Research Aviation Facility

Algorithm Memorandum:

Subject: Temperature Measurements
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5 March 2013

Background

On RAF aircraft, the primary temperature measurements are produced by platinum-resistance thermometers carried in housings just outside the skin of the aircraft. The housing protects the sensing elements from damage by hydrometeors and other objects while channeling outside air past the sensing wires. The measured variables have names like TTx where x describes the measurement with characteristics like HR1 for the heated sensor #1 mounted on the right side of the fuselage. These measurements undergo considerable processing in order to produce variables characterizing the actual air temperature, with names like ATx where x has a similar meaning and will be the same as used for TTx for a given sensor. That processing takes into account that the air temperature as sensed has been heated from the temperature in the ambient atmosphere and so a correction to the measured temperature must be made to obtain a measurement of the ambient temperature.

The Present Processing Chain:

The processing chain from sensor to archived variable includes the following steps:

1. Determine the resistance-temperature relationship for the sensor. This has historically been done by immersing the sensor in a stirred bath along with a high-quality stem PRT, setting the bath at a series of different temperatures (as indicated by the PRT), and at each temperature measuring the resistance of the sensor. The result is a calibration that consists of a set of corresponding measurements of temperature and resistance, $\{T_i, R_i\}$.

2. Calibrate the onboard data-acquisition system. That system consists of a special circuit to pass a known current through the sensor and to pre-amplify the resulting voltage. That voltage is then digitized by an A-D board and recorded. For this calibration, to substitute for the need to subject the sensors to various temperatures, the sensor is removed and a resistance box is placed at its place in the circuit. Calibration consists of setting the resistances from step 1 $\{R_i\}$ into the resistance box and recording the corresponding voltages that are provided by the data system. This calibrates the entire data-acquisition chain including exciter-preamp and A-D conversion. The result is a set of corresponding measurements of resistance and voltage $\{R_i, V_i\}$ or, using the correspondence from step 1, a set of correspondong measurements of temperature and voltage $\{T_i, V_i\}$.

3. A quadratic fit to these measurements, in the form $T = c_0 + c_1V + c_2V^2$, then is the calibration used to deduce temperature from recorded voltage for measurements acquired in flight. These fits are often of marginal quality, with standard errors of >0.1°C and clear evidence in the residuals of a need for higher-order terms in the polynomial. However, this magnitude of error has been deemed acceptable because other sources of error make larger contributions

to the net uncertainty. The temperature T determined as above is a measurement of TTx, the total temperature sensed by the sensor. The resulting temperatures are the values archived in final data sets and used for subsequent calculation of the ambient temperature.

Reasons For Proposing Changes

There are some weaknesses in the historical approach, especially as applied to the GV:

1. Weaknesses in the bath calibrations:

The bath cals have come into question as we compared results from the RAF cals to those obtained by ISF, NIST, and DLR. The HARCO and Goodrich (formerly Rosemount) heated sensors are high-quality platinum resistance thermometers (PRTs) that should seldom need recalibration and that should conform to known properties of PRTs even without calibration, unless the probe is damaged or stressed, in which case the element should be discarded rather than recalibrated.

HARCO and Goodrich state that their probe conforms to MIL-P-27723E. [1] The specification states that the temperature-resistance relationship shall be as given by the following equation, called the Callendar - Van Dusen equation,[2] *with the following coefficients:* $\alpha = 0.003925$, $\delta = 1.45$, *and* $\beta = 0.1$ *for T<0 but 0 otherwise:*

$$\frac{R_T}{R_0} = 1 + \alpha \left[T - \delta \left(\frac{T}{100} - 1 \right) \left(\frac{T}{100} \right) - \beta \left(\frac{T}{100} - 1 \right) \left(\frac{T}{100} \right)^3 \right] \tag{1}$$

The tolerance allowed by the MIL spec is $\Delta T = \pm(0.25 + 0.005\,|T|)$; i.e., 0.25°C at 0°C increasing to 0.5 at -50°C. This specification applies to the normal flight environment, not just to lab tests. Thus even without calibration the sensor should satisfy (1) to this level of accuracy. Once calibrated, the sensor should be capable of lower uncertainty than this, and unstressed platinum has very stable resistance, so one could argue that calibrations should only be used when it is expected that their accuracy exceeds the tolerance of the MIL SPEC.

