Research Aviation Facility

Algorithm Memorandum:

Subject: Corrections to Pressure for Airflow Effects
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Background

Ambient pressure is measured by accurate transducers connected to pressure ports called "static buttons" that are designed and located on the fuselage to provide pressure sources that are close to the ambient. In the case of the GV, these pressure sources are connected to a Parascientific Model 1000 quartz transducer that has nominal accuracy of 0.1 mb. Dynamic pressure is then measured by a differential sensor connected to measure the difference between the static ports and the total pressure at the tip of a a pitot tube. For the GV, this measurement is made by a Honeywell Model PPT0001 differential pressure transducer having nominal accuracy of 0.05 mb.

To account for possible errors in these measurements, "Pcorr" values $\{\Delta p, \Delta q\}$ are added to the measurements $\{p_m, q_m\}$ of pressure and dynamic pressure, respectively, to obtain estimates of the true values $\{p_a, q_a\}$:

$$p_a = p_m + \Delta p \tag{1}$$

$$q_a = q_m + \Delta q \tag{2}$$

One common assumption about the origin of these errors is that the total pressure at the tip of a pitot tube is correct, but that errors in both p_m and q_m arise from errors in the pressure present at the static ports. If that is the case, then $\Delta q = -\Delta p$, so

$$q_a = q_m - \Delta p\ . \tag{3}$$

The variables PSXC and QCXC then represent values of the static and dynamic pressure that are corrected by estimates of Δp and Δq, respectively. Note that, with this assumption and using p_t to denote the pressure at the tip of the pitot tube,

$$p_t = p_a + q_a = p_m + q_m \tag{4}$$

so p_t would be measured correctly.

Redundant measurements provide a test of this assumption because they should provide the same total pressure. There are two sets of measurements of static and dynamic pressure on the C-130, (PSFD/QCF) and (PSFRD/QCFR), so if both provide accurate measurement of the total pressure then PSFD+QCF should equal PSFRD+QCFR. The figure below shows that this is valid to high accuracy: The best fit to the measurements is (PSFRD+QCFR)=α(PSFD+QCF)+0.10 where $\alpha =$ 1.0000, and the RMS deviation from this best-fit line is 0.04 mb. However, the total pressures differ on average by almost 3 mb and a best fit shows more scatter (about 0.6 mb). This is supporting evidence for the assumption that p_t is measured correctly and that the errors in p_m and q_m arise from errors in the pressure provided by the static buttons.

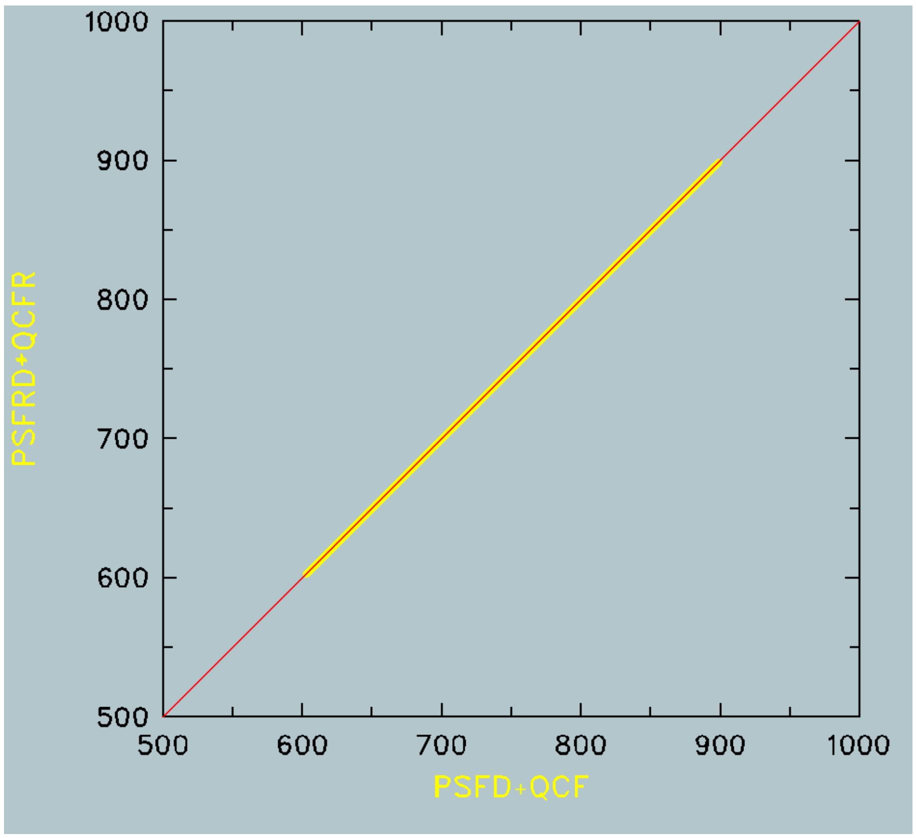

Figure 1: Measurements made at 1 Hz during flight 8 of IDEAS-4. All measurements from 1955-2453 are included for times when the true airspeed TASX exceeded 50 m/s (to exclude a short period with flaps deployed at the end of the flight). The measurements plotted are the total pressure p_t measured by two independent systems using two different pitot tubes and sets of static buttons. The quantities plotted are (PSFDR+QCFR) vs (PSFD+QCF), both before any corrections beyond calibration coefficients are applied to the raw measurements.

Present Processing Code

In current processing, different equations are used for the C-130 and the GV.

