$
        For $0 \leq l < n_C$
            If $clutter\_trips(l) = t_A$
                (The strongest trip contains clutter; therefore, set clutter trip to the strongest trip)
                $t_C = t_A$
            End
        End
    Else
        (The strongest power is not K_2-times stronger than the out-of-trip total powers; therefore, strongest trip is not recoverable)
        $t_A = -1$
        (Ignore clutter)
        $t_C = -1$
    End
End

Note: Censoring due to clutter strength and location is handled in step 21.

If $t_A = -1$ and $t_B = -1$, none of the trips are recoverable and the algorithm continues at step 6.

3) Windowing (Input: V. Output: V_W)

$$V_W(m) = \frac{V(m)h(m)}{\sqrt{G_h}}$$, for $0 \le m < M$, where h is the window function.

The use of the Blackman window is recommended to achieve required clutter suppression. In the previous equation, the signal is normalized by the square root of the window gain, G_h, in order to preserve its power. The window gain is computed from the window function as

$$G_h = \frac{1}{M}\sum_{m'=0}^{M-1} \left|h(m')\right|^2 .$$

If $t_C = -1$, there is no clutter (or clutter is ignored) and the algorithm continues at step 6.

4) Ground clutter trip cohering (Inputs: V_W, t_C, ψ. Output: V_{CW})

Time series data are cohered to trip t_C to filter ground clutter:

$$V_{CW}(m) = V_W(m)\exp[-j\phi_{t_C,0}(m)]$$, for $0 \le m < M$,

where ϕ_{k_1,k_2} is the modulation code for the k_1-th trip with respect to the k_2-th trip, obtained from the measured switching code ψ. In general,

$$\phi_{k_1,k_2}(m) = \psi(m-k_1) - \psi(m-k_2)$$, for $0 \le m < M$.

5) Ground clutter filtering (Inputs: V_{CW}. Outputs: V_{CF}, k_{GMAP})

Time series data V_{CW} are filtered using the GMAP ground clutter filter to get V_{CF} as follows:

i) Discrete Fourier Transform

$$F_{CW}(k) = \sum_{m=0}^{M-1} V_{CW}(m) e^{-j\frac{2\pi mk}{M}}$$, for $0 \le k < M$.

ii) Power spectrum

$$S_{CW}(k) = \left|F_{CW}(k)\right|^2$$, for $0 \le k < M$.

iii) Ground Clutter Filtering

$$S_{CF} = \text{GMAP}\left(S_{CW}\right)$$

Note: The receiver noise power is not provided to GMAP. In addition, GMAP should be modified to return the number of spectral coefficients with clutter (k_{GMAP}). Note that k_{GMAP} is `iGapPoints` in SIGMET's `fSpecFilterGMAP()` function.

iv) Phase reconstruction

Use the original phases except in those spectral components notched and reconstructed by GMAP:

$$\varphi_{CF}(k) = \begin{cases} 0, & k \le (k_{GMAP} - 1)/2 \text{ or } k \ge M - (k_{GMAP} - 1)/2 \\ Arg[F_{CW}(k)], & \text{otherwise} \end{cases}, \text{ for } 0 \le k < M,$$

where $Arg(.)$ indicates the complex argument or phase.

v) Inverse Discrete Fourier Transform

$$V_{CF}(m) = \frac{1}{M} \sum_{k=0}^{M-1} \sqrt{S_{CF}(k)} e^{j\varphi_{CF}(k)} e^{j\frac{2\pi mk}{M}}, \text{ for } 0 \le m < M.$$

6) Trip A and trip B cohering (Inputs: V_W, V_{CF}, t_A, t_B, t_C, ψ. Outputs: V_A, V_B)

The original (cohered to the 1st trip: $t = 0$) or ground-clutter-filtered (cohered to trip t_C) signal is now cohered to trip t_A and trip t_B using the proper modulation code.

If $t_A \neq -1$
 (Strongest trip is recoverable; therefore, cohere to trip A)
 If $t_C = -1$
  (Signal was not clutter filtered; therefore, cohere from the 1st trip)
  $V_A(m) = V_W(m)\exp[-j\phi_{t_A,0}(m)]$, for $0 \le m < M$
 Else
  (Signal was clutter filtered; therefore, cohere from trip t_C)
  $V_A(m) = V_{CF}(m)\exp[-j\phi_{t_A,t_C}(m)]$, for $0 \le m < M$
 End
Else
 (Signal was unrecoverable)
 $V_A(m) = 0$, for $0 \le m < M$
End

If $t_B \neq -1$

(Second strongest trip is recoverable; therefore, cohere to trip B)