As an example of a calibration that shows all indications of providing the highest quality, consider the calibration of sensor S/N 630393 by ISF in 2012 in the ISF stirred bath, with calibration points shown in Fig. 1. For the fit shown in this figure, all the parameters in (1) were allowed to vary (with the restriction that $\beta = 0$ for T>0). The fit is exceptionally good, matching the measured points to a degree that appears higher than reasonable expected accuracy of the temperature measurement. This is strong support both for the validity of the calibration and for the representativeness of the Callendar-Van Dusen equation. There are much more accurate interpolation formulas in use for

[1] see http://mil-spec.tpub.com/MIL-P/MIL-P-27723E/MIL-P-27723E00006.htm
[2] Van Dusen, M. S., 1925: *J. Am. Chem. Soc.,* **47,** 326-332

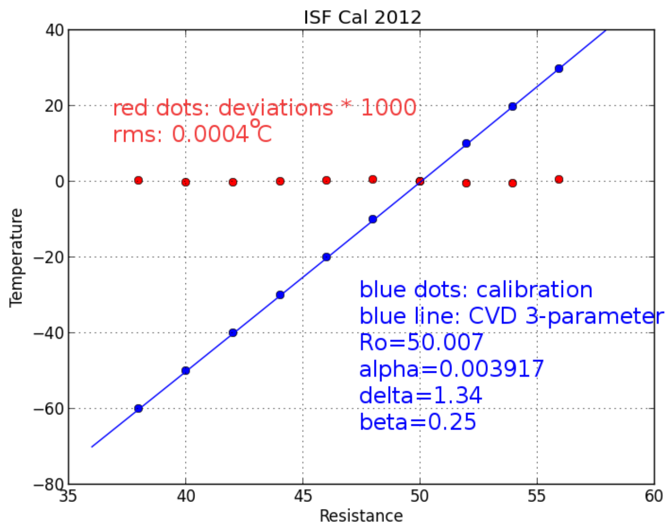

Figure 1: ISF calibration of sensor 630393, 2012. The blue line is the fit for which all four parameters in the Callendar-Van Dusen equation were allowed to vary. The red dots show the deviations multiplied by 1000; even with this amplification factor, the residuals from the fit are negligible, as is consistent with the very small standard error for the fit (0.0004°C).

precise work, including 9th-order and 12-th order polynomials, but the high accuracy achieved with (1) argues that for our work the added complexity is unnecessary.[3]

The second figure shows the result for a fit in which the parameters δ and β in (1) were fixed at the nominal values and only R_0 and α were allowed to vary. The standard error remains very small, about 0.001°C, so the results are quite insensitive to these small changes in the values of the parameters δ and β. Even if the nominal value of α, 0.003925, is used, the resulting fit to R_0 alone gives a maximum error of only about 0.03°C. Thus this calibration is quite consistent with expected values for a high-quality PRT, and the results can be represented quite well either with the unadjusted Callendar-Van Dusen equation using nominal coefficients or with a fit that slightly improves the representation of the measurements by adjusting the coefficient of thermal resistivity.

This result suggests a way to characterize the various bath calibrations that have been performed in terms of just one parameter, the value of α obtained from a fit of (1) to the measurements with R_0 adjusted to match the particular sensor and δ and β held at their nominal values. Table 1 shows a summary of the coefficients of thermal resistivity obtained from such fits to many of the recent bath calibrations.

The consistency among the ISF 2012, ISF TORERO, and DLR 2011-6 calibrations, and also their consistency with the expected nominal values, strongly suggests that these are reliable calibrations. Conversely, the low values of the RAF calibrations suggests that these are not reliable. A likely explanation is that heat losses through the supporting structure during calibration and inadequate stirring reaching the sensing element is responsible, because the coefficient of thermal resistivity would then be too low as the sensor is actually exposed to a warmer temperature (for the low-T

[3]This conclusion may need further consideration because the comparison is between two PRTs that can have the same inaccuracies while agreeing with each other to high precision. It will be worth revisiting the ITS-90 temperature scale and associated interpolation formulas at some point, but all indications and some specific guidelines from NIST suggest that within 0.1C there won't be any important differences.

