# Measuring State Parameters of the Atmosphere Some Applications of Atmospheric Thermodynamics

Al Cooper

Earth Observing Laboratory, NCAR

IDEAS-4 Tutorial

#### Introduction

#### Goals of This Presentation

Present two complementary aspects related to atmospheric thermodynamics:

- Discuss some basics regarding how measurements of thermodynamic state variables are measured by a research aircraft
- ② Show some useful applications of atmospheric thermodynamics to how those measurements are made

## STATE VARIABLES

#### What Are State Variables?

- Those variables needed to specify the thermodynamic state of the system, in this case the atmosphere.
- If we consider a moist atmosphere, in general we need three variables to specify the state. They may be taken, for example, to be *temperature*, *pressure*, and *water vapor pressure*.

Other variables can then be determined from these, for example:

- density from the perfect gas law
- relative humidity from knowledge of the equilibrium vapor pressure vs T for water
- dew point also from knowledge of the equilibrium vapor pressure vs T for water



## **TEMPERATURE SENSORS**

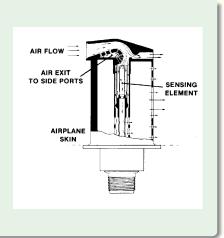
#### Types of Temperature Sensors

- Resistive-element sensors:
  - often, platinum wire
  - resistance varies with temperature
- 2 radiometric:
  - CO<sub>2</sub> absorbs and re-emits radiation in short distances at some specific wavelengths
  - 2 The intensity of such radiation varies with the temperature
- Others sensors are also sometimes used, including thermocouple junctions and thermistors



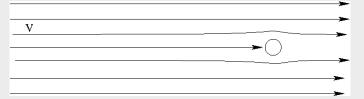
## THE STANDARD SENSOR





## EFFECT OF AIRSPEED ON IN SITU SENSORS

## Airflow Approaching a Stagnation Point



- At boundaries, airspeed tends to zero relative to the boundary.
- The result is compressional adiabatic heating of the air
- The sensing wire therefore is in contact with warmed air, not ambient air

## WHAT IS TTX?

#### Definition

TTX is the measured temperature determined from the resistance of the wire.

- It is named "total" temperature because it is approximately the total temperature of air brought to a stagnation point.
- It is actually closer to the "recovery" temperature, defined below

## HOW IS TTX RELATED TO ATX?

## Conservation of Energy

First Law:  $dU = \delta Q - \delta W$ , and for a perfect gas  $dU = c_V dT$ 

On a streamline starting with varying speed V,

$$\delta Q = 0$$
 and  $-\delta W = p\delta V$ 

To U, must add kinetic energy  $\frac{1}{2}\rho\,V_a^2$  , with air density  $\rho_a$ :

$$\frac{1}{2}V^2 + c_v T + \frac{\rho}{\rho_a} = \text{Constant}$$

Because  $rac{
ho}{
ho_{a}}=R_{d}T$  and  $c_{v}+R_{d}=c_{
ho}$ ,

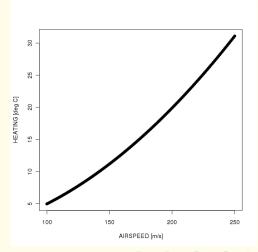
$$\frac{1}{2}V^2 + c_p T = \text{Constant}$$



## MAGNITUDE OF THE HEATING

Heating is about  $5^{\circ}C$  at 100 m/s, increasing to about  $30^{\circ}C$  at airspeeds reached by the GV.

Accurate correction for this airspeed thus is quite important, especially at high airspeed.



## THE RECOVERY FACTOR

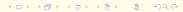
## A Further Correction That Depends On Probe Geometry

- The air does not reach complete stagnation at a distance in thermal contact with the sensing wire. One might expect that the temperature that affects the wire is that present at a distance from the sensor of about a mean free path for air molecules.
- This is usually dealt with via a "recovery factor" that varies with sensor but may be as high as 0.98 (where 1.0 would apply for a stagnation point).
- Often this is determined from flight maneuvers where the aircraft varies airspeed while flying through a region of uniform temperature so the effect of airspeed on the measurement can be detected.