C-130:

For the C-130, a parameterization based on tests with a trailing cone is used. The correction Δp is applied to both the measured static pressure and, with reverse sign, to the measured dynamic pressure. The equations are:

$$\begin{aligned}\Delta p &= 4.66+11.4405(ADIFR/QCRC) \quad \mathrm{for\,PSFD\,and\,QCF} \\ &= 4.66+11.4405(AKRD) \quad for\,QCR \\ &= 3.29+0.0273q \quad \mathrm{for\,PSFRD,\,QCFR,\,but\,steady\,4.7915\,for\,q} < 55\end{aligned}$$

notes: the second line appears to be an error, and different corrections are applied to the two pairs of measurements, {PSFD/QCF} and {PSFRD/QCFR}.

GV:

Fits have been determined using the trailing code as reference and using the avionic-system pressure as reference, but they differ significantly. Present processing uses the following, based on using the avionic systems as the standard:[1]

$$\begin{aligned}\Delta p &= -1.02+0.1565*q)+q1*(0.008+p*(7.1979e-09*p-1.4072e-05) \quad \mathrm{for\,PSF} \\ &= 2.00-q*(0.023809+q*0.0001361) \quad \mathrm{for\,QCR} \\ &= 1.02+(ADIFR/QCR)*(0.215-0.04*p/1000.) \\ &\quad +p*(-0.003266+p*1.613e-06) \quad \mathrm{for\,QCF}\end{aligned}$$

trailing cone parameterization:

$$\Delta p = -1.02+0.1565*(ADIFR/QCR))+q*(0.008+q*(7.1979e-09*q-1.4072e-05)$$

Friehe parameterization:

$$\Delta p = -2.089+ADIFR*(0.196+0.00138*ADIFR)+MACH\_A*(9.609-8.307*MACH\_A)$$

[1] the listed quantities for Δp are added to static pressure but subtracted from dynamic pressure

Reasons For Proposing Changes

New representations of these corrections can be found from the LAMS. Because that provides an absolute measurement, it should be more reliable than other calibration sources. If the LAMS measurements of relative wind give consistent parameterizations in terms of other variables, those parameterizations can be used when the LAMS is not present, but when present it may be preferable to use the LAMS determination of Δp directly in processing. The next section analyzes this possibility.

The absolute accuracy of the measurements from LAMS suggests that it may be possible to calibrate several other measurements by using LAMS:

1. LAMS determines TAS directly and so is a check on alternate calculations of TAS.
2. Equivalently, LAMS can be used to determine dynamic pressure.
3. Because the error in measured dynamic pressure is probably a result of erroneous static pressure, LAMS, by determining this error, can determine an accurate value of static pressure.
4. LAMS can be used in this way to provide a reference from which to determine parameterizations for the errors in static and dynamic pressure, for use when the LAMS is not present or not operational.
5. Because LAMS determines airspeed, it can be used with measurements of static and dynamic pressure to determine the temperature, *independent of any temperature sensor.*
6. This measurement of temperature will be valid in cloud and rain, so valid measurements of in-cloud temperature will be possible. LAMS operates well in cloud unless the windows become covered with ice.
7. Improved accuracy in the measurement of pressure makes it possible, with accurate differential GPS, to integrate the hydrostatic equation and use the difference in height between to levels to determine the mean temperature in the interval between those levels, thus checking or calibrating the temperature measurement from either LAMS or the immersion sensors.

Analysis

Equations for Correcting Static and Dynamic Pressure

Refer to the Lenschow Tech Note describing the Buffalo air motion system for the basis of the following derivation. For compressible flow,

$$\frac{v^2}{2} + c_v T + \frac{p}{\rho_a} = \frac{v^2}{2} + c_p T = constant$$

However, for adiabatic compression, with$\gamma = c_p/c_v$

$$Tp^{-R/c_p} = Tp^{(1-\gamma)/\gamma} = const$$

or, if T^* is the temperature at stagnation where the pressure is $(p+q)$,

$$T^* = T\left(\frac{p+q}{p}\right)^{R/c_p}$$

$$\frac{v^2}{2} + c_pT = c_pT\left(\frac{p+q}{p}\right)^{R/c_p} \tag{5}$$

In terms of Mach number ($M = v/\sqrt{\gamma R_a T}$, so that $v^2 = M^2\gamma R_a T$), and

$$\frac{M^2\gamma R_a T}{2} + c_pT = c_pT\left(\frac{p+q}{p}\right)^{R/c_p} \tag{6}$$

If solved for Mach number, this gives

$$M = \left\{\left(\frac{2c_v}{R_a}\right)\left[\left(\frac{p+q}{\mathrm{p}}\right)^{R_a/c_p} - 1\right]\right\}^{1/2} \tag{7}$$

which shows that Mach number can be found without knowing the temperature.[2] Solving (5) instead for q gives:

$$q = p\left\{\left(\frac{v^2}{2c_pT} + 1\right)^{c_p/R_a} - 1\right\} \tag{8}$$

which shows that, with knowledge of p and T, LAMS can provide an independent measurement of the dynamic pressure q. That will lead to a direct measurement of the PCORs also:

$$\Delta p == \Delta q = q_m - q = q_m - p\chi \tag{9}$$

where, to simplify the notation, χ is written for

$$\chi = \left(\frac{v^2}{2c_pT} + 1\right)^{c_p/R_a} - 1 \tag{10}$$

[2]An nteresting application of LAMS can then be that, with v measured by LAMS, the temperature can be determined from $T = v^2/(\gamma R_a M^2)$ without any input from a temperature sensor, just from accurate measurements of p and q.