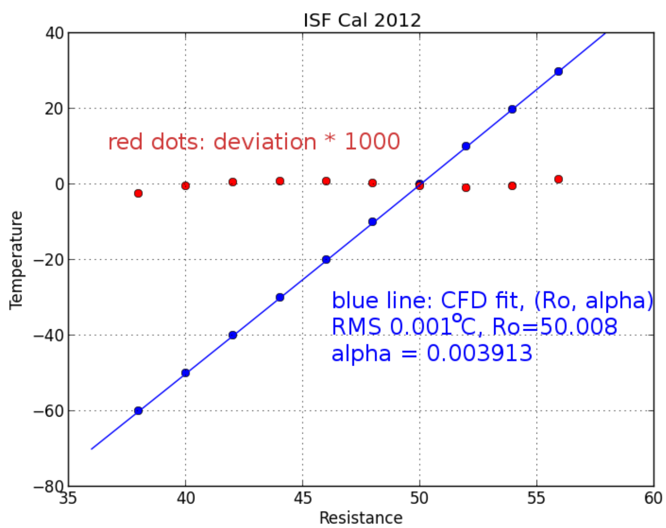

Figure 2: ISF calibration as shown in Fig. 1 but with a fit in which the parameters δ and β in (1) were restricted to the nominal values of $\delta = 1.45$ and $\beta = 0.1$for T<0 and 0 otherwise.

points) than measured by the calibrating PRT immersed alongside the sensor in the bath. It is not clear why the ISF-2011 or NIST-2011 calibrations are not consistent with the top group, but at least in the NIST case there is some suspicion that the immersion of the probes was not adequate. The test set-up they used is designed for stem PRTs, not our sensor configuration.

2. Temperature-dependence performance of the Analog-to-Digital conversion boards

It was learned in 2010 that the A-D boards used for digitizing signals have a significant temperature dependence, and this is particularly important for boards like those sampling the temperature probes because they are located where they encounter significant temperature changes. This is difficult to remove for early projects because each board has its own calibration and there is enough board-to-board variability to be significant. However, it appears that the variation with temperature is universal among boards, so if it can be determines which boards were used for sampling in each project these corrections can be made.

3. Record-keeping for calibrations

A recent system has been built to enable routine recording and archiving of calibrations. This is an opportunity to incorporate old calibrations, often recorded in technician log books, into a central repository and to try to recover as much old data into that archive as possible.

Table 1: Coefficients of thermal resistivity (multiplied by 1000) for some of the bath calibrations. HARCO S/N 630393 unless otherwise specified.

Bath Calibration	$\alpha\times$**1000**
nominal	3.925
ISF 2012	3.914
DLR 2011-6 S/N 708904 #1	3.916
" " #2	3.916
ISF TORERO 708094 #1	3.917
" " #2	3.912
ISF TORERO 708094 #1 post-cal	3.918
" " #2 post-cal	3.913
ISF Rosemount 2884 TORERO pre-cal	3.919
" " post-cal	3.920
NIST S/N 708904 2011-11	3.813
ISF 2011	3.754
ISF 2011-3 Rosemount Heated #1	3.615
" " #2	3.635
DLR Rosemount E102AL S/N 2603 (unheated)	3.744
" " S/N 2943	3.748
" " S/N 2980	3.741
" " S/N 3109	3.745
" " S/N 3241	3.774
RAF Low-T bath 2011-3	3.665
RAF Low-T bath 2010-6[4]	3.683
RAF Low-T bath 2010-6 RSMT heated #1	3.615
" " #2	3.635
RAF old bath 2010-6	3.715
RAF old bath 2009-03	3.708

4. Extreme GV flight conditions

Procedures appropriate for the C-130 may not be for the GV because of its high flight speed (creating 20-25°C of dynamic heating) and the low temperature in the upper parts of the troposphere or the lower stratosphere where the GV can fly. Calibration procedures appropriate for ±20°C can fail when the total temperature is -50°C, as is common during GV flight. More attention to calibration and recovery factors is needed, and there is evidence that the recovery factor will change with Mach number within the flight envelope of the GV.

5. Recovery-factor dependence on Mach number

At Mach numbers typical of GV flight, studies documented in Goodrich Technical Report 5755 indicate that, for the heated probe, the recovery factor can be dependent on Mach number. The recovery factor is defined from

$$\alpha_R = \frac{T_r - T_0}{T_t - T_0} \tag{2}$$

where T_r is the recovery temperature (i.e., the measurement, apart from errors that might arise from self-heating and similar causes of deviation from the accurate value), T_t is the total temperature (if the air were slowed to rest), and T_0 is the true ambient temperature. From this definition,

$$T_r = T_0(1 + \alpha_R \frac{(\gamma - 1)}{2} M^2) \tag{3}$$

where γ is the ratio of specific heats for air (1.4 for dry air, only slightly lower for moist air) and M is the Mach number. If α_R is constant, it is straightforward to fit to speed runs, but if it depends on Mach number then the approach is more complicated. There is evidence that it does, especially near Mach 1, where separation changes the flow through the housing.

