Research Aviation Facility

Algorithm Memorandum:

Subject: Temperature Measurements in CONTRAST
Al Cooper
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Background

Recently, there have been two significant improvements in RAF calibrations of temperature sensors:

1. Bath calibrations have been improved by transferring responsibility to the ISF calibration facility, where the low-temperature bath is much better circulated than the RAF low-temperature bath. The result has been that most calibrations of Rosemount or HARCO heated temperature sensors produce consistent results that are consistent with the nominal calibration of the probe as given by the Callendar-Van Dusen equation (as described in other recent notes). Bath calibrations were performed after CONTRAST and produced normal results in good agreement with the previous calibrations, so the association between resistance and temperature is known with confidence for these probes.

2. Tracking of calibrations has been improved through implementation of the calibration editor, which maintains a history of the on-aircraft calibrations in terms of corresponding temperature, resistance, and voltage measurements. In the past, the on-aircraft calibrations were often recorded in terms of temperature vs voltage without recording the actual resistances used, so it was difficult to determine which bath calibration was being used for T vs R. That uncertainty is now removed, and the on-aircraft calibrations prior to CONTRAST appear to have been done correctly.

An additional change recently implemented is that the recovery factor has been made dependent on Mach number, as determined by Rosemount in wind-tunnel tests. This is also documented in another note.

The result of the above has been that temperature measurements in recent projects such as DC-3 appear much improved over earlier projects, and in fact the errors identified in earlier projects were so significant that reprocessing of those data sets is being considered. In addition, a study conducted and documented in a paper describing the LAMS measurements (Cooper et al., 2014) used integrations of the hydrostatic equation to deduce that the temperature measurements in one recent project (HIPPO-5), after correction for calibration errors, were within about 0.5°C of the temperatures deduced from that study. These results have led to increased confidence in the temperature measurements, especially from the G-V where potential errors are large and calibration at low temperature is particularly difficult.

Despite these steps, it was evident throughout the CONTRAST project that there was a problem with the temperature measurements. Particularly troublesome was the difference between the measurements from the HARCO heated sensors (ATHR) and the Rosemount heated sensor (ATFH). Two sensors in each of these probes agreed well with each other, but the two probes differed by as much a 4°C, especially for low temperature. This note documents an effort to study this problem and arrive at a recommendation for CONTRAST processing.

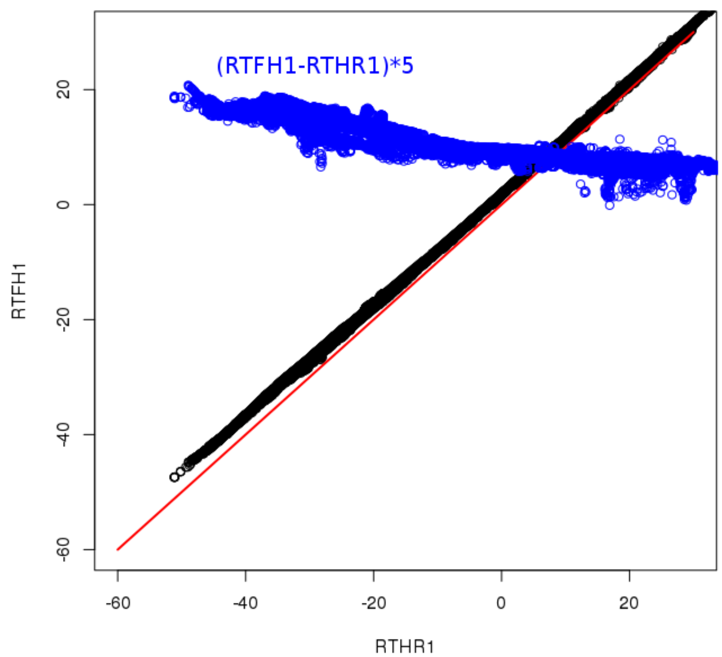

Figure 1: *Comparison of the measurements TTHR1 and TTFH1 (black symbols) for all measurements in CONTRAST except those excluded as described in the text. The blue symbols show the difference in measurements (RTFH1-RTHR1)*5 as a function of RTHR1, with the factor of 5 used to magnify the difference for plotting. The red line is a reference line representing agreement between the measurements.*