Then, because

$$p + p_m + \Delta p \ ,$$

$$\Delta p = \frac{q_m - p_m \chi}{1 + \chi} \tag{11}$$

which gives the pressure correction directly in terms of the measured quantities $\{p_m, q_m\}$. However, the temperature is needed to calculate χ, so there are several possible approaches to evaluating (10), including::

1. Solve the complicated equations for T, p, and q simultaneously, obtaining an immediate result for Δp.
2. Obtain the solution iteratively be calculating Δp as from (11) but with uncorrected values of $\{p, q\}$ used in the calculation of T, then improve the calculation of T using the derived corrections and iterate.
3. Use a measurement of temperature (e.g., from a radiometric sensor) that does not involve knowing static pressure and dynamic pressure.
4. Use the measurement of temperature obtained solely from LAMS without reference to any temperature sensor, using only the measurements $\{p_m, q_m\}$ and (7), where again either iteration or complicated solution of simultaneous equations is needed because Δp as found from (11) is needed for accurate calculation of M:

$$T = v^2 / (\gamma R_a M^2) \tag{12}$$

Supporting Evidence for Accuracy of p_t

There are general studies of pitot tubes that suggest their typical sensitivity to flow angle is <1% at angles up to 10°, 0.2% up to 5°. They are oriented so that they face the mean flow, so normal departures from the mean flow angle during flight are usually much smaller than this. It is critical to the analysis approach taken here that the measured total pressure be accurate, so some test of this assumption is useful.

On the C-130, there are redundant sensor pairs that sense the dynamic pressure q and the static pressure p. The total pressure is measured by adding two sets of sensors, either PSFD and QCF or PSFRD and QCRF. The total pressure is measured as PSFD+QCF or PSFRD+QCRF, where the two sets use independent static ports and independent pitot-tube sources. Comparing these provides a test of the assumption that the total pressure is delivered accurately by the pitot tubes and is measured accurately by the transducers, because, even if the two measurements of static pressure differ (as they do) because of their different sensing positions on the aircraft, the sums should be the same. The following figure shows a comparison of the two independent measurements of p_t on the C-130:

[——————————— insert Figure from PCOR presentation here ———————]

A fit to these measurements gave PSFRD+QCFR=$a_o + a_1$(PSFD+QFC) with $a_0 = 0.10$ mb and $a_1 = 1.0000$, with an RMS scatter about this fit of 0.04 mb. This provides support for the assumption that both of these measurements are accurate.

On the GV, there is only one set of sensors in the NCAR/RAF-provided data system, but there are pressure measurements available from the avionics package also that can be used in the same way. In this case, the total pressure measured by the avionic-system pair of measurements is about 1.1 mb higher than the corresponding measurements from the GV RAF data system, and the best fit is PS_A+QC_A=1.2 mb + 0.9996(PSF+QCF), with an RMS error from this fit of 0.2 mb. This error arises partly from filtering applied to the avionics-system measurements, which is evident in spectral characteristics of those measurements and leads to obvious differences in cases where measured quantities are changing rapidly. Nevertheless, this comparison is better than the comparison of either static or dynamic pressure alone and so provides some evidence that the measurement of total pressure is more accurate than those measurements. Therefore, in the following, it will be assumed that the total pressure is measured accurately, and corrections will be developed to adjust q to be that required by the measurements from LAMS.

Moist air correction

For accurate calculation in humid air, the values used in the preceding should be those for moist air, although the density of dry air at the same pressure and temperature is often used. The density of moist air having vapor pressure e (and so mixing ratio $r = \varepsilon e/(p-e)$ where ε is the ratio of the molecular weight of water to that of dry air) is

$$\rho_a = \frac{(p-e)}{R_d T} + \frac{e}{R_w T} = \frac{p}{R_d T}\left(1 - \frac{e}{p} + \frac{e\varepsilon}{p}\right) = \frac{p}{R_d T}\left(1 + (\varepsilon - 1)\frac{e}{p}\right) \tag{13}$$

so the gas constant that should be used is

$$R_a = R_d/[1 + (\varepsilon - 1)\frac{e}{p}] \tag{14}$$

Because for air the specific heats are very close to those for a diatomic molecule with five degrees of freedom, while for water the values are approsimately those for six degrees of freedom (i.e., $c_v = 3R_w$), similar results for c_p, c_v, and γ for humid air are averages weighted by the mass fraction of each constituent, as follows:

$$c_v = \frac{(p-e)R_a}{pR_d}\frac{5R_u}{2M_d} + \frac{eR_a}{pR_w}\frac{3R_u}{M_w} = c_{vd}\frac{R_a}{R_d}\left(1 + (\frac{6}{5} - 1)\frac{e}{p}\right) = c_{vd}\frac{R_a}{R_d}(1 + \frac{e}{5p}) \tag{15}$$

$$c_p = c_{pd}\frac{R_a}{R_d}\left(1+(\frac{8}{7}-1)\frac{e}{p}\right) = c_{pd}\frac{R_a}{R_d}(1+\frac{e}{7p}) \tag{16}$$

$$\gamma = \gamma_d\frac{1+\frac{e}{7p}}{1+\frac{e}{5p}} \tag{17}$$

These values for R_a, c_p and γ should be used when evaluating (10-12).

When LAMS is present, this estimate of Δp could be used to determine adjustments to the static pressure and the dynamic pressure, with further correction for the effect of measurement angle as discussed below.

Effect of LAMS Orientation

The pitot tube is relatively insensitive[3] to flow angles and so measures the total dynamic pressure, but LAMS measures the relative wind in a specific direction. For LAMS, the effect of a flow angle θ relative to the beam is that it measures $v_l = v\cos(\theta)$. The beam is oriented close to but slightly offset from the longitudinal axis of the aircraft, at viewing angles θ_1 above and θ_2 to the starboard side of the longitudinal axis. On the C-130 for IDEAS-4, these angles are Then, with sideslip β positive for relative wind approaching from the starboard side of the aircraft, to a sufficient approximationcos $\theta = \cos(\theta_1+\alpha)\cos(\theta_2-\beta)$ with α the angle of attack. The resulting equation for Δp is then (11) where, in (10), v is replaced by $v_l/\cos\theta$ and the moist-air values are used for R_a and c_p.