This is sometimes handled through definition of a "recovery correction," defined as

$$\eta(M) = \frac{T_t - T_r}{T_t} \tag{4}$$

Then the recovery temperature is

$$T_r = T_0 \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2 (1 - \eta) - \eta\right] \tag{5}$$

which also could be used to fit to speed runs to find η, although the fit is more complicated than the usual approach to determine the recovery factor. An advantage of this form is that the recovery correction is available, for some sensors, from wind-tunnel tests, e.g., for the Rosemount 101 non-deiced and the Rosemount 102 deiced sensor. Two plots that show such calibration, taken from the Goodrich descriptions (Technical Report 5755, 2003), are shown below:

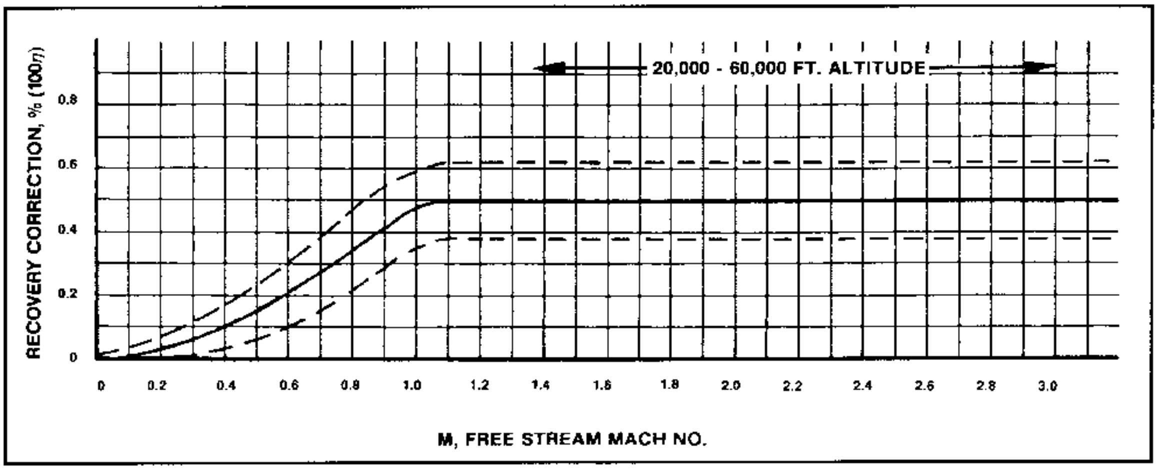

Figure 7: Wind Tunnel Data; Model 101 Recovery Corrections

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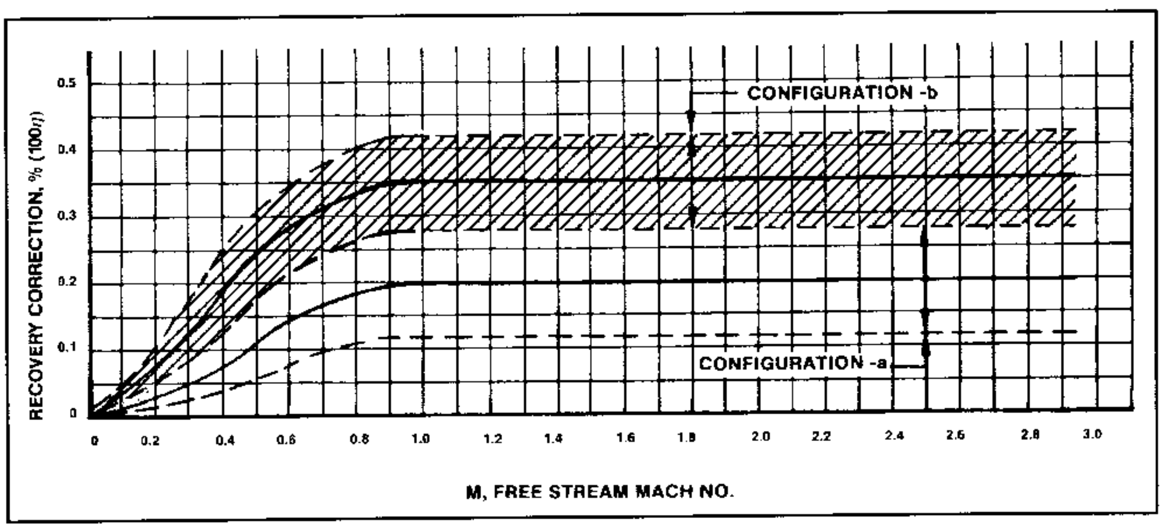

Figure 14: Wind Tunnel Data; Model 102 Recovery Corrections

These generally show quadratic behavior at low Mach number but level out to a constant correction at high Mach number. The HARCO probe differs slightly in geometry from the Rosemount probe, so use of this information for the HARCO probes may introduce an error, but similar information is not available for the HARCO geometry. It may be that the best approach is to use the Goodrich/Rosemount information as guidance to the functional form expected but fit to flight data to determine or check the details.