Problems with the temperature measurements in CONTRAST

Difference Among Measurements

Figure 1 illustrates the problem by comparing the recovery temperatures (approximately equal to the total temperatures) from two sensors, RTHR1 (from the HARCO heated probe) and RTFH1 (from the Rosemount heated probe). The same recovery factors are used to find ambient temperature from these measurements, so the comparison between ATHR1 and ATFH1 is similar. The black symbols in this plot are all measurements from all 17 flights in CONTRAST, except for some points where the airspeed was low that are excluded to eliminate measurements on the ground or during initial takeoff. Measurements in water cloud (flagged by CONC_RWOO > 1) were also excluded to avoid possible effects of wetting of the sensors. The plotted discrepancy approaches 4°C for low temperature and remains significant also for the highest temperatures. The comparison is quite consistent throughout the project with little change from the first flight to the last in the average, and the scatter evident in the difference (blue symbols in Fig. 1) is not associated with change from flight to flight but rather temporary differences arising when the temperatures were

Table 1: *Linear-fit coefficients for ATx=$a_0 + a_1$AT_A, for each of the 17 flights in CONTRAST and for ATx={ATHR1, ATFH1). Results are similar for ATHR1 and ATFH2.*

Flight	ATHR1	ATFH1
1	0.60+1.026X	1.79+1.002X
2	0.56+1.028X	1.801+1.004X
3	0.43+1.024X	1.74+1.003X
4	0.39+1.023X	1.748+1.000X
5	0.37+1.021X	1.775+1.001X
6	0.61+1.028X	1.93+1.004X
7	0.62+1.023X	2.05+1.003X
8	0.71+1.025X	2.11+1.003X
9	0.87+1.025X	2.14+1.002X
10	1.01+1.024X	2.36+1.001X
11	1.02+1.028X	2.35+1.006X*
12	1.12+1.025X	2.21+0.999
13	1.28+1.027X	2.52+1.001X
14	1.39+1.0311	2.66+1.007X*
15	1.23+1.028X	2.43+1.002X
16	1.28+1.028X	2.61+1.005X
17	1.24+1.027X	2.41+0.995X

*The two flights marked with asterisks had more than usual scatter and might be biased by response-time differences or other problems with the measurements.

changing rapidly and perhaps arising because spectral analysis indicates that the Rosemount sensor responds faster than the HARCO sensor.[1]

Change in the A/D card:

After the project, John Cowan reported that the calibration of the A/D card had shifted by about 30 mv, a shift that corresponds to about 0.7°C. To investigate this, the regression fit between ATx and AT_A (where ATx is either ATHR1 or ATFH1) was examined flight-by-flight. If AT_A remains consistent throughout the project, this comparison should help determine if the shift occurred discretely at some time or gradually throughout the project. Table 1 shows the results. The first five flights and the last five flights are reasonably consistent, but there is a gradual shift evident from flight 6 to flight 13 (red entries in the table). The change is primarily a change in the coefficient a_0 in the expression $ATx = a_0 + a_1 AT_A$), with relatively steady slope throughout the project for each sensor, which suggests an offset in the A/D card. The shift is consistent with a change of 0.7°C as expected from the measured change in the calibration of the card.

[1] If the measurements from the avionics system were added to this plot as (TT_A-RTHR1), they would lie below and approximately parallel to the blue symbols, with differences changing to positive for the highest temperatures but otherwise intermediate between ATFH1 and ATRH1.