Fits to Δp:

C-130

For IDEAS-4 flight 8, times 2000-2440, fits to the values of (11) obtained as above were tried as function of various measured variables. One fit that seems to represent the primary source of variability was the following:

$$\frac{\Delta p}{\text{PSFD}} = a_0 + a_1\frac{\text{ADIFR}}{\text{QCR}} + a_2\frac{\text{QCF}}{\text{PSFD}} \tag{18}$$

with$\{a_0,a_1,a_2\}$={0.004127875,0.021279816,0.030849643). The RMS error for this fit was 0.00045, corresponding to a pressure uncertainty at 700 hPa of about 0.3 hPa for the individual measurements.[4] Other fits also gave good representations of the measurements, including $\Delta p =$

[3]typical sensitivity is less than 1% at flow angles up to 10° and less than 0.2% for flow angles up to 5°. The error is in the direction of measuring too low a total pressure, and to some extent it is compensated by orientation of the pitot tubes along the flow angle expected in normal flight.

[4]The data constrained the mean of the function to more than a factor of 100 smaller uncertainty than this.

$3.963228 + 10.529772(\mathrm{ADIFR}/\mathrm{QCR})$ which accounted for almost as much of the variability. While the residuals from these fits are small, the mean offset was about 2 mb.

Another, perhaps preferable, approach is to determine sensitivity to angle of attack and sideslip from periods when there are maneuvers, fix these, and then determine any additional variability from fits to entire flights. IDEAS RF05 had two periods with sideslip maneuvers and two with pitch maneuvers; for former suggested that sensitivity of $\Delta p/\mathrm{PSFD}$ to (BDIFR/QCR) is about -0.001773 and the latter indicated sensitivity to (ADIFR/QCR) of 0.01890. When these are forced and the flight data are used to determine further dependence, the residual significant dependence was on either XMACH2 or (QCF/PSFD), suggesting the following fit:

$$\frac{\Delta p}{\mathrm{PSFD}} = b_0 + b_1\frac{\mathrm{ADIFR}}{\mathrm{QCR}} + b_2\mathrm{XMACH2} + b_3\frac{\mathrm{BDIFR}}{\mathrm{QCR}} \tag{19}$$

where $b_0 = 0.004199$, $b_1 = 0.01891$, $b_2 = 0.01890$, and $b_3 = -0.001773$. The RMS error for this fit was only slightly higher, for IDEAS flight 8, than that given by (18), but the representation of the maneuvers in flight 5 was improved. Under normal conditions the last term makes negligible contribution, but it has some effect during the high-sideslip portions of maneuvers so could be included without harm.

A similar approach can be taken to determining Δp for other sensors. The results are the same for QCR because, by assumption, the errors arise from the static source and this is common between QCF and QCR. However, QCR has additional corrections that arise from the flow dependence of the dynamic pressure measurement, because this is a radome port instead of a pitot-tube measurement and is more sensitive to flow angle. On the C-130, another pair of measurements is provided by PSFRD and QCFR; these use a different set of static buttons and so Δp_R may have a different functional dependence. The same procedure was used to determine fits for this sensor, with these results (where PSFD and QCF are replaced in the equations by PSFRD and QCFR): $\{a_0, a_1, a_2\}$={0.002047, 0.008890, 0.07119} and $\{b_0, b_1, b_2, b_3\}$={0.001897, 0.01083, 0.07254, -0.0007951}.

My recommendation is the set $\{a_0, a_1, a_2\}$ and equation (18) or the corresponding equation for PSFRD/QCFR.

One check on this procedure is that, after application of these corrections, PSFD and PSFRD should produce the same corrected values. The following plot shows a comparison for flight 8 of IDEAS-4:

GV

In the case of the GV, a different parameterization was useful, probably because of the wider range of flight conditions of that aircraft. The measurements from PREDICT ferry flight #1 (from Colorado to St. Croix) included many altitude changes, many of the constant-altitude legs included