## WHAT HAPPENS IN CLOUD?

#### If the sensor becomes wet, the measurement will be wrong

- Suppose the relative humidity is 100% in the cloud and the sensor becomes covered with water
- As the air is heated on approach to the sensor, the humidity falls below 100% because the cloud droplets cannot evaporate fast enough to raise the humidity
- The water on the sensor will evaporate into the subsaturated air, cooling the sensor
- At the extreme, the sensor will approach the wet-bulb temperature appropriate for the near-stagnation air, potentially introducing errors of a few degrees Celsius



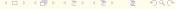
## THE WET-BULB TEMPERATURE

#### Basic Formula

$$rL_V + c_p T = \text{Constant}$$

r is the water-vapor mixing ratio,  $L_{\nu}$  the latent heat of vaporization,  $c_{p}$  the specific heat at constant pressure, and T the temperature.

- As the air evaporates from the sensing wire and cloud drops,
   T decreases and r increases.
- At saturation,  $r = r_s(T_{WB})$ . The wet-bulb temperature is the temperature for which this condition is satisfied.
- Conceptually, one could plot the quantity  $r_s(T')L_v + c_pT'$  vs T' and find the point at which that curve intersects the value specified by the formula above for ambient conditions  $\{r, T\}$ . In practice, the equation is usually solved iteratively.



## **HUMIDITY SENSORS**

#### Basic Sensor Types:

- Dew point hygrometers: Devices that detect the presence of condensate on a chilled mirror
- Light-absorption hygrometers: Devices that measure the absorption of radiation at a wavelength where there is strong water absorption
- Wet-bulb thermometers: Devices that measure the cooling of a wetted sensor
- Capacitance measurements or hygristor (resistance) devices: Common in radiosondes, seldom used in research aircraft

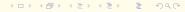


## A CHILLED-MIRROR HYGROMETER

The sensor housing



in stitution - lo



## A CHILLED-MIRROR HYGROMETER

Photograph as mounted on the GV



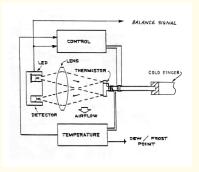




## A CHILLED-MIRROR HYGROMETER

### The control process

- Reflected light from the mirror is measured
- If the reflected light decreases, the mirror is heated, and v.v.
- The control loop is adjusted to keep just threshold condensation on the mirror

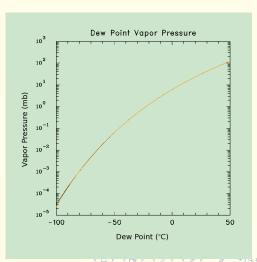


## USING CHILLED-MIRROR MEASUREMENTS

Finding the Water Vapor Pressure

#### Definition

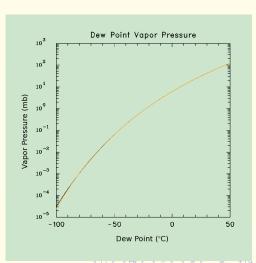
The dew point is the temperature at which the water vapor pressure would be in equilibrium with a plane water surface.



## USING CHILLED-MIRROR MEASUREMENTS

Finding the Water Vapor Pressure

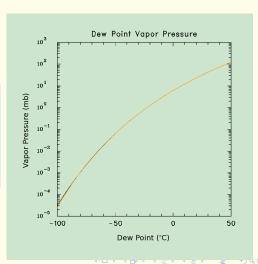
The functional dependence is usually expressed as  $e=e_s(T_{DP})$  where  $e_s$  is the equilibrium vapor pressure function, e is the vapor pressure, and  $T_{DP}$  is the dew point.



## USING CHILLED-MIRROR MEASUREMENTS

Finding the Water Vapor Pressure

Formulas exist to express the function  $e_s$ , including the Clausius-Clapeyron equation, the Goff-Gratch formula, or the Murphy-Koop formula. We now use the latter.