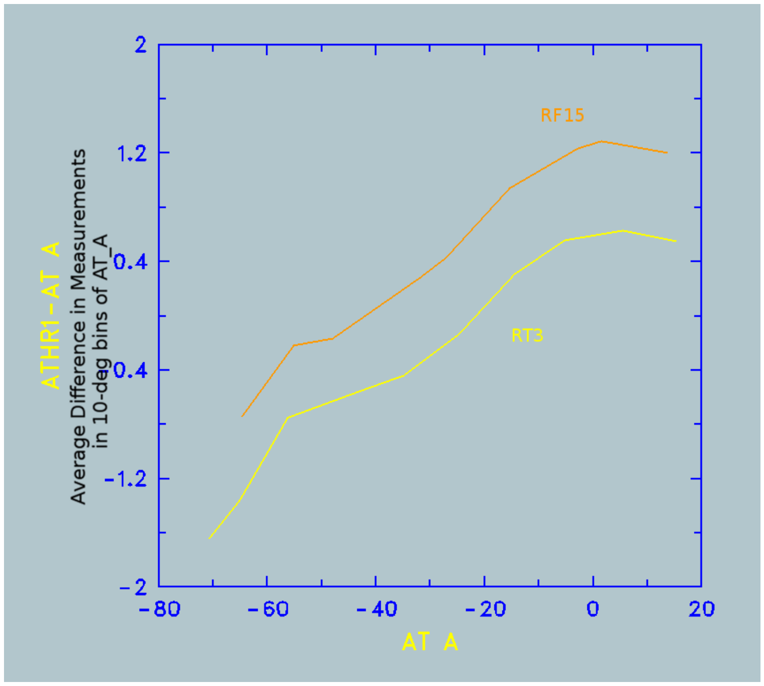

Figure 2: *Difference between ATHR1 and AT_A for flight 3 and flight 15, showing the shift that occurred during the project. All measurements were averaged in bins of AT_A that were 10°C wide and the resulting points were connected in this graph.*

A further illustration of this offset is shown in Fig. 2. The plot is consistent with an offset of about 0.7°C as expected. Based on Table 1, it appears that the first calibration coefficient should be adjusted as follows: before RF6: no change; flights RF6-12: adjust by −0.7°C * (flight number - 5) / 8 ; After RF12: adjust by -0.7°C. This would compensate for the apparent change in the calibration coefficients as deduced from the comparison to AT_A, assumed unchanged through the project. This does not remove the difference between ATHR1 and ATFH1, though; they remain the same with this suggested adjustment to both. Both are sampled by the same A/D card and the measured shift in the card is about the same for each channel, so the discrepancy between measurements from the Rosemount vs HARCO sensors is not resolved by this adjustment.

Analysis

The estimation of calibration coefficients from the hydrostatic equation

To try to determine which, if either, sensor should be used for temperature measurement, an approach documented in the recent LAMS paper was repeated for the CONTRAST data. In this approach, the hydrostatic equation is used to deduce what the temperature should be by relating the temperature to the ratio of the height changes to the changes in the logarithm of the pressure. The following equations are used to deduce the (absolute) temperature from measurements of pressure (p) and altitude (z):

$$\delta p_i = -\frac{g\,p_i}{R_a T_i}\delta z_i \ . \tag{1}$$

The solution for temperature is

$$T_i = -\frac{g}{R_a}\frac{\delta z_i}{\delta \ln p_i} \tag{2}$$

where g is the acceleration of gravity and R_a is the gas constant for air. For accuracy, the latitude and altitude dependence of g were incorporated and R_a was calculated considering the moisture content of the air. Measurements from a global positioning system receiver were used to determine δz_i with high accuracy, and LAMS-calibrated corrections to the pressure measurements (as described in Cooper et al., 2014) were used for p_i.

The estimated precision in p is about 0.1 mb, and this is too large to permit accurate deduction of the temperature from any individual 1-s measurement. However, it is possible to use all measurements from the 17 CONTRAST flights to define a measure of disagreement between the temperature measurements and the estimates derived from (2), as follows:

$$\chi^2 = \sum_i \frac{1}{\sigma_z^2}(h_i - z_i)^2 \tag{3}$$

where z_i is the height measurement from GPS and h_i is the height predicted from the following integration step:

$$h_i = Z_{i-1} - \frac{R_a f(V_i)}{g}\ln\frac{p_i}{p_{i-1}} \tag{4}$$

$$f(V_i) = \frac{\left(c_0 + c_1 V_i + c_2 V_i^2 + T_0\right)}{1 + \alpha \frac{R_a}{2C_v} M^2} \tag{5}$$

with V_i the voltage measured for the *ith* second, α the recovery factor, M the Mach number, T_0 the conversion from units of Celsius to kelvin (273.15°C), and $\{c_0, c_1, c_2\}$ calibration coefficients used to convert from V_i to recovery temperature (e.g., RTHR1).