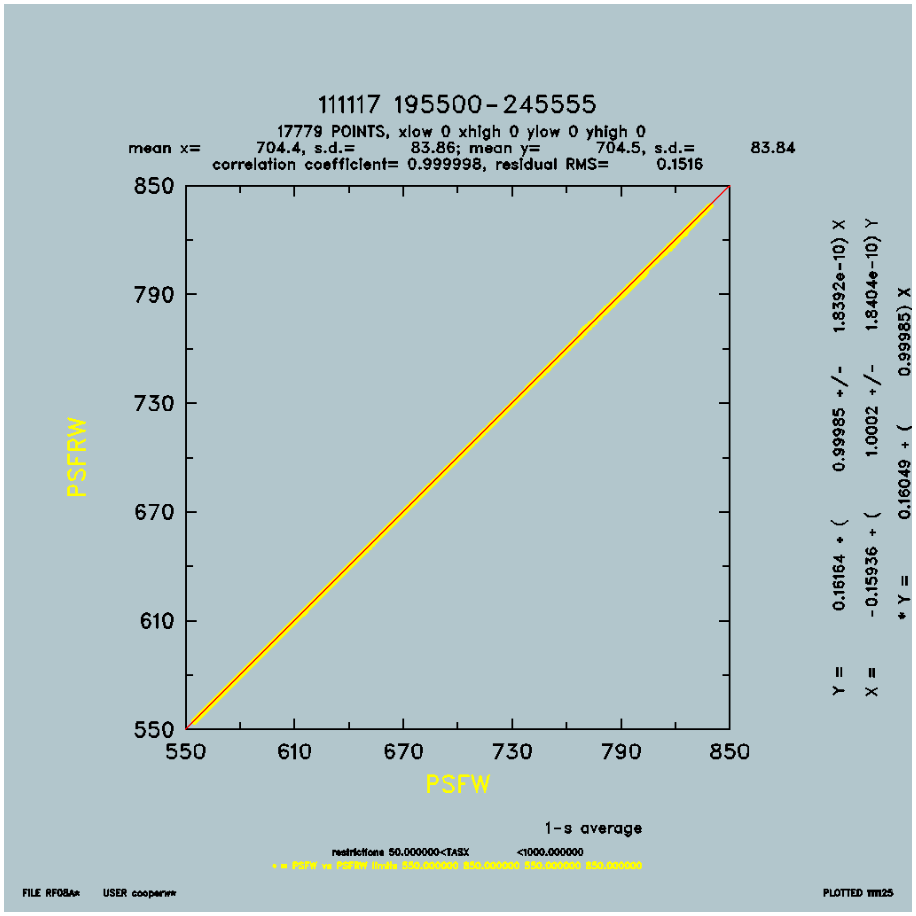

Figure 2: Comparison of independent measurements of static pressure obtained after correction using (18) with the coefficients given in the text, for PSFD corrected to PSFW and PSFRD corrected to PSFRW. All measurements from flight 8 of IDEAS-4 are included, except those with TASX<50 m/s, to exclude a period with flaps at the end of the flight and the large difference in pressures that occurs after landing when the props are reversed. The best fit that minimizes the distance to the plotted points is PSFRW=0.18+0.99985PSFW, and the RMS deviation of the points from this fit is 0.15 mb. The mean values of pressure differ by only 0.08 mb

speed changes, and LAMS provided good measurements for most of the flight, so this flight was used for fitting to determine a functional representation of the PCORs for use when LAMS is not present.

The best representation of Δp, obtained after trying many options, was

$$\frac{\Delta p}{p} = a_0 + a_1 \frac{q}{p} + a_2 M + a_3 M^2 + a_4 M^3 \tag{20}$$

where p is the direct measurement of pressure (PSF), q the dynamic pressure (QCF), and M is the Mach number (obtained from $\sqrt{\text{XMACH2}}$). The dimensionless coefficients for the best fit to this flight were as given in the following table:

coefficient	**value**
a_0	0.00696
a_1	0.6678
a_2	-0.05965
a_3	-0.2833
a_4	-0.2437

The fit RMS error vs the LAMS-determined PCORs was 0.00081 or, for a typical pressure of about 350 mb, about 0.3 mb. Some part of this scatter resulted from variance in the LAMS measurement, perhaps arising from sensing a different air volume vs that arriving at the pitot tube, but more likely indicating some resolution problem in determining the LAMS prediction because the variance spectrum showed some indication of resolution noise at 1 Hz. This result did not improve significantly with the inclusion of ADIFR/QCR or BDIFR/QCR or abs(BDIFR/QCR), and it did not improve significatly with higher powers of QCF/PSF or of M. Unlike the C-130 result, however, this higher-degree polynomial in M was required; omitting any of the terms led to significant reduction in the correlation coefficient and significant increase in the RMS error of the fit. The correlation coefficient between the LAMS prediction and this formula was 0.97, showing that this parameterization accounts for around 94% of the variance in Δp. This formula with these coefficients thus appears to be a good representation of PCOR on a flight with many altitude changes and maneuvers.

Tests Using Flight Maneuvers:

A test of the accuracy of the measurement of dynamic pressure is that the longitudinal component of the wind should not change in reverse-heading maneuvers in which the aircraft is flown over the same (drifting) flight leg twice with opposite headings. To isolate the effect of the measurement of q and hence true airspeed, the best wind component to use is that along the axis of the aircraft, which is $v_g \cos\beta - \text{TAS}$ where v_g is the groundspeed of the aircraft and β is the angle between the groundspeed vector and the heading of the aircraft. The GPS system provides the groundspeed in variables GGSPD and GGTRK (magnitude and direction), so β =GGTRK-THDG and the wind component along the aircraft axis, as a new variable called UXW, is:

$$\mathrm{UXW} = \mathrm{GGSPD}\cos(\mathrm{GGTRK}-\mathrm{THDG}) - \mathrm{TASW}$$

where TASW is the true airspeed calculated from the corrected measurement of dynamic pressure, QCW:

$$\begin{aligned} \mathrm{QCW} &= \mathrm{QCF} - \Delta p_1 \\ \mathrm{or} &= \mathrm{QCFR} - \Delta \mathrm{p}_2 \end{aligned}$$

The true airspeed is a function of the measurements $\{q,\, p,\, T_r\}$ of dynamic pressure, static pressure, and measured temperature:

$$T_a = \frac{T_r}{1+\dfrac{\alpha M^2(\gamma-1)}{2}}$$

where $T_a$is the air temperature, $\gamma = c_p/c_v$, and the Mach number M is determined from p and q via (7). Then TASW=$v = M\sqrt{\gamma R_a T_a}$.

C-130:

The following table shows results for two reverse-heading maneuvers flown on flight 5 of IDEAS-4:

Time Interval	UXW	GGTRK	THDG	WDC	WSC	UX
220000–230530	1.56±0.91	241	242	65	3.31	2.89
230700–231200	-1.08±0.55	59	59	224	1.43	-1.47
225100–225300	1.975±0.40	148	149	312	2.58	2.40
225500–225700	0.99±0.52	328	329	122	1.09	1.21

The angles show that the maneuvers were flown well, with close to 180° difference in heading between the two legs and an orientation close to the mean wind (although the second is about 20° offset). Because UXW represents the wind along the longitudinal axis of the aircraft, it should reverse sign for the two legs of the maneuver. The first pair above has a difference of 0.48 m/s, significantly reduced from the standard processing (UX) which shows a difference of 1.42 m/s. The second pair gives a larger difference: 2.97 m/s for the new processing vs 3.61 m/s for the standard processing. Both old and new processing give a larger error than is expected for the second pair, although the first pair leads to a quite acceptable difference. Analysis of more reverse-heading maneuvers would help evaluate if these corrections are acceptable, but the improvement produced in both cases by the new fit suggests it is appropriate for use in processing.