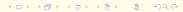


## FURTHER CONSIDERATIONS

#### Three additional considerations when using these formulas:

- 1 The condensate on the mirror may be frost, not dew.
- The pressure in the sensing chamber may differ from the ambient.
- In the presence of dry air, the equilibrium vapor pressure over a plane surface is slightly higher than the equilibrium value in the absence of air.

All these require corrections. The first two are often substantial.



## **HUMIDITY SENSORS THAT MEASURE ABSORPTION**

#### Beer's Law

$$\frac{dI}{I} = -\sigma n d\ell$$

[I is the intensity of radiation,  $\ell$  distance,  $\sigma$  the molecular cross-section for absorption, and n the number density of molecules.]

$$I = I_0 e^{-\sigma \ell n}$$

- The quantity measured by instruments using absorption is then n or  $\rho_w = m_w n$ , the mass density of water vapor.
- Two types of radiometric hygrometer are in common use:
  - Uyman-alpha hygrometers based on absorption of the Lyman-alpha line of hydrogen, which lies in the UV
  - 2 Tunable diode laser (TDL) hygrometers that work in the near IR

## THE HUMIDITY VARIABLES

- Original Measurements: DPB, DPT (mirror temperatures)
- Corrected for frost-dew difference, etc: DPBC, DPTC, DPXC
- Derived:

EDPC water vapor pressure

MR water vapor mixing ratio

RHUM relative humidity

RHODT water vapor density

 Experimental: MIRRORT\_CR2 (cryogenic hygrometer) and some measurements from a TDL hygrometer. These are not yet processed to final engineering-unit form.

## CALCULATING THE DERIVED VARIABLES

#### Preferred dew-point sensor

One of the dew point measurements is designated as the preferred measurement; e.g., DPXC=DPTC. Derived measurements are determined from this basic measurement.

- EDPC: determined from  $e=e_s(T_{DP})$  after corrections as discussed earlier
- RHUM:  $e/e_s(T)$  where T is the ambient temperature
- MR:  $r=\frac{\varepsilon e}{p-e}$  where  $\varepsilon=M_W/M_a$  is the ratio of the molecular weight of water to that of air and p is the total pressure
- RHODT:  $\rho_w = \frac{e}{R_w T}$  where  $R_w$  is the gas constant for water vapor. This equation is also used to obtain e from measurements of  $\rho_w$  or n, such as provided by the radiometric hygrometers.

## PRESSURE MEASUREMENT

#### The Sensors

- Many different transducers are available to measure absolute or differential pressure.
- Among the most accurate are digital quartz crystal sensors that change oscillation frequency with pressure.
- Others are capacitative, piezoelectric, piezoresistive, ...
- On aircraft, these attach to static ports designed to provide pressure close to the flight-level pressure

## HOW DO STATIC PORTS WORK?

## The Key Problem

Airflow around the surfaces of the aircraft creates a varying pressure field that makes accurate measurement difficult.

 An example was encountered earlier in connection with temperature measurement:

$$\Delta p = \rho_a \frac{V^2}{2}$$

which can produce an error of about 70 hPa under the following conditions: p=200 hPa,  $T=-40^{\circ}$ C, V=220 m/s.

- Pressure ports are located at special locations where this effect is minimized. Corrections are still necessary.
- Calibration: "trailing cone" and flight maneuvers to test the effects of angle of attack and sideslip

## MAPPING PRESSURE FIELDS

#### Heights on a constant-pressure surface show pressure gradients

- GPS measurements give the height of the aircraft to few-cm accuracy. (Synoptic maps of pressure fields often use contour increments of 50 m or more.)
- It is possible to map mesoscale pressure fields by flying on constant-pressure surfaces.
- Accuracy considerations: If uncertainty in p is 0.5 mb, then the corresponding uncertainty in height can be estimated from the hydrostatic equation:

$$\frac{dp}{p} = -\frac{g}{RT}dZ$$

For  $\delta p = 0.5$  mb, p = 500 mb, T = 263 K, gives  $\delta Z = 7.5$  m.