With this definition of χ^2, summed over all measurements from the 17 flights of CONTRAST, it is possible to vary the calibration coefficients in order to minimize the errors and so to determine new calibration coefficients that are independent of the normal calibration procedure. This takes into account the recovery factor and humidity dependence of the air properties and so finds the best fit subject to the assumptions made for those quantities. As a result, the minimization produces the best calibration consistent with all those assumptions.

There is another adjustment that should be introduced, however, The voltage measurements apparently changed during the project, as described on the preceding page. This would introduce

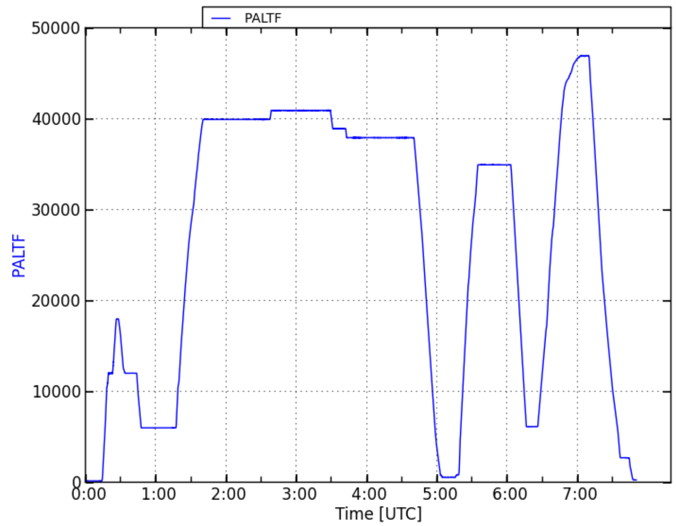

Figure 3: *Representative height profile for one of the CONTRAST flights (flight #5).*

additional variance and an error in the deduced results, so the flight-dependent adjustment described earlier was added to the voltages entering (5). This was accomplished by adjusting the temperature as described on page 4 before deducing voltages from the calibration coefficients used to calculate recovery temperature as recorded in the processed files. This produced a substantial (ca. 30%) reduction in χ^2, so this provides support for introducing this shift.

There is a subtlety involved in this calculation: If the atmosphere is baroclinic, the pressure gradient present along the flight track can introduce a false variation into the relationship between height and pressure that is not simply that provided by the hydrostatic equation. A procedure to account for such variations was described by Cooper et al., 2014, so that should be consulted for documentation of that correction. It was used in this study in the same form documented in that reference.

This analysis succeeds because of the large number of altitude changes present in the CONTRAST flights. Figure 3 shows an example. For success, it is important to have enough measurements in climbs and descents, because the uncertainty associated with each individual measurement is only adequate to constrain the temperature to within a few degrees and significant averaging is needed to reduce this uncertainty to acceptable limits. In the case of CONTRAST, there were more than 450,000 measurements, so it was possible to obtain a highly constrained result. This was also true of the HIPPO-5 measurements where this was applied previously (although in a different form, to validate measurements rather than deduce new calibration coefficients).

It is perhaps worth documenting some details regarding how this minimization was performed:

1. The needed measurements from the processed netCDF files were extracted to a form that could be read into 'R'.
2. All the measurements were then read sequentially to initialize an R session. The measurements were concatenated into single series for each variable, and the offsets described above

were introduced for the A/D drift.

3. An R routine was defined to evaluate χ^2 by considering all the measurements and applying qualification tests (to ensure some climb was involved and the flight speed was not too slow, e.g.). About 450,000 1-s measurements were used, giving minimum $\chi^2 \simeq 450,000$ for $\sigma_z \simeq$ 1.2 m.

4. The R routine 'optim' was used to find the values of the calibration coefficients that minimized χ^2 for this data set.