Speed runs also provide some indication of the quality of the corrections. For the speed changes in flight 8 of IDEAS-4, 2413-2420, the standard deviation in DVALUE calculated from the new pressures was 3.9 m, compared to 5.3 m for the standard processing, and the mean value of the DVALUE was almost 6 m smaller. Also, UXW calculated as above varied only about 1 m/s during the speed run in either processing, as required if the results for q are accurate.

GV:

One of the PREDICT test flights, #4, had a large number of reverse-heading maneuvers flown at different altitudes, so these can be used to test if this representation of $\Delta p = -\Delta q$ gives satisfactory agreement between the longitudinal component of the wind measured on the reverse-heading maneuvers. As for the C-130, the appropriate variable to compare is called UXW (analogous to the standard variable UX). The following table shows the results for 18 reverse-heading pairs of legs from this flight. The mean difference on legs along opposing headings is +0.6±0.2 m/s, suggesting that the error in measurement of longitudinal wind is +0.3±0.1m/s. Because this result is dependent on the measurement of temperature, it will be important to iterate this entire procedure before considering this a correction that might be applied to the measurements; instead, it is an indication that the accuracy of the measurement of horizontal wind is on the order of 0.3 m/s.

The suggested conclusion for both the C-130 and GV is that the LAMS provides a valid calibration source for both static and dynamic pressure, and that fits to the resulting corrections predicted by LAMS can be used to improve the corrections that are applied to static and dynamic pressure.

Determining the Air Temperature

Once confidence in the pressure measurements has been established as in the preceding sections, a check on the absolute accuracy of the temperature measurement can be obtained from integration of the hydrostatic equation, expressed in this form:

$$\delta p_i = -\frac{g\,p_i}{R_a T_{a,i}}\delta z_i \tag{21}$$

where $\{p_i, T_{a,i}\}$ are the values of air pressure and temperature for the i-th measurement and δp_i is the change in pressure for the i-th step, during which the geometric altitude changes by δz_i. This equation can be rearranged to obtain an estimate of the temperature:

$$T_{a,i} = -\frac{g}{R_a}\frac{\delta z_i}{\delta \ln p_i} \tag{22}$$

The altitude change $\delta z_i$is provided with high accuracy by GPS measurements: for a climb rate of 10 m/s, measurement accuracy of 1% in derived temperature requires at least 1% accuracy in the measurement of the 1-s change, or accuracy of 0.1 m. To determine the temperature to 0.1% (as required for a typical error of about 0.3°C), the altitude change must be measured to 1 cm. The accuracy approaches this for differential measurements, but a better approach is to use longer intervals so that the altitude difference is greater – perhaps 10 s or more. The requirement is more stringent on the measurement of pressure. In 1 s at 10 m/s climb, the pressure change is less than 1 mb, and it seems likely that even differences in pressure cannot be measured confidently to better than 0.05 mb, so this would introduce an error of >5% in the deduced (absolute) temperature. This is inadequate because it leads to errors in the temperature of >13°C. Instead, either longer intervals are needed or many measurements must be averaged.

C-130:

About 30 min of flight during flight #8 of IDEAS-4 (2309-2341) was devoted to repeated climbs and descents, so there are about 1800 measurements and it might be expected that the standard error in the determination of temperature from (22) could be reduced by $\sqrt{1800} = 42$, or to around 0.5°C, by this procedure. Alternately, a "mean" temperature between two levels can be determined from (22); for this flight segment, climbs were repeated from about 12-16000 ft, or over a pressure range of about 100 mb. An uncertainty of 0.1 mb in pressure leads to about an uncertainty of 0.1% or, in absolute temperature, an uncertainty of about 0.3°C in the mean temperature between the layers. It should therefore be possible to test the temperature measurements with about this level of confidence.

To test this, three sums were used between different flight levels:

$$S_1 = \sum_i \frac{R_{a,i}}{g_i} \ln\left(\frac{p_i}{p_{i-1}}\right)$$

$$S_2 = \sum_i (z_i - z_{i-1})$$

$$S_3 = \sum_i \frac{z_i - z_{i-1}}{T_{m,i}}$$

where $R_{a,i}$ and g_i are respectively the gas constant (adjusted for humidity) and the acceleration of gravity (adjusted for latitude and altitude) and $T_{m,i}$ is the temperature in absolute units, corrected for airspeed but based on the measured value. The predicted mean temperature for the layer, weighted by altitude, is given by $T_p = -S_1/S_2$, while the corresponding weighted-mean measured temperature is $\overline{T}_m = S_2/S_3$. The following table shows some measurements from selected flight legs in IDEAS-4:

Flight Segment	T_p	$\overline{T}_m$	ΔT
RF05, 205800–211100	-10.98	-10.37	-0.5
RF07, 212510–213300	-6.36	-5.89	-0.47
RF07, 212510–212900	2.27	2.42	-0.15
RF07, 212900–213300	-12.85	-12.15	-0.70
RF08, 214500–215300	-0.9	-0.5	-0.4
RF08, 233700–234130	-6.5	-6.3	-0.4
RF08, 234500–235000	-9.4	-8.8	-0.6
RF08, 235600–240100	-9.5	-8.4	-1.1
mean offset[a], $T_p - \overline{T}_m$			-0.55

[a] excluding the first listed value for RF07 because the next two break this climb segment into two segments

The evidence from these climbs then indicates that the temperature is about 0.5°C too high and that the offset perhaps increases as the temperature decreases.[5]

GV:

A similar approach can be taken for the GV, with promise of a larger range of calibration points because of the large altitude changes present on many of the flights. However, because of the existence of many flights with altitude changes (e.g., from HIPPO), it was decided to take a different approach in an attempt to determine a polynomial-fit correction to the temperature via minimization of the error between actual altitude changes and those predicted from integration of the hydrostatic equation. The procedure used was as follows:

1. Calculate new data files that have corrected values of pressure and dynamic pressure, Mach number, and temperature based on the fits that give Δp and Δq. In addition, add the altitude variable "ALTV" described in a separate memo.