# Pseudo-adiabatic EQUIVALENT POTENTIAL TEMPERATURE

(advanced topic)

Call attention to the new equation of Davies-Jones (2009)

#### Rossby Form

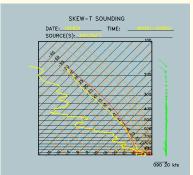
•  $L_v$  and  $c_{pd}$  are kept constant.

$$\Theta_p^{[Rossby]} = \Theta_d \exp\left\{\frac{L_V r}{c_{pd} T}\right\}$$

#### Rossby Form

- $L_v$  and  $c_{pd}$  are kept constant.
- Basis for thermodynamic diagrams

$$\Theta_p^{[Rossby]} = \Theta_d \exp\left\{\frac{L_V r}{c_{pd} T}\right\}$$



## Rossby Form

- $L_v$  and  $c_{pd}$  are kept constant.
- Basis for thermodynamic diagrams

#### Revised Forms Like Bolton

 Take into account the temperature dependence of L<sub>V</sub> and c<sub>pd</sub>

$$\Theta_p^{[Rossby]} = \Theta_d \exp\left\{\frac{L_V r}{c_{pd} T}\right\}$$

#### Rossby Form

- $L_v$  and  $c_{pd}$  are kept constant.
- Basis for thermodynamic diagrams

#### Revised Forms Like Bolton

- Take into account the temperature dependence of L<sub>v</sub> and c<sub>pd</sub>
- Often adjust L<sub>v</sub> to minimize errors

$$\Theta_p^{[Rossby]} = \Theta_d \exp\left\{\frac{L_V r}{c_{pd} T}\right\}$$

#### Rossby Form

- $L_v$  and  $c_{pd}$  are kept constant.
- Basis for thermodynamic diagrams

#### Revised Forms Like Bolton

- Take into account the temperature dependence of L<sub>V</sub> and c<sub>pd</sub>
- Often adjust L<sub>v</sub> to minimize errors
- New: Davies-Jones, 2009

$$\Theta_p^{[Rossby]} = \Theta_d \exp\left\{\frac{L_v r}{c_{pd} T}\right\}$$

Davies-Jones (2009):

$$\Theta_E^* = \Theta e^{\left\{ \frac{[L_0^* - L_1^*(T_L - T_0)]r + \kappa_2 r^2}{c_{pd} T_L} \right\}}$$

where  $L_0^*$ ,  $L_1^*$ , and  $K_2$  are coefficients that are adjusted to minimize errors.

```
Wet vs Dry
```



#### Wet vs Dry

• wet-equivalent: carry all water with parcel  $(\Theta_q)$ 



#### Wet vs Dry

• wet-equivalent: carry all water with parcel  $(\Theta_q)$ 

# AMS Glossary: Equivalent Potential Temperature

"A thermodynamic quantity, with its natural logarithm proportional to the entropy of moist air, that is conserved in a reversible moist adiabatic process. "

#### Wet vs Dry

- wet-equivalent: carry all water with parcel  $(\Theta_q)$
- pseudo-adiabatic equivalent: all water is removed as it condenses

# AMS Glossary: Equivalent Potential Temperature

"A thermodynamic quantity, with its natural logarithm proportional to the entropy of moist air, that is conserved in a reversible moist adiabatic process."

## Wet vs Dry

- wet-equivalent: carry all water with parcel  $(\Theta_q)$
- pseudo-adiabatic equivalent: all water is removed as it condenses

#### "Pseudo-adiabatic"

- Preferred terminology for all water removed when condensed
- Equivalent to neglecting the specific heat of liquid

# AMS Glossary: Equivalent Potential Temperature

"A thermodynamic quantity, with its natural logarithm proportional to the entropy of moist air, that is conserved in a reversible moist adiabatic process. "

### Wet vs Dry

- wet-equivalent: carry all water with parcel  $(\Theta_q)$
- pseudo-adiabatic equivalent: all water is removed as it condenses

#### "Pseudo-adiabatic"

- Preferred terminology for all water removed when condensed
- Equivalent to neglecting the specific heat of liquid water

