Calibrating {p, q, T} Using LAMS



RAF Algorithm Review

December 2 2011

Al Cooper LAMS Applications

・ロト ・部 ・ ・ ヨ ・ ・ ヨ ・

Э

Finding "PCORRS" Flight Maneuvers LAMS-Based Temperature Measurement Conclusions

ROADMAP

Steps:

• $p_t = p + q$ is accurate

p_t

- *p_t*=total pressure, *p*=static,
 q=dynamic
- pitot tube insensitive to flow angle (<0.1%)
- orientation removes normal offset
- redundant measurements ⇒consistency
- still, AN ASSUMPTION

< D > < P > < P > < P >

Finding "PCORRS" Flight Maneuvers LAMS-Based Temperature Measurement Conclusions

ROADMAP

Steps:

- $p_t = p + q$ is accurate
- Errors in p and q arise from error at static sources

Consequences

- *p* and *q* are the measured quantities
- plumbing: q is difference, $q = p_t - p$
- therefore $\Delta p = -\Delta q$
- can predict q from LAMS, hence Δq
- can use Δp to correct p

(日) (同) (三) (

Finding "PCORRS" Flight Maneuvers LAMS-Based Temperature Measurement Conclusions

ROADMAP

Steps:

- $p_t = p + q$ is accurate
- Errors in p and q arise from error at static sources
- Find Δq required to match LAMS; hence Δp

What *q* is required to match LAMS?

- TAS: depends on q, p, T
- relatively insensitive to T (iterate...)
- find Δq s.th. $q_m + \Delta q$, $p_m - \Delta q$, T gives v_{LAMS}

Finding "PCORRS" Flight Maneuvers LAMS-Based Temperature Measurement Conclusions

ROADMAP

Steps:

- $p_t = p + q$ is accurate
- Errors in p and q arise from error at static sources
- Find Δq required to match LAMS; hence Δp
- Refinements for accuracy

Adjustments

- correct because p enters the prediction of Δp
- calculate humidity influences on R_a , C_p , C_v , $\gamma = c_p/c_v$
- correct for offset angle of LAMS

< D > < P > < P > < P >

 recalculate T using corrected pressures

Finding "PCORRS" Flight Maneuvers LAMS-Based Temperature Measurement Conclusions

ROADMAP

Steps:

- $p_t = p + q$ is accurate
- Errors in p and q arise from error at static sources
- Find Δq required to match LAMS; hence Δp
- Refinements for accuracy
- Δp is a function of measured quantities like p_m , q_m , α_m

Fit Correction to p and q given by LAMS:

- use second-by-second prediction of Δq from LAMS
- try fits like $(\Delta p/p) = a_0 + a_1q + a_2(ADIFR/QCR)$
- find $\Delta p(measurements)$

Finding "PCORRS" Flight Maneuvers LAMS-Based Temperature Measurement Conclusions

ROADMAP

Steps:

- $p_t = p + q$ is accurate
- Errors in p and q arise from error at static sources
- Find Δq required to match LAMS; hence Δp
- Refinements for accuracy
- Δp is a function of measured quantities like p_m , q_m , α_m
- Test using flight maneuvers, also calibrating T

Maneuvers for testing results

- reverse-heading maneuvers
- climbs and descents to calibrate temperature via integration of the hydrostatic equation

< D > < P > < P > < P >



Al Cooper

Finding "PCORRS" Flight Maneuvers LAMS-Based Temperature Measurement Conclusions

ROADMAP

Steps:

- $p_t = p + q$ is accurate
- Errors in p and q arise from error at static sources
- Find Δq required to match LAMS; hence Δp
- Refinements for accuracy
- Δp is a function of measured quantities like p_m , q_m , α_m
- Test using flight maneuvers, also calibrating T
- Use LAMS with the above results to measure T directly.

LAMS-based measurement of temperature

- LAMS provides TAS = v
- *p* and *q* determine Mach number *M* = *v*/*v_s*
- $v_s = \sqrt{\gamma R_a T}$ so measured Mand $v \Rightarrow T$ without reference to a temperature sensor

ヘロト ヘヨト ヘヨト ヘ

ROADMAP

Steps:

- $p_t = p + q$ is accurate
- Errors in p and q arise from error at static sources
- Find Δq required to match LAMS; hence Δp
- Refinements for accuracy
- Δp is a function of measured quantities like p_m , q_m , α_m
- Test using flight maneuvers, also calibrating T
- Use LAMS with the above results to measure T directly.