The following equations provide conversion between the recovery factor α and the recovery correction η:

$$\alpha_R = 1 - \eta\left(1 + \frac{2}{(\gamma-1)M^2}\right) \tag{6}$$

$$\eta = \frac{(1-\alpha_R)\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M^2}\ . \tag{7}$$

Normally α_R is near unity and η is small (<1%).

Alternate approaches using this information include:

1. Apply the recovery-correction ($\eta(M)$) data from the wind-tunnel tests, using checks vs our speed-run data;
2. Fit the recovery factor α_R to a functional form like $\alpha_R = \alpha_0 + \alpha_1 M + \ldots$ and check that this form matches the wind-tunnel data adequately. This has the advantage of simplicity in the low-Mach-number range, where a constant recovery factor is expected to provide a good representation of the needed correction. However, this form will not extrapolate well through the range spanning $M \approx 1$, so a different functional form may work better.
3. Fit instead to a functional form expected to give the observed leveling of η above $M = 1$, for example by using inverse powers of M or more complicated functions.

To help determine which approach to take, first consider fits to the available data (Figs. and), either as presented ($\eta(M)$) or converted to $\alpha_R(M)$. For this purpose, the data from these plots were used to determine tabular relationships, and from those the recovery factor was determined as a function of Mach number, as shown in figures 3 and 4:

These figures suggest that the unheated probe has, within the accuracy of this determination, a recovery factor that can be taken as constant with Mach number, with the approximate value of 0.97 throughout the normal GV flight range. However, the heated probe (102, configuration a) shows a substantial dependence of recovery factor on Mach number, such that neglect of this dependence would introduce an error. As shown in Fig. 5, the dependence is represented well by the following equation, where M is the Mach number:

$$\alpha_R = 0.988 + 0.053\log_{10}M + 0.090(\log_{10}M)^2 + 0.091(\log_{10}M)^3 \tag{8}$$