# AMS Glossary: Equivalent Potential Temperature

"A thermodynamic quantity, with its natural logarithm proportional to the entropy of moist air, that is conserved in a reversible moist adiabatic process. "

## Wet vs Dry

- wet-equivalent: carry all water with parcel  $(\Theta_q)$
- pseudo-adiabatic equivalent: all water is removed as it condenses

### "Pseud<u>o-adiabatic"</u>

- Preferred terminology for all water removed when condensed
- Equivalent to neglecting the specific heat of liquid

# AMS Glossary: Equivalent Potential Temperature

"A thermodynamic quantity, with its natural logarithm proportional to the entropy of moist air, that is conserved in a reversible moist adiabatic process."

- pseudo-adiabatic equivalent
- $\Theta_p$  instead of  $\Theta_e$
- $\bullet$   $\Theta_q$  for wet-equivalent



## Wet vs Dry

- wet-equivalent: carry all water with parcel  $(\Theta_q)$
- pseudo-adiabatic equivalent: all water is removed as it condenses

### "Pseud<u>o-adiabatic"</u>

- Preferred terminology for all water removed when condensed
- Equivalent to neglecting the specific heat of liquid

# AMS Glossary: Equivalent Potential Temperature

"A thermodynamic quantity, with its natural logarithm proportional to the entropy of moist air, that is conserved in a reversible moist adiabatic process."

- pseudo-adiabatic equivalent
- $\Theta_p$  instead of  $\Theta_e$
- $\bullet$   $\Theta_q$  for wet-equivalent



## Wet vs Dry

- wet-equivalent: carry all water with parcel  $(\Theta_q)$
- pseudo-adiabatic equivalent: all water is removed as it condenses

### "Pseud<u>o-adiabatic"</u>

- Preferred terminology for all water removed when condensed
- Equivalent to neglecting the specific heat of liquic water

# AMS Glossary: Equivalent Potential Temperature

"A thermodynamic quantity, with its natural logarithm proportional to the entropy of moist air, that is conserved in a reversible moist adiabatic process."

- pseudo-adiabatic equivalent
- $\Theta_p$  instead of  $\Theta_e$
- $\bullet$   $\Theta_q$  for wet-equivalent



## Wet vs Dry

- wet-equivalent: carry all water with parcel  $(\Theta_q)$
- pseudo-adiabatic equivalent: all water is removed as it condenses

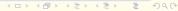
### "Pseud<u>o-adiabatic"</u>

- Preferred terminology for all water removed when condensed
- Equivalent to neglecting the specific heat of liquid

# AMS Glossary: Equivalent Potential Temperature

"A thermodynamic quantity, with its natural logarithm proportional to the entropy of moist air, that is conserved in a reversible moist adiabatic process."

- pseudo-adiabatic equivalent
- $\Theta_p$  instead of  $\Theta_e$
- ullet  $\Theta_q$  for wet-equivalent



$$\Theta_{q} = T \left(\frac{p_{0}}{p_{d}}\right)^{R_{d}/c_{pt}} \exp\left(\frac{L_{v}r}{c_{pt}T}\right)$$
 (1)

- Quantities in red vary with temperature.
- Equation (1) is a straightforward definition if  $L_v$  and  $c_{pd}$  (entering  $c_{pt} = c_{pd} + r_t c_w$ ) are taken at the level of the LCL

$$\Theta_{q} = T \left(\frac{p_{0}}{p_{d}}\right)^{R_{d}/c_{pt}} \exp\left(\frac{L_{v}r}{c_{pt}T}\right)$$
 (1)

- Quantities in red vary with temperature.
- Equation (1) is a straightforward definition if  $L_v$  and  $c_{pd}$  (entering  $c_{pt} = c_{pd} + r_t c_w$ ) are taken at the level of the LCL

$$\Theta_{q} = T \left(\frac{p_{0}}{p_{d}}\right)^{R_{d}/c_{pt}} \exp\left(\frac{L_{v}r}{c_{pt}T}\right)$$
 (1)