Results:

 Calibration of dynamic pressure, hence true airspeed, hence longitudinal component of wind

ROADMAP

Steps:

- $p_t = p + q$ is accurate
- Errors in p and q arise from error at static sources
- Find Δq required to match LAMS; hence Δp
- Refinements for accuracy
- Δp is a function of measured quantities like p_m , q_m , α_m
- Test using flight maneuvers, also calibrating T
- Use LAMS with the above results to measure T directly.

Results:

- Calibration of dynamic pressure, hence true airspeed, hence longitudinal component of wind
- ② Calibration of pressure

Al Cooper

LAMS Applications

ROADMAP

Steps:

- $p_t = p + q$ is accurate
- Errors in p and q arise from error at static sources
- Find Δq required to match LAMS; hence Δp
- Refinements for accuracy
- Δp is a function of measured quantities like p_m , q_m , α_m
- Test using flight maneuvers, also calibrating T
- Use LAMS with the above results to measure T directly.

Results:

- Calibration of dynamic pressure, hence true airspeed, hence longitudinal component of wind
- 2 Calibration of pressure
- Calibration of temperature via accurate measurements of pressure + GPS

Roadmap Finding "PCORRS" Flight Maneuvers

LAMS-Based Temperature Measurement Conclusions

ROADMAP

Steps:

- $p_t = p + q$ is accurate
- Errors in p and q arise from error at static sources
- Find Δq required to match LAMS; hence Δp
- Refinements for accuracy
- Δp is a function of measured quantities like p_m , q_m , α_m
- Test using flight maneuvers, also calibrating T
- Use LAMS with the above results to measure T directly.

Results:

- Calibration of dynamic pressure, hence true airspeed, hence longitudinal component of wind
- 2 Calibration of pressure
- Calibration of temperature via accurate measurements of pressure + GPS
- Provide new independent temperature measurement that *should* work in cloud

Al Cooper

The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

THE MEASUREMENT OF TOTAL PRESSURE

Basic Measurements

static pressure PSFD, PSFRD
absolute sensors
±0.1 mb (Parascientific Model 1000)

dynamic pressure QCF, QCRF

- differential sensors, total vs static
- ±0.05 mb

hypothesis:

 $p_t = p + q$ is accurate

イロト イポト イヨト イヨト

The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

THE MEASUREMENT OF TOTAL PRESSURE

Basic Measurements

static pressure PSFD, PSFRD

- absolute sensors
- ±0.1 mb (Parascientific Model 1000)

dynamic pressure QCF, QCRF

 differential sensors, total vs static

• ±0.05 mb

hypothesis:

 $p_t = p + q$ is accurate

pitot tube performance

• typical sensitivity <1% at angles up to 10°, 0.2% up to 5°.

< ロ > < 同 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >

 mean orientation chosen along expected flow direction, not centerline



The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

THE MEASUREMENT OF TOTAL PRESSURE

Basic Measurements

static pressure PSFD, PSFRD

- absolute sensors
- ±0.1 mb (Parascientific Model 1000)

dynamic pressure QCF, QCRF

 differential sensors, total vs static

• ±0.05 mb

hypothesis:

 $p_t = p + q$ is accurate

 $PSFRD+QCFR=a_1(PSFD+QCF)+a_2$ $a_1=1.0000, a_2=0.10 mb$





The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

THE EQUATIONS

Energy conservation in compressible flow: $\frac{v^2}{2} + c_p T = \text{constant}$ For adiabatic compression to stagnation (v = 0),

$$M^{2} = \frac{v^{2}}{\gamma R_{a} T} = \left\{ \left(\frac{2c_{v}}{R_{d}} \right) \left[\left(\frac{p+q}{p} \right)^{R_{a}/c_{p}} - 1 \right] \right\}$$
$$q = p \left\{ \left(\frac{v^{2}}{2c_{p} T} + 1 \right)^{c_{p}/R_{a}} - 1 \right\} = p\chi$$

where the last equality defines $\chi(v, T)$. Write in terms of measured quantities $p_m = p - \Delta p$ and $q_m = q - \Delta q$ and unknown $\Delta p = -\Delta q$:

$$\Delta p = \frac{q_m - p_m \chi}{1 + \chi}$$

・ 同 ト ・ ヨ ト ・ ヨ ト

The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

HOW TO ADDRESS NEED FOR TEMPERATURE

Need Temperature:

Have v from LAMS

$$\chi(\mathbf{v}, T) = \left\{ \left(\frac{\mathbf{v}^2}{2c_p T} + 1 \right)^{c_p/R_a} - 1 \right\}$$

- Not very sensitive: Fractional error in *T* is small
- Use available-processed T as the first approximation
- Then, iterate both in calculation and in calibration

Determining "PCORR" Function

$$\Delta p = \frac{q_m - p_m \chi}{1 + \chi}$$

- Airspeed from LAMS gives second-by-second estimates of Δp
- Can fit those values to get Δp as function of other measurements

э

The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

REFINEMENTS FOR ACCURACY

Moist Air Corrections

- humidity matters at this level of precision
- Use R_a , c_p , γ adjusted for humidity

Pointing Angle Corrections

- $v_l = v \cos(\theta)$ where v_l is the speed measured by LAMS
- $\cos \theta \simeq \cos(\theta_1 + \alpha) \cos(\theta_2 \beta)$
 - $heta_1$ is the pointing angle above the longitudinal axis=0.1°
 - θ_2 is the pointing angle to starboard of the longitudinal axis=-0.2°
 - if $lpha=-4^\circ,\cos \theta\simeq 0.9976$ and at 130 m/s $\delta
 u=0.3$ m/s.
- Therefore, use $v=v_{l}/\cos(heta)$ in the preceding equation

The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

FITS TO Δp

Candidate Fit Parameters

- p: Effects on Δp probably scale with pressure, so fitting functions of the form $\frac{\Delta p}{p} = f(...)$ seems appropriate and provided improved fits
- q or q/p
- $\odot \alpha$ or ADIFR/QCR
- () β or BDIFR/QCR
- M (but M^2 represents much of the same dependence as q/p)

< ロ > (同 > (回 > (回 >))

The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

PREFERRED FIT

Best Option?

$$\frac{\Delta p}{\text{PSFD}} = a_0 + a_1 \frac{\text{ADIFR}}{\text{QCR}} + a_2 \frac{\text{QCF}}{\text{PSFD}}$$

Fit procedure:

- Determine a_1 by fit to pitch maneuvers
- Keep a_1 constant and fit for a_0 and a_2 using data from all times when LAMS provides a measurement of v.
- Obtain coefficients by separate fits for {PSFD, QCF} and {PSFRD, QCFR}

RMS error vs LAMS measurements: 0.3 mb, 1-Hz measurements.

- (Different sample volumes)
- Mean correction: uncertainty < 0.01 mb (>10,000 measurements)



The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

FIT THAT IS BEST FOR MANEUVERS

Another Good Option

$$\frac{\Delta \rho}{\text{PSFD}} = b_0 + b_1 \frac{\text{ADIFR}}{\text{QCR}} + b_2 \text{XMACH2} + b_3 \frac{\text{BDIFR}}{\text{QCR}}$$

Procedure:

- Determine b_2 from sideslip maneubers
- **2** Determine b_1 from pitch maneuvers
- With b_1 and b_2 fixed, find b_0 and b_2 using data from all times when LAMS provides a measurement of v.

Result:

- Better performance in maneuvers
- Slightly higher overall RMS

ヘロト ヘポト ヘヨト ヘヨト

The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

FIT (cyan) VS. LAMS PREDICTION (yellow)



The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

CHANGE FROM STANDARD?

PSFW: new calibration

- mean pressure is about 0.5 mb lower
- low scatter (<0.2 mb) ⇒ functional dependence is similar



The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

COMPARISON OF CORRECTED PSFD and PSFRD





Al Cooper

LAMS Applications

The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

SOURCE OF NOISE?