If $t_C = -1$

(Signal was not clutter filtered; therefore, cohere from the 1st trip)

$V_B(m) = V_W(m)\exp[-j\phi_{t_B,0}(m)]$, for $0 \le m < M$

Else

(Signal was clutter filtered; therefore, cohere from trip t_C)

$V_B(m) = V_{CF}(m)\exp[-j\phi_{t_B,t_C}(m)]$, for $0 \le m < M$

End

Else

(Signal was unrecoverable)

$V_B(m) = 0$, for $0 \le m < M$

End

In the previous algorithm, ϕ_{k_1,k_2} is the modulation code for the k_1-th trip with respect to the k_2-th trip, obtained from the switching code ψ as in step 4.

7) Power computation (Input: V_A. Output: $\tilde{P}_T$)

$$\tilde{P}_T = \tfrac{1}{M}\sum_{m=0}^{M-1}\left|V_A(m)\right|^2 .$$

Note: ideally, this is the short-PRT total power in all trips with the clutter power in trip t_C removed; i.e., $\tilde{P}_T \approx P(0) + P(1) + P(2) + P(3) + NOISE$ (this assumes no overlaid clutter).

8) Computation of lag-one correlation for trips A and B (Inputs: V_A, V_B, t_A, t_B. Outputs: R_A, R_B)

If $t_A \neq -1$

(Strongest trip is recoverable; therefore, compute lag-one autocorrelation)

$$R_A = \tfrac{1}{M-1}\sum_{m=0}^{M-2} V_A^*(m)V_A(m+1)$$

Else

(Strongest trip is not recoverable)

$R_A = 0$

End

If $t_B \neq -1$

(Second strongest trip is recoverable; therefore, compute lag-one autocorrelation)

$$R_B = \tfrac{1}{M-1}\sum_{m=0}^{M-2} V_B^*(m)V_B(m+1)$$

Else
    (Second strongest trip is not recoverable)
    $R_B = 0$
End

9) Strong/weak trip determination (Inputs: V_A, V_B, R_A, R_B, t_A, t_B. Outputs: V_S, R_S, t_S, t_W)

The final strong/weak trip determination is done using the magnitude of the lag-one autocorrelation estimates (equivalent to using the spectrum widths) from the actual phase-coded data.

If $|R_A| \geq |R_B|$
    (Trip A is strong, trip B is weak)
    $t_S = t_A$
    $t_W = t_B$
    $R_S = R_A$
    $V_S(m) = V_A(m)$, for $0 \leq m < M$
Else
    (Trip B is strong, trip A is weak)
    $t_S = t_B$
    $t_W = t_A$
    $R_S = R_B$
    $V_S(m) = V_B(m)$, for $0 \leq m < M$
End

If $t_S = -1$ and $t_W = -1$, none of the trips are recoverable and the algorithm continues at step 21.

10) Strong trip velocity computation (Input: R_S. Output: v_S)

$$v_S = -\frac{v_a}{\pi} Arg(R_S),$$

where v_a is the maximum unambiguous velocity corresponding to the short PRT ($v_a = \lambda/4T_s$, where λ is the radar wavelength).

11) Discrete Fourier Transform (DFT) (Input: V_S. Output: F_S)

$$F_S(k) = \sum_{m=0}^{M-1} V_S(m) e^{-j\frac{2\pi mk}{M}}, \text{ for } 0 \leq k < M.$$

12) Processing notch filtering (Inputs: F_S, v_S, t_S, t_W, t_C, k_{GMAP}. Outputs: F_{SN}, NW)

The PNF is an ideal bandstop filter in the frequency domain; i.e., it zeroes out the spectral components within the filter's cutoff frequencies (stopband) and retains those components outside the stopband (passband). With the PNF center (v_S) in m s^{-1} units, the first step consists of mapping the center velocity into a spectral coefficient number. Next, the stopband is defined by moving half the notch width above and below the central spectral coefficient (these are wrapped around to the fundamental Nyquist interval) and adjusting the position to always include those coefficients that originally had ground clutter. However, the notch width depends on the strong- and weak-trip numbers. For strong and weak trips that are one or three trips away from each other, the modulation code is the one derived from the SZ(8/64) switching code. On the other hand, for strong and weak trips that are two trips away from each other, the modulation code is the one derived from the SZ(16/64) switching code. While the processing with a SZ(8/64) code requires a notch width of 3/4 of the Nyquist interval, the SZ(16/64) is limited to a notch width of one half of the Nyquist interval.