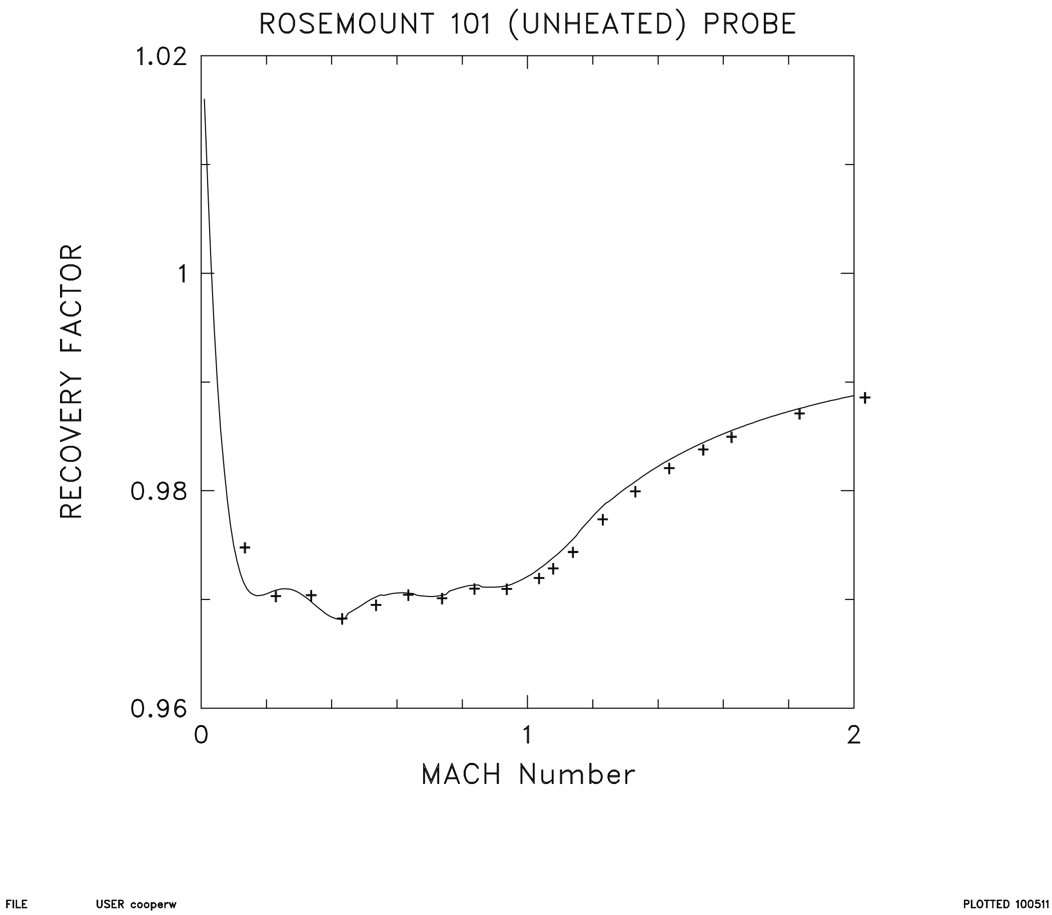

Figure 3: Recovery factor for the Rosemount 101 (unheated) sensor

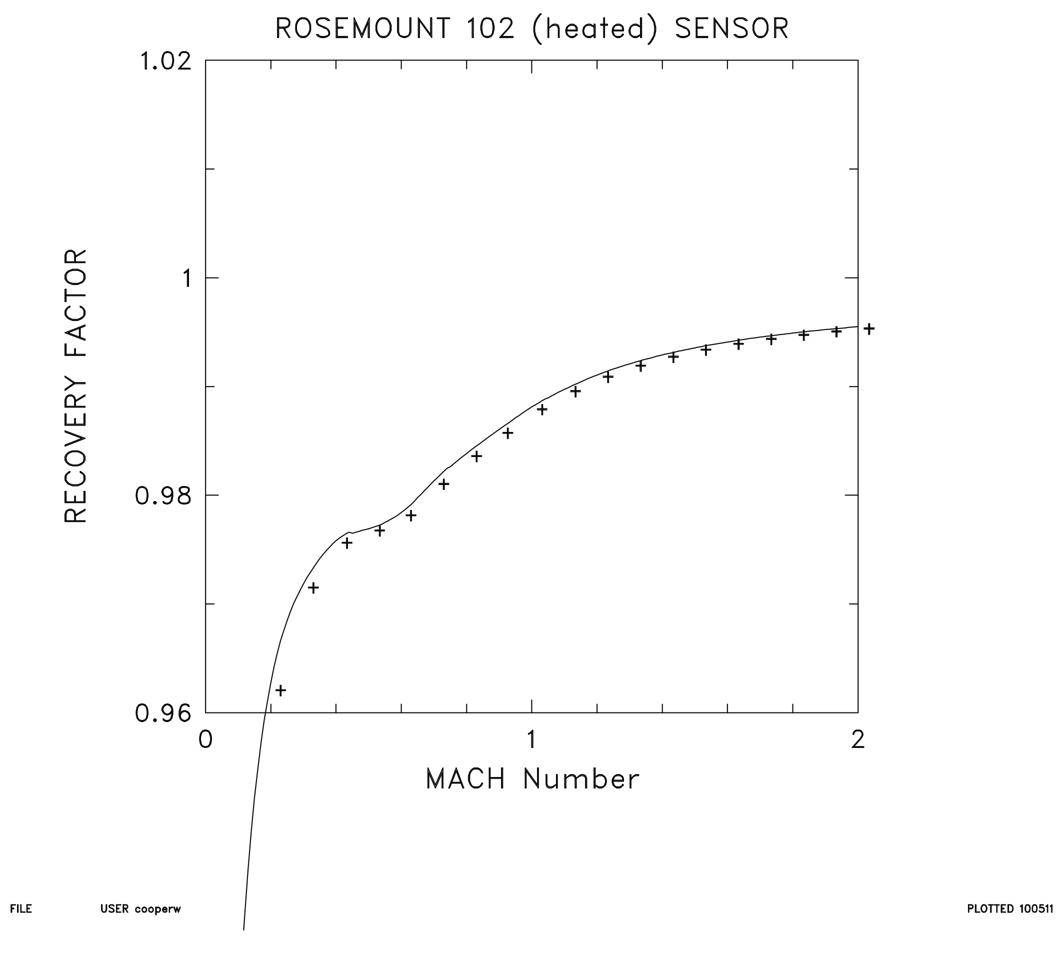

Figure 4: Recovery factor for the Rosemount 102a (heated) sensor.