Al Cooper



The Total Pressure (p+q)Finding Δp using LAMS Fitting LAMS Predictions

VARIANCE SPECTRA



Al Cooper L/

LAMS Applications

Checking PCORs Checking Layer Temperatures

< = >

REVERSE-HEADING FLIGHT SEGMENTS

Reverse-heading maneuvers flown on flight 5 of IDEAS-4:

Time Interval	UXW	GGTRK	THDG	WDC	WSC	UX
220000-230530	$1.56{\pm}0.91$	241	242	65	3.31	2.89
230700-231200	-1.08 ± 0.55	59	59	224	1.43	-1.47
225100-225300	$1.975 {\pm} 0.40$	148	149	312	2.58	2.40
225500-225700	$0.99{\pm}0.52$	328	329	122	1.09	1.21

Test of Accuracy: Longitudinal Wind Should Reverse

- Set #1: Δ UXW \simeq 0.5 m/s (vs. 1.4 m/s, std)
- Set #2: $\Delta UXW \simeq 3.0 \text{ m/s}$ (vs 3.6 m/s, std)

Checking PCORs Checking Layer Temperatures

CALIBRATING TEMPERATURE USING LAMS

Integrating the Hydrostatic Equation

- Assumption: absolute pressure is accurate as calibrated, LAMS
- GPS provides acurate measurement of geometric altitude
- Integrate the hydrostatic equation between two pressure levels:
 - Predicted altitude change depends on the "mean" temperature.
 - Comparison to a similar "mean" of the measurements checks the accuracy of the temperature measurement.

ヘロト ヘヨト ヘヨト ヘ

Finding "PCORRS" Finding "PCORRS" Flight Maneuvers LAMS-Based Temperature Measurement Conclusions

Checking PCORs Checking Layer Temperatures

EQUATIONS USED

Three Sums Are Needed: S_1 , S_2 , S_3 :

$$\delta p_{i} = -\frac{g p_{i}}{R_{a} T_{a,i}} \delta z_{i} \quad (\text{too noisy, second} - \text{by} - \text{second})$$
$$S_{1} = \sum_{i} \frac{R_{a,i}}{g_{i}} \ln \left(\frac{p_{i}}{p_{i-1}}\right)$$

$$S_2 = \sum_i (z_i - z_{i-1})$$

$$S_3 = \sum_i \frac{z_i - z_{i-1}}{T_{m,i}}$$

Then compare prediction (T_p) to observed (T_m)

$$T_p = -S_1/S_2$$
 and $\overline{T}_m = S_2/S_3$, weighted appropriately
Al Cooper LAMS Applications



Checking PCORs Checking Layer Temperatures

RESULTS FOR MEAN TEMPERATURES BETWEEN LAYERS

Flight Segment	T _p	\overline{T}_m	ΔT
RF05, 205800-211100	-10.98	-10.37	-0.5
RF07, 212510-213300	-6.36	-5.89	-0.47
RF07, 212510-212900	2.27	2.42	-0.15
RF07, 212900-213300	-12.85	-12.15	-0.70
RF08, 214500-215300	-0.9	-0.5	-0.4
RF08, 233700–234130	-6.5	-6.3	-0.4
RF08, 234500–235000	-9.4	-8.8	-0.6
RF08, 235600-240100	-9.5	-8.4	-1.1
mean offset ^a , $T_p - \overline{T}_m$			-0.55

^aexcluding the first listed value for RF07 because the next two break this climb segment into two segments



< ロ > (同 > (回 > (回 >))

DETERMINING THE MACH NUMBER

$$M^{2} = \left\{ \left(\frac{2c_{v}}{R_{a}}\right) \left[\left(\frac{p+q}{p}\right)^{R_{a}/c_{p}} - 1 \right] \right\}$$

- LAMS provides a correction Δp to be added to p and subtracted from q (affecting only the denominator).
- Measured temperatue is not needed (except indirectly as it enters fitting to find Δp).
- Once calibrated, the above equation for M² can be used to find the temperature, independent of a temperature probe, using only pressure measurements and v determined by LAMS::

$$T_{LAMS} = v^2 / (\gamma R_a M^2)$$

AN EXAMPLE OF T FROM LAMS



Al Cooper LAMS Applications

AN EXAMPLE OF T FROM LAMS



Al Cooper LAMS Applications

Key Results:

- LAMS provides a direct calibration of airspeed.
 - TAS_LAMS is slightly higher than TASX
- 2 This calibrates dynamic pressure:
 - q should be increased by about 0.5 mb on average.
- If total pressure is accurately measured, this calibrates pressure:
 - p should be decreased by about 0.5 mb on average.
- Accurate pressure supports integration of the hydrostatic equation to calibrate temperature:
 - Results indicate that temperature should be decreased by about 0.5°C.
- It is then possible to derive a new temperature measurement solely from p, q, and v_{LAMS} .