• Bolton: If  $\Theta_D$  is the dry-air potential temperature at the LCL, e the vapor pressure in mb,  $T_K$  the air temperature in kelvin,  $T_L$  the temperature at the LCL in kelvin and r the mixing ratio

$$T_L = \frac{2840}{3.5 \ln T_K - \ln e - 4.805} + 55$$

$$\Theta_p^{Bolton} = \Theta_D \exp\left\{ \left( \frac{3.376}{T_L} - 0.00254 \right) r (1 + 0.81 \times 10^{-3} r) \right\}$$

$$\Theta_{q} = T \left(\frac{p_{0}}{p_{d}}\right)^{R_{d}/c_{pt}} \exp\left(\frac{L_{v}r}{c_{pt}T}\right)$$
 (1)

$$\frac{dT}{dp_d} = \frac{TR_d + \frac{L_v r}{p_d}}{p_d} \left[ \left( \frac{c_{pd} + r_t c_w}{p_d} \right) + \frac{T\varepsilon}{p_d} \left( \frac{\partial \left( \frac{L_v e_s(T)}{T} \right)}{\partial T} \right)_{p_d} \right]^{-1}$$
(2)

Do revised vapor-pressure equations matter?

- Oo revised vapor-pressure equations matter?
  - Davies-Jones (2009) still uses an approximate formula instead of a more accurate representation of  $e_s(T)$ .

- On revised vapor-pressure equations matter?
  - Davies-Jones (2009) still uses an approximate formula instead of a more accurate representation of  $e_s(T)$ .
  - Expect small effect because differences are at low temperature where vapor pressure is low.

- Oo revised vapor-pressure equations matter?
  - Davies-Jones (2009) still uses an approximate formula instead of a more accurate representation of  $e_s(T)$ .
  - Expect small effect because differences are at low temperature where vapor pressure is low.
- What is the effect of including the temperature dependence of the specific heats, especially for supercooled water?

- On revised vapor-pressure equations matter?
  - Davies-Jones (2009) still uses an approximate formula instead of a more accurate representation of  $e_s(T)$ .
  - Expect small effect because differences are at low temperature where vapor pressure is low.
- What is the effect of including the temperature dependence of the specific heats, especially for supercooled water?
  - Neglected in previous studies, although Murphy and Koop show that variation in  $c_w$  is particularly significant.

- On revised vapor-pressure equations matter?
  - Davies-Jones (2009) still uses an approximate formula instead of a more accurate representation of  $e_s(T)$ .
  - Expect small effect because differences are at low temperature where vapor pressure is low.
- What is the effect of including the temperature dependence of the specific heats, especially for supercooled water?
  - Neglected in previous studies, although Murphy and Koop show that variation in  $c_w$  is particularly significant.
  - Applies to both  $\Theta_p$  and  $\Theta_q$ .

- O Do revised vapor-pressure equations matter?
  - Davies-Jones (2009) still uses an approximate formula instead of a more accurate representation of  $e_s(T)$ .
  - Expect small effect because differences are at low temperature where vapor pressure is low.
- What is the effect of including the temperature dependence of the specific heats, especially for supercooled water?
  - Neglected in previous studies, although Murphy and Koop show that variation in  $c_w$  is particularly significant.
  - Applies to both  $\Theta_p$  and  $\Theta_q$ .
- How accurate is the Bolton formula for the temperature at the LCL?

- Do revised vapor-pressure equations matter?
  - Davies-Jones (2009) still uses an approximate formula instead of a more accurate representation of  $e_s(T)$ .
  - Expect small effect because differences are at low temperature where vapor pressure is low.
- What is the effect of including the temperature dependence of the specific heats, especially for supercooled water?
  - Neglected in previous studies, although Murphy and Koop show that variation in  $c_w$  is particularly significant.
  - Applies to both  $\Theta_p$  and  $\Theta_q$ .
- How accurate is the Bolton formula for the temperature at the LCL?
  - How much error is introduced?