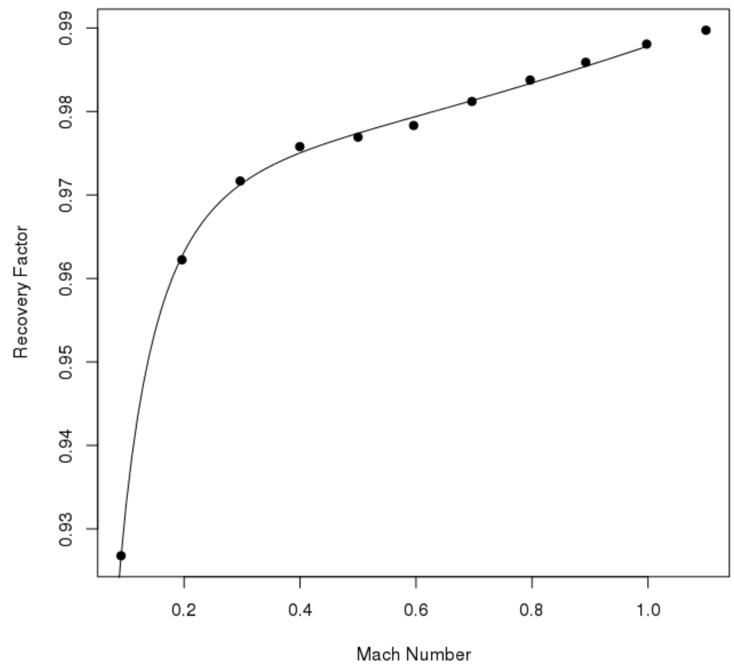

Figure 5: Fit to recovery factor for the Rosemount 102a (heated) sensor.

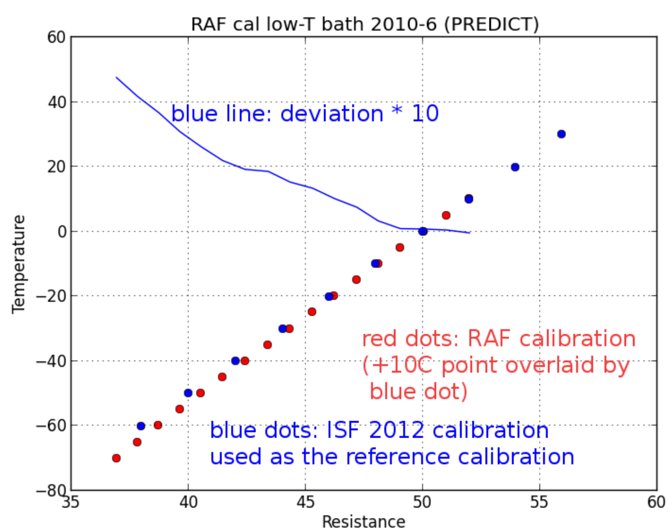

Figure 6: RAF calibration of 2010-6 in the low-T RAF bath, as used for PREDICT (red dots), in comparison to the ISF-2012 calibration (blue dots). The blue line represents 10 times the error that results if the ISF-2012 calibration is correct and the RAF calibration is used instead; e.g., a typical error at -50°C is about 3°C. The sign is such that the temperature produced by the RAF calibration would be too low by the indicated amount.

Magnitude of Errors: PREDICT as an Example

In table 1, the bath calibration that was the basis for the PREDICT on-board calibrations is marked by a footnote. The error introduced by this calibration, if it is assumed that the 2012 ISF calibration is correct, is shown in Fig. 6. The errors are substantial at low T, amounting to about 3°C at typical low GV total temperature around -50°C. The errors at higher temperature are dependent on how the fit is propagated through the system via the onboard calibration because there are no measurements above 10°C in the RAF calibration so the onboard fit will have to be extrapolated to higher temperature, with associated extrapolation errors likely because the assumed coefficient of thermal resistivity is too low.

Recommendations

Bath and on-board calibrations

1. The evidence seems clear that the calibrations in RAF baths are not adequate and probably cannot be improved to the degree needed. We should cease doing these calibrations and, when calibration is necessary, we should rely on the ISF calibration facility. NIST also does not appear to be a reliable option. However, the calibrations are expected to be stable so calibrations should be repeated only as occasional checks or in cases where there is suspicion that the probe has been damaged.