2. Construct "R" data files that have the preceding variables and in addition include EDPC (for calculation of the humidity-adjusted specific heats), LATC (for calculation of the latitude-adjusted acceleration of gravity), GGALT (as a backup to ALTC), TASX (for validation that the aircraft is above flight speeds where the flaps might be extended), and a total temperature measurement like TTHR1. Include also DVALUE measurements, for information on the horizontal pressure gradient as explained below.

[5] For the record: I did try to fit a linear correction using the measurements of RF08, IDEAS-4. A least-squares minimization procedure was used: Find the values of c_0 and c_1 that minimize the variance $\chi^2 = \sum_i (T_{m,i} + c_0 + c_1 T_{m,i} - T_{a,i})^2$. However, the 1-s values were so variable that the attempt to calculate fite coefficients led to a singular inversion, which presumably arose from the large numbers entering the fit. It may be possible to return to this, but the preceding evidence suggested that the temperature measurements are approximately within expected errors so I didn't pursue this for the C-130.

(a) Use the "R" routine "optim" to minimize a χ^2 function defined as the difference between the predicted and measured altitude, where the predicted altitude is determined by integration of the hydrostatic equation in this form (with T_t the temperature in °C, $T_0 = 273.15$, and $\{a_0, a_1, a_2, ...\}$ fit coefficients used to minimize the value of χ^2:

$$\chi^2 = \sum_i (h_i - Z_i)^2 \tag{23}$$

where

$$h_i = h_{i-1} - \frac{R_a}{g} \ln \frac{p_i}{p_{i-1}} (f(T_t) + T_0) \tag{24}$$
$$f(T_t) = T_t + a_0 + a_1 T_t + a_2 T_t^2 + ...$$

The vertical integration will match the pressure change only if the atmosphere is horizontally homogeneous. If not, the results will be biased as the fit attempts to compensate for horizontal gradients. This can introduce a serious error into the minimization results.

To consider how serious this problem is, it is useful to assess how a pressure gradient will affect the results. Suppose the pressure gradient along the flight path is $dp/ds = G_p$. Then there will be a contribution to the pressure change over a period ΔT not associated with an altitude change, of magnitude $G_p V \Delta T$, arising just from the pressure gradient, so in (24) the pressure ratio must be modified to be $(p_i - G_p V)/p_{i-1}$. Fortunately, there is a way to measure the horizontal pressure gradient because it will be reflected in the change in DVALUE. For example, in level flight the change in DVALUE will measure the pressure gradient; similarly, in a climb, the change in DVALUE can be compared to that expected for the measured temperature vs that of a standard atmosphere, so that also can be used as a measure of the pressure gradient. For horizontal or nearly horizontal motion (e.g., along pressure surfaces), $(dp/ds = -\rho g dH_p/ds$ where H_p is the height at pressure p. The change dH_p is the same as the change in DVALUE. The term needed to correct the pressure used in the hydrostatic equation is thus $G_p V \Delta T = -\rho g V \Delta T (dH_p/ds) = -\rho g \Delta H_p$. In a climb, however, the same correction is not possible because a change caused by a horizontal pressure gradient can't be distinguished from one caused by a different temperature along the flight path. Even at 40,000 ft, though, the GV climbs at above 5 m/s, so the ratio of contributions from the horizontal and vertical gradients will be

$$\frac{-\rho g \Delta H_p}{-\rho g \Delta z} = \frac{V(dH_p/ds)}{W} \simeq 40 \frac{dH_p}{ds}$$

For example, an upper limit to the horizontal height gradient might be 10 m/100 km or <10^{-4}; most level flight segments in the HIPPO flights show less than this gradient. In this case the vertical gradient would be changed by about 0.004, and a temperature change of around 1°C would be required to produce the same change in the gradient.

At this level of effect, it seems reasonable to expect that the measurements will tend to average toward the correct answer, especially if repeated climbs and descents are flown in the same direction so that the effects of the horizontal pressure gradient on climbs and descents tend to cancel.

There are several steps that can be taken to compensate for the horizontal pressure gradient:

i. Use only periods when the climb rate exceeds some threshold (e.g., 2 m/s);
ii. Use DVALUE measurements from level flight segments to estimate the gradient;
iii. Use a comparison between pressure measured during climb and descent to estimate and remove the gradient;
iv. Use the whole-flight average DVALUE (perhaps only from level legs) to determine the pressure gradient, and apply a correction.
v. Use only flights not returning to the takeoff point, to avoid cases where it is not possible to determine the gradient

A LAMS-based Measurement of Temperature:

When (12) is used with the fit to Δp from (18), the result for temperature is shown as the cyan line in the plot below. The plotted result overlies the yellow line showing the result for ATX but is now independent of that measurement except that the measurement of ATX affects the data on which the fit was based. These results therefore are not independent; indeed, if (11) , shown also in the plot, is used to extimate Δp point-by-point and then (12) is used to calculate the temperature, the result is exactly equal to ATX because then the reference really is just recalculation of the same value that entered the estimation of Δp via (11). Once a fit is found to predict Δp from measurements like ADIFR, QCR, and PSFD, then the results become independent of the measurements of ATX, except to the extent that ATX affects the coefficients in the fit.