i) Central spectral coefficient computation:

$$k_o = \begin{cases} [\![-v_S \frac{M}{2v_a}]\!], & \text{if } v_S \le 0 \\ [\![M - v_S \frac{M}{2v_a}]\!], & \text{if } v_S > 0 \end{cases}$$

ii) Notch width determination:

$$NW = \begin{cases} M/2, & \text{if } |t_S - t_W| = 2 \\ 3M/4, & \text{otherwise} \end{cases}$$

iii) PNF center adjustment (perform only if clutter was with the strong signal)

If $t_C = t_S$ and $k_{GMAP} > 0$

$k_{ADJ} = (k_{GMAP} - 1)/2 + k_{GMAP\_EXTRA}$

if $\lfloor \frac{NW-1}{2} \rfloor - k_{ADJ} < k_o < \frac{M}{2}$

$k_o = \lfloor \frac{NW-1}{2} \rfloor - k_{ADJ}$

ElseIf $\frac{M}{2} \le k_o < M - \lceil \frac{NW-1}{2} \rceil + k_{ADJ}$

$k_o = M - \lceil \frac{NW-1}{2} \rceil + k_{ADJ}$

End

End

Note: The computation of k_{ADJ} includes an empirical constant k_{GMAP_EXTRA}. Simulations suggest that k_{GMAP_EXTRA} should be set to 1 to obtain better results.

iv) Cutoff frequency computation:

$$k_a = \begin{cases} k_o - \left\lfloor \frac{NW-1}{2} \right\rfloor, & \text{if } k_o - \left\lfloor \frac{NW-1}{2} \right\rfloor \geq 0 \\ k_o - \left\lfloor \frac{NW-1}{2} \right\rfloor + M, & \text{if } k_o - \left\lfloor \frac{NW-1}{2} \right\rfloor < 0 \end{cases},$$

$$k_b = \begin{cases} k_o + \left\lceil \frac{NW-1}{2} \right\rceil, & \text{if } k_o + \left\lceil \frac{NW-1}{2} \right\rceil < M \\ k_o + \left\lceil \frac{NW-1}{2} \right\rceil - M, & \text{if } k_o + \left\lceil \frac{NW-1}{2} \right\rceil \geq M \end{cases}.$$

v) Notch filtering:

$$F_{SN}(k) = \begin{cases} \dfrac{F_S(k)}{\sqrt{1-\frac{NW}{M}}}, & \begin{array}{l}\text{if } k_b < k < k_a \text{ for } k_b < k_a \text{ or} \\ \text{if } 0 \leq k < k_a \text{ or } k_b < k < M \text{ for } k_a < k_b\end{array} \\ 0, & \text{otherwise} \end{cases}, \text{ for } 0 \leq k < M.$$

Note: The factor $\sqrt{1-\frac{NW}{M}}$ normalizes the filtered signal in order to preserve its power.

In the previous equations $[\![x]\!]$ is the nearest integer to *x*, $\lfloor x \rfloor$ is the nearest integer to *x* that is smaller than *x*, and $\lceil x \rceil$ is the nearest integer to *x* that is larger than *x*; k_o, k_a, and k_b are zero-based indexes.

13) Inverse DFT (Input: F_{SN}. Output: V_{SN})

$$V_{SN}(m) = \frac{1}{M}\sum_{k=0}^{M-1} F_{SN}(k) e^{j\frac{2\pi mk}{M}}, \text{ for } 0 \leq m < M.$$

14) Weak trip power sum computation (after notching) (Input: V_{SN}. Output: $\tilde{P}_W$)

$$\tilde{P}_W = \tfrac{1}{M}\sum_{m=0}^{M-1} \left| V_{SN}(m) \right|^2 .$$

Note: ideally, this would be the short-PRT total power in all trips except the strong trip; i.e., $\tilde{P}_W \approx P(1) + P(2) + P(3) + NOISE$ (this assumes no overlaid clutter).

If $t_W = -1$, only one trip is recoverable and the algorithm continues at step 19.

15) Weak trip cohering (Inputs: V_{SN}, t_S, t_W, ψ. Output: V_W)

$$V_W(m) = V_{SN}(m)\exp\left[-j\phi_{t_W,t_S}(m)\right], \text{ for } 0 \le m < M,$$

where ϕ_{k_1,k_2} is the modulation code for the k_1-th trip with respect to the k_2-th trip, obtained from the switching code ψ as in step 4.