- On revised vapor-pressure equations matter?
  - Davies-Jones (2009) still uses an approximate formula instead of a more accurate representation of  $e_s(T)$ .
  - Expect small effect because differences are at low temperature where vapor pressure is low.
- What is the effect of including the temperature dependence of the specific heats, especially for supercooled water?
  - Neglected in previous studies, although Murphy and Koop show that variation in  $c_w$  is particularly significant.
  - Applies to both  $\Theta_p$  and  $\Theta_q$ .
- How accurate is the Bolton formula for the temperature at the LCL?
  - How much error is introduced?
  - Do we need to use a numerical solution to obtain better accuracy?

 For all three questions, the approach was to compare solutions from equations to numerical solutions





- For all three questions, the approach was to compare solutions from equations to numerical solutions
  - Include new vapor pressure formulas

- For all three questions, the approach was to compare solutions from equations to numerical solutions
  - Include new vapor pressure formulas
  - Allow specific heats and the latent heat of vaporization to vary with temperature

- For all three questions, the approach was to compare solutions from equations to numerical solutions
  - Include new vapor pressure formulas
  - Allow specific heats and the latent heat of vaporization to vary with temperature
- Example: Adiabatic motion from in initial point with  $(T_1, p_1)$  to a new point with  $(T_2, p_2)$ : Given  $\{T_1, p_1, p_2\}$ , find  $T_2$  two ways:

- For all three questions, the approach was to compare solutions from equations to numerical solutions
  - Include new vapor pressure formulas
  - Allow specific heats and the latent heat of vaporization to vary with temperature
- Example: Adiabatic motion from in initial point with  $(T_1, p_1)$  to a new point with  $(T_2, p_2)$ : Given  $\{T_1, p_1, p_2\}$ , find  $T_2$  two ways:
  - Integrate the exact equation for  $dT/dp_d$  from point 1 to point 2 to find  $T_2$ .

- For all three questions, the approach was to compare solutions from equations to numerical solutions
  - Include new vapor pressure formulas
  - Allow specific heats and the latent heat of vaporization to vary with temperature
- Example: Adiabatic motion from in initial point with  $(T_1, p_1)$  to a new point with  $(T_2, p_2)$ : Given  $\{T_1, p_1, p_2\}$ , find  $T_2$  two ways:
  - ① Integrate the exact equation for  $dT/dp_d$  from point 1 to point 2 to find  $T_2$ .
  - 2 Evaluate the equation for potential temperature at point 1, then invert it at point 2 to find  $T_2$ .

## **NEW VARIABLES**

1. Change to (6.5) of Davies-Jones (2009), and change the variable name to "pseudo-adiabatic equivalent potential temperature". Continue to use (21) of Bolton (1980) to determine the saturation temperature  $T_L$ .

$$\Theta_p^{[DJ]} = \Theta_{DL} \exp\left\{ \frac{\left(L_0^* - L_1^* (T_L - T_0) + K_2 r\right) r}{c_{pd} T_L} \right\}$$

$$\Theta_{DL} = T_k \left(\frac{1000}{p_d}\right)^{0.2854} \left(\frac{T_K}{T_L}\right)^{0.28 \times 10^{-3} r}$$

$$T_L = \frac{2840}{3.5 \ln T_K - \ln e - 4.805} + 55$$

## **NEW VARIABLES**

2. Add a new variable "wet-equivalent potential temperature" and use the standard equation for its evaluation. .

$$\Theta_{q} = T \left( \frac{p_{0}}{p_{d}} \right)^{R_{d}/c_{pt}} \exp \left( \frac{L_{v}r}{c_{pt}T} \right)$$

where  $c_{pt} = c_{pd} + r_t c_w$  and  $r_{tot}$  is the total water mixing ratio,  $r_{tot} = r + r_w$  where  $r_w = \chi/\rho_d$  with  $\chi$  the liquid water content and  $\rho_d$  the density of dry air:  $\rho_d = (p-e)/(R_d T)$ .

# MIXING DIAGRAMS

(Advanced Topic)

[to be added later]





## More Information:

#### Contact Information:

email: cooperw@ucar.edu

phone: 303 497 1600