The variance is higher in ATLAMS for the flight segment in the boundary layer (near 2100Z) because the flow conditions at the pitot tube and in the air sampled by LAMS tend to have lower coherence at high rate, leading to a noisier estimate of the temperature. Techniques for reducing this variance will be explored in the studies of the GV system that follow.

Recommendations

1. For the C-139, use (18) with the coefficients given above to correct PSFD and PSFDR, and with reversed sign to correct QCF and QCFR, to adjust for errors in the static pressure delivered by the static sources. The same analysis needs to be added for the GV.

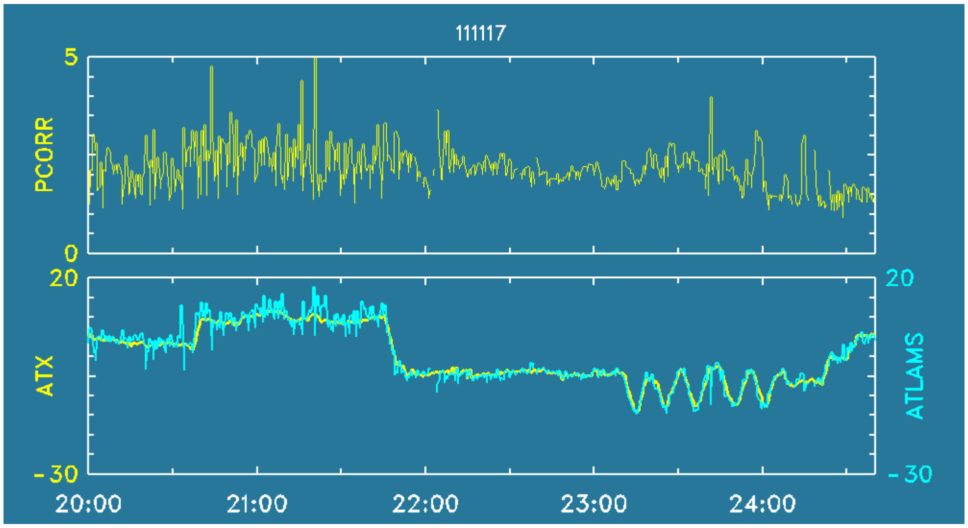

Figure 3: Temperature determined from LAMS (ATLAMS, cyan line) overlaid on the standard measurement of temperature, ATX, for a flight segment from IDEAS-4 Flight 8. Also plotted is an estimate of Δp (PCORR) determined from LAMS for the pressure measurement PSFD.

2. Consider additional flight data to learn if the indicated offset in temperature, indicating that the measurements are too high by about 0.5°C and that the offset is perhaps increasing as the temperature decreases, are consistent and should be applied to the measurements.
3. Repeat the temperature-calibration study with GV data from PREDICT, for flights where the LAMS was operational.
4. When evaluating the consistency of reverse-heading maneuvers, compare only UX to test the accuracy of the pressure corrections. Calm air is best, and the maneuvers are best flown along and against the wind, but exactly complementary headings are more important than orientation. Consistency of VY isolates the accuracy of any offset in radome sideslip angle but is also affected by any error, changing between the legs, in heading.
5. Further study is needed of the temperature measurement from LAMS in cloud, because the measurements from IDEAS-4 flight 7, 2200–2230, look suspect.
6. It will be appropriate to consider appropriate filtering applied to the LAMS measurement of temperature to reduce the noise arising from minor incoherence between the dynamic pressure at the pitot tubes and in the volume sensed by LAMS at periods of less than about 3 s.

GV addition:

Reconsider a fit to determine a correction function to the temperature:

$$S_1 = \sum_i \frac{R_{a,i}}{g_i} \ln\left(\frac{p_i}{p_{i-1}}\right)$$

$$S_2 = \sum_i (z_i - z_{i-1})$$

$$S_3 = \sum_i \frac{z_i - z_{i-1}}{T_{m,i}}$$

$$\chi^2 = \sum_i (T_{m,i} + c_0 + c_1 T_{m,i} - T_{a,i})^2$$

where $T_{m,j} = S_2/S_3$ and $T_{a,j} = -S_1/S_2$. Options are:

1. Limit values entering the fit to ones with some minimum change in z or p, to avoid undefined divide-by-near-zero errors;
2. Select (perhaps automatically) segments with some minimum change in pressure or z, calculate individual sums for those, and then minimize the Chisquare;
3. Low-pass-filter results before calculating sums to avoid problems introduced by small fluctuations;

These all are not optimal, though, because the error characteristics are not represented well by the Chisquare function (I think). A maximum-likelihood solution seems called for, in which the steps might be as follows:

1. Use the integrated hydrostatic equation, with measured temperature adjusted by fit coefficients, to predict the altitude z_c continuously, starting from some reference time, chosen after flaps/gear are up at the start of the flight (and restarted anytime there are missing values). The equation is:

$$\delta z_i = -\frac{R_a T_{a,i}}{g\, p_i} \delta p_i$$

   so it is useful to archive the quantity $R_a \delta p_i / (g\, p_i)$ to use for recalculating δz_i with adjustment of $T_{a,i}$. "newdap" processing must then include this quantity in data files that can then be translated to "R" data files, to be read into the routine that will process a large number of flights simultaneously (perhaps all PREDICT, HIPPO-4, and HIPPO-5 flights – HIPPO is especially good for this purpose because the flights include many climbs and descents).
2. Use as reference the GPS-measured altitude z_{GPS}, and find the best fit coefficients that match those measurements throughout the flight.

3. Assume a Gaussian error distribution (probably a better assumption for z than for the deduced T), and calculate a likelihood based on the difference between $z_{GPS} - z_c$. Use statistical properties of this difference to estimate the appropriate standard deviation to use when calculating the likelihood.

4. Maximize the likelihood by adjusting the fit coefficients.

5. Estimate the uncertainty in the result obtained from the likelihood

—- END —-