16) Computation of weak trip lag-one correlation (after notching) (Input: V_W. Output: R_W)

$$R_W = \tfrac{1}{M-1}\sum_{m=0}^{M-2} V_W^*(m)V_W(m+1)\,.$$

17) Weak trip velocity computation (Input: R_W. Output: v_W)

$$v_W = -\frac{v_a}{\pi}Arg(R_W)\,,$$

where v_a is the maximum unambiguous velocity corresponding to the short PRT ($v_a = \lambda/4T_s$, where λ is the radar wavelength).

18) Weak trip spectrum width computation (Input: w_L, t_W. Output: w_W)

(Retrieve long-PRT spectrum width estimate)
$w_W = w_L(n + t_W N)$.

19) Power Adjustments (Inputs: P, $\tilde{P}_T$, $\tilde{P}_W$, t_W. Outputs: P_S, P_W)

i) Strong trip power adjustment:

(Subtract short-PRT out-of-trip powers and noise power from total power)
$P_S = \tilde{P}_T - \tilde{P}_W$
If $P_S < 0$
(Clip negative powers to zero)
$P_S = 0$
End

ii) Weak trip power adjustment:

If $t_W \neq -1$

(Weak trip is recoverable; therefore, subtract long-PRT out-of-trip powers and noise power from adjusted weak power)

$P_W = \tilde{P}_W - [P(2) + P(3) + NOISE]$

If $P_W < 0$

(Clip negative powers to zero)

$P_W = 0$

End

End

In the previous equations *NOISE* is the receiver noise power.
Note: while P_S is used both for censoring and in the computation of the strong-trip spectrum width, P_W is used solely for censoring purposes.

20) Strong trip spectrum width computation (Inputs: P_S, R_S. Output: w_S)

The following algorithm is the one used in the legacy WSR-88D:

If $|R_S| = 0$

(Lag-one correlation is zero; therefore, signal is like white noise having the maximum possible spectrum width)

$w_S = v_a / \sqrt{3}$

ElseIf $P_S < |R_S|$

(Lag-one correlation is larger than the power; therefore, signal is very coherent having the minimum possible spectrum width)

$w_S = 0 \text{ (m s}^{-1})$

Else

(Spectrum width computation)

$$w_S = \frac{v_a}{\pi}\left[\ln\left(\frac{P_S{}^2}{|R_S|^2}\right)\right]^{1/2}$$

End

If $w_S > v_a / \sqrt{3}$

(Clip large values of spectrum width)

$w_S = v_a / \sqrt{3}$

End

Here v_a is the maximum unambiguous velocity corresponding to the short PRT ($v_a = \lambda/4T_s$, where λ is the radar wavelength).

21) Censoring and determination of outputs (Inputs: P, Q, r, P_S, P_W, v_S, v_W, w_S, w_W, t_S, t_W, n_C. Outputs: v, w, $type_v$, $type_w$)

(Go through 4 trips)
For $0 \leq l < 4$
    If $P_L(n + lN) > NOISE.K_{SNR}$ *(If #1)*
        (Long-PRT power is significant)
        If $t_S = l$ *(If #2)*
            (Trip was recovered as strong trip)
            If $P_S > NOISE.K_{SNR}$ *(If #3)*
                (Short-PRT strong-trip power is significant; determine if censoring is needed)
                (Initially tag for no censoring)
                CENSOR = FALSE
                (SNR censoring)*
                If $P[r(t_S)] < \{P[r(t_W)] + P(2) + P(3) + NOISE\}K_s$
                    (Strong-trip long-PRT power is not above K_s-times the sum of the powers of the out-of-trip trip signals plus noise; therefore, censor)
                    CENSOR = TRUE
                End
                If $t_C \neq -1$
                    (Clutter was not ignored)
                    (CSR censoring)
                    If $\{Q[r(t_C)] - P[r(t_C)]\} > P[r(t_S)]\, K_{CSR1}$
                        (Clutter is much stronger than strongest signal; therefore, censor)
                        CENSOR = TRUE
                    End
                End
                If *CENSOR = TRUE*
                    (Censor data)
                    $type_v(l)$ = *OVERLAID_LIKE*
                    $type_w(l)$ = *OVERLAID_LIKE*
                    $v(l) = 0$
                    $w(l) = 0$
                Else
                    (Do not censor data)
                    $type_v(l)$ = *SIGNAL_LIKE*
                    $type_w(l)$ = *SIGNAL_LIKE*
                    $v(l) = v_S$
                    $w(l) = w_S$
                End
            Else *(If #3)*
                (Short-PRT power is not significant; therefore, tag as noise)
                $type_v(l)$ = *NOISE_LIKE*
                $type_w(l)$ = *NOISE_LIKE*
                $v(l) = 0$
                $w(l) = 0$