5. The search procedure was required to converge to the default tolerance, which ended the iteration when steps reducing the χ^2 became smaller than about 0.01. About 200 function evaluations (looping over >450,000 measurements for each evaluation) were required, and the process typically took about 10 min.

6. The routine also estimates the Hessian of the dependence of χ^2 on the coefficients, and from that the following error matrix was calculated for RTHR1:

$$\mathrm{H}^{-1} = \begin{pmatrix} 0.001510 & -0.000970 & 0.0001484 \\ -0.0009704 & 0.0006341 & -0.00009827 \\ 0.0001484 & -0.00009827 & 0.00001541 \end{pmatrix}$$

indicating standard errors in the coefficients of {0.04°C, 0.025°C V^{-1}, 0.004°C V^{-2}}. Because the measured voltages range from 0–5 V, this error matrix indicates that the uncertainty in temperature resulting from uncertainty in the fit coefficients is < 0.07°C. Note that the errors are negatively correlated to a high degree, which still further reduces the uncertainty in any application.

Results

The minimization resulted in the new calibration coefficients listed in the following table:

sensor	RTHR1			RTFH1		
coefficient	c_0	c_1	c_2	c_0	c_2	c_2
deduced	-80.29	20.867	0.57181	-86.60	23.037	0.37657
original	-82.40	22.658	0.29320	-81.89	22.729	0.29122

In Addition: Add these corrections to the first coefficient in the new set, for both sensors:
-0.7°C * (flight number - 5) / 8 for flights 6–12
-0.7°C for flights 13–17

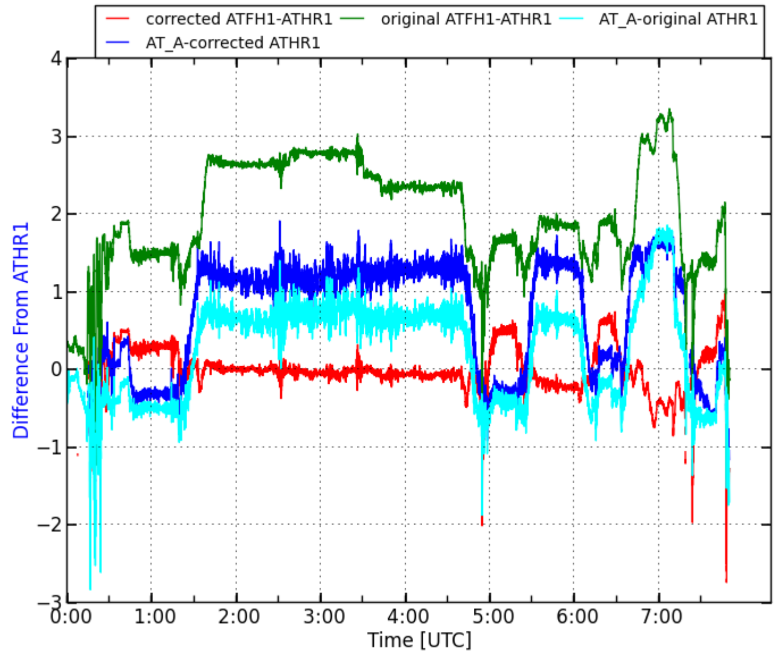

Figure 4: *Original and reprocessed results, flight #5.*

Figure 4 compares the differences for the original (green and cyan lines) and the reprocessed (red and blue lines) temperatures. The large difference between ATFH1 and ATRH1 (green line) has mostly been removed (red line). A smaller difference between ATHR1 and AT_A (originally, cyan line) and remained or increased (blue line). If it is assumed, because of the absolute nature of the minimization process, that the reprocessed values are correct, then the result is that the new values of ATHR1 and ATFH1 are mostly within about 0.5°C of each other and are usually still closer, while AT_A is different by variable amounts from +1.5 to -0.3°C. The systematic variation with flight number in plots like this has also been removed by the adjustment introduced to the calibration coefficients.

Figure 5 shows that the adjustment introduced to RTHR1 is fairly minor, amounting to less than 1°C over the full range of measurements. A similar plot for RTFH1 (Fig. 6) shows that the adjustment required for this sensor is much larger, up to 4°C at low temperature.