2. Onboard calibrations are valuable checks and should be continued. However, resistance values taken from reference calibrations fitted to the Callendar - Van Dusen equation should be used. For example, Table 2 lists corresponding values based on a fit to the ISF 2012 calibration. The values in this table are the resistance values that result from (1) at the specific temperatures listed and with α as determined by ISF 2012, so these resistance values can be set into the calibrating resistance box attached to the sensor location on the aircraft and the corresponding voltages read by the data system to provide the desired onboard calibration. This appears to be applicable to heated HARCO and heated Rosemount temperature probes but not to the unheated Rosemount probe, which DLR found to have a value of $\alpha \approx 0.0037$ (see Table 1) but exhibiting enough probe-to-probe variability to be worth including the probe-specific calibration.

Table 2: Corresponding values of temperature and resistance determined from Eq. (1) using the fit parameters determined from the 2012 ISF calibration. These would be appropriate values to use during on-board calibration.

Temperature, °C	Resistance, Ω
-70	35.968
-60	37.991
-50	40.008
-40	42.020
-30	44.025
-20	46.025
-10	48.020
0	50.008
10	51.991
20	53.968
30	55.939
40	57.905

Recovery factor

For unheated Rosemount probes, use a constant recovery factor of 0.97. For heated Rosemount or HARCO probes, use (8). Check these using speed-run data.

Reprocessing

Reprocessing archived datasets, or evaluation to determine if reprocessing is necessary, requires a different approach. The easiest is to tabulate the resistances used for the project onboard calibration, use (1) to determine the correct temperatures that should have been used with those resistances, and repeat the quadratic fit to the resulting values. For example, for PREDICT the RAF low-T bath cal of 2010-6 was used to determine the resistances used for the onboard calibration, and the temperatures and resistances as determined in that calibration are as listed in the first two columns of Table 3. However, from (1) with the coefficients as determined from the ISF 2012 calibration, the actual temperatures for those resistances are as listed in the 3rd column labeled "corrected temperature". The voltages measured during the onboard calibration before the project are listed in the fourth column; for these voltages and the temperatures in the first column, the quadratic calibration coefficients were (-89.225, 25.933, -0.07941).[5] However, when a quadratic fit is performed instead to the corrected temperature vs voltages, the result is (-82.44, 22.71, 0.297)

[5]Strangely, the archived files list (-89.2, 25.923, -0.07941) which differs from both the pre-project calibration and the post-project calibration, which was (-89.235, 25.925, -0.08888). I haven't yet learned the source of the calibration used for production processing.

with a standard error of 0.024°C. This standard error is much smaller than that of the original calibration, which was 0.12°C. This is an indication of a problem with the functional form of the fit in the original calibration, for which a much higher order polynomial would have been needed; even cubic still gave a standard error larger than that from the new calibration. This new quadratic calibration is then the one that should be used for PREDICT re-processing, if it is accepted that the ISF-2012 values should be used. Alternately, of course, the archived temperature values can be used with the calibration used in processing to determine the original voltages, from which the corrected temperature can be obtained using the new calibration.

Table 3: The RAF bath calibration of 2010-6 in the low-T bath, as used for PREDICT pre-project and post-project calibrations, with the pre-project voltages and the corrected temperatures determined from the resistances under the assumption that the ISF-2012 calibration is correct.

bath temperature as set	measured resistance	corrected temperature[a]	voltages
10	51.983	9.961	3.8709
5	51.010		
0	50.024	0.080	3.4764
-5	49.033		
-10	48.086	-9.667	3.0807
-15	47.174		
-20	46.229	-18.981	2.6988
-25	45.294		
-30	44.331	-28.473	2.3061
-35	43.395		
-40	42.404	-38.082	1.9042
-45	41.453		
-50	40.532	-47.389	1.5124
-55	39.618		
-60	38.725	-56.344	1.130
-65	37.815		
-70	36.919	-65.267	0.74733

[a]

The calculation of the corrected temperatures in Table 3 is not immediately straightforward because (1) provides resistance as a function of temperature while the inverse is needed to determine temperature from resistance. Numerical methods can be used for this, but rewriting (1) in the following form provides a good basis for iteration:

$$T=\left(\frac{R_T}{R_0}-1\right)\frac{1}{\alpha\left(1+(\delta/1.e2)\right)}+\frac{1}{(1+\delta/1.e2)}\left\{\frac{\delta}{1.e4}T^2-\frac{\beta}{1.e6}T^3+\frac{\beta}{1.e8}T^4\right\}$$

Using the first term on the right side as an initial estimate and then iterating quickly converged, to machine double-precision in six iterations, so this is a practical equation to use iteratively for calculating temperature from resistance.

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