End *(If #3)*

ElseIf $t_W = l$ *(If #2)*

(Trip was recovered as weak trip)

If $P_W > NOISE.K_{SNR}$ *(If #4)*

(Short-PRT weak-trip power is significant; determine if censoring is needed)

(Initially tag for no censoring)

CENSOR = FALSE

(SNR censoring)*

If $P[r(t_W)] < [P(2) + P(3) + NOISE]K_w$

(Long-PRT weak-trip power is not above K_w-times the sum of the powers of the out-of-trip trip signals plus noise; therefore, censor)

CENSOR = TRUE

End

(Power-ratio recovery-region censoring)

If $P[r(t_S)] > P[r(t_W)]\ K_r(w_S/2v_a, w_W/2v_{a,L})$

(The strong-weak power ratio is outside the recovery region for the weak trip; therefore, censor)

CENSOR = TRUE

End

If $t_C \neq -1$

(Clutter was not ignored)

(Clutter-not-with-strong-trip censoring)

If $t_C \neq t_S$

(Clutter was not with the strong-trip signal; therefore, censor)

CENSOR = TRUE

End

(CSR censoring)

If $\{Q[r(t_C)] - P[r(t_C)]\} > P[r(t_W)]\ K_{CSR2}$

(Clutter was much stronger than weak-trip signal; therefore, censor)

CENSOR = TRUE

End

End

If *CENSOR = TRUE* *(If #5)*

(Censor data)

$type_v(l)$ = *OVERLAID_LIKE*

$type_w(l)$ = *OVERLAID_LIKE*

$v(l) = 0$

$w(l) = 0$

Else *(If #5)*

(Do not censor data)

$type_v(l)$ = *SIGNAL_LIKE*

$v(l) = v_W$

(long-PRT-spectrum-width censoring)

If $w_W/2v_{a,L} > w_{n,max}$

(Spectrum width is wide; therefore, long-PRT estimate is saturated and the spectrum width is censored)

type$_W$(l) = OVERLAID_LIKE
w(l) = 0
Else
(Spectrum width is narrow; therefore, long-PRT estimate should be fine)
type$_W$(l) = SIGNAL_LIKE
w(l) = w$_W$
End
End *(If #5)*
Else *(If #4)*
(Short-PRT power is not significant; therefore, tag as noise)
type$_v$(l) = NOISE_LIKE
type$_w$(l) = NOISE_LIKE
v(l) = 0
w(l) = 0
End *(If #4)*
Else *(If #2)*
(Trip was not recovered but long-PRT power is significant; therefore, tag as overlaid)
type$_v$(l) = OVERLAID_LIKE
type$_w$(l) = OVERLAID_LIKE
v(l) = 0
w(l) = 0
End *(If #2)*
Else *(If #1)*
(Long-PRT power is not significant; therefore, tag as noise)
type$_v$(l) = NOISE_LIKE
type$_w$(l) = NOISE_LIKE
v(l) = 0
w(l) = 0
End *(If #1)*
End *(For l)*

In the previous algorithm, K_{SNR} is the SNR threshold to determine significant returns. This should be obtained from the VCP definition as in the legacy WSR-88D. K_s and K_w are the minimum SNRs needed for recovery of the strong and weak trips, respectively. Here, the noise consists of the whitened out-of-trip powers plus the system noise. K_r is the maximum p_1/p_2 ratio for recovery of the weaker trip. K_r is a function of the normalized strong and weak trip spectrum widths $w_{n1} = w_1/2v_a$ and $w_{n2} = w_2/2v_{a,L}$, and is defined as

$$K_r(w_{n1}, w_{n2}) = \begin{cases} 10^{C_T(w_{n2})/10}, & w_{n1} < C_I(w_{n2}) \\ 10^{\{C_S(w_{n2})[w_{n1} - C_I(w_{n2})] + C_T(w_{n2})\}/10}, & w_{n1} \geq C_I(w_{n2}) \end{cases},$$

where C_T is the threshold, C_S is the slope and C_I is the intercept all of which depend on w_{n2}. v_a and $v_{a,L}$ are the maximum unambiguous velocities corresponding to the short and long PRT, respectively. K_{CSR1} and K_{CSR2} are the clutter-to-signal ratio (CSR) thresholds for determination of recovery of the strong and weak trip, respectively ($K_{CSR2} \leq K_{CSR1}$). K_2 is the power ratio threshold for the determination of significant clutter in the overlaid case. Lastly, $w_{n,max}$ is the maximum valid normalized spectrum width estimated from the long-PRT data.