Figures 5 and 6 are strong evidence that the HARCO measurement is more reliable while the Rosemount measurements have substantial error.

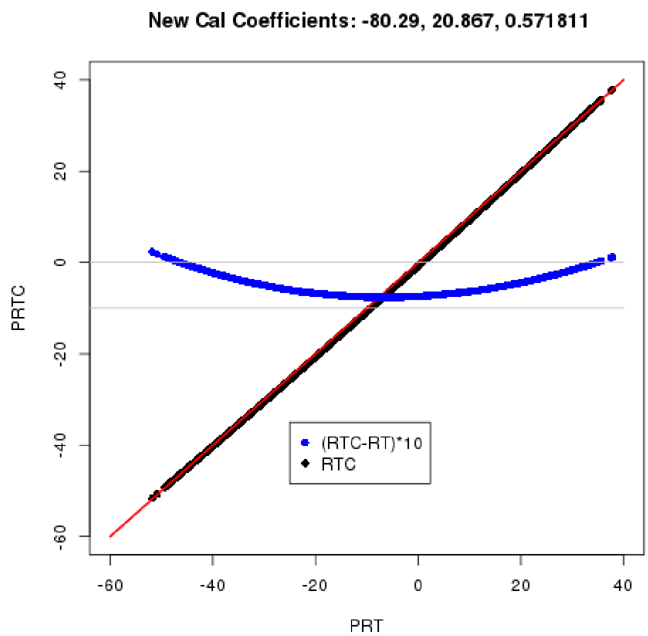

Figure 5: *Comparison of the original (PRT=RTHR1) and new (RTC) measurement of total temperature from the HARCO sensor. Data from all 17 CONTRAST flights are combined in this plot. The gray lines denote the range from 0 to 1°C for the blue symbols, which show the difference introduced in RTHR1 by the new calibration.*

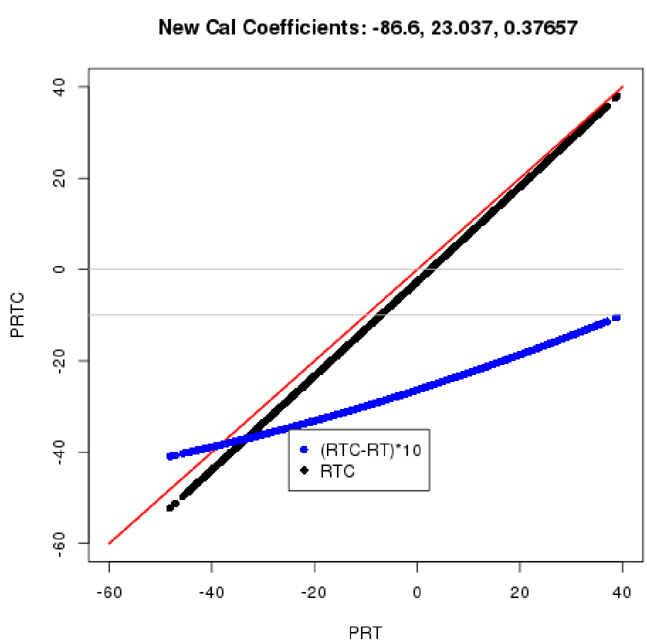

Figure 6: *As in Fig. 5 but for RTFH1.*

Recommendations

1. For CONTRAST, all indications are that the HARCO measurement should be the primary temperature. To compensate for drift in the A/D card, it may be preferable to make the following adjustment to the first cal coefficient:
   (a) For flights before 6: no adjustment
   (b) For flights 6–12: adjust by -0.7°C * (flight number - 5) / 8
   (c) For flights 13 and beyond: adjust by -0.7°C
2. Effort to identify the cause of disagreement among temperatures in CONTRAST should now focus on the Rosemount sensor. For now, treat measurements from that sensor as unreliable.
3. Consider a new temperature variable given by the cal coefficients listed in the "RESULTS" subsection (on page 7). I think it can be argued that this is our best temperature measurement, with uncertainty of about 0.3°C.
4. For future test flights, use checks based on the hydrostatic equation to identify major problems such as the one that occurred in CONTRAST.

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