From LWP to COT: \(k\) coefficient

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Experimental and Instrumental Research Group
Parameterization of Cloud Radiative Transfer in GCM

Cloud Module $\Rightarrow$ LWP
Aerosol Module $\Rightarrow$ $N_{\text{act}}$ $\Rightarrow$ COT

$\Rightarrow$ Radiative transfer module
Parameterization of Cloud Radiative Transfer in GCM

\[ W = \frac{4}{3} \pi \rho_w \int_0^H N(h) r_3^3(h) dh = \frac{4}{3} \pi \rho_w \int_0^H M_3(h) dh \]

\[ \tau = \int_0^H \pi Q_{ext} \left( \bar{x} \right) N(h) r_2^2(h) dh = \int_0^H \pi Q_{ext} \left( \bar{x} \right) M_2(h) dh \]

\[ k = \left( \frac{r_2}{r_3} \right)^6 = \left( \frac{r_3}{r_e} \right)^3 \]

Martin et al. 1994

\[ \tau = A(kNH)^{1/3} W^{2/3} \quad \text{or} \quad r_e = \left( \frac{3W}{4\pi \rho_w kNH} \right)^{1/3} \]

\[ k = M_2^3 / N M_3^2 \]
During the course of this work it was found that the most suitable parameterization for effective radius of droplets in layer clouds is

\[
 r_e [\mu m] = 10^3 \left( \frac{3L \text{ [g m}^{-3}] }{4 \pi \rho_w k N_{\text{TOT}} \text{ [cm}^{-3}] } \right)^{1/3}, \tag{14}
\]

where the values of \( k \) and \( N_{\text{TOT}} \) are

(i) in maritime airmasses:

\[
 k = 0.80 \pm 0.07 \quad (1 \text{ standard deviation})
\]

\[
 N_{\text{TOT}} = -1.15 \times 10^{-3} A^2 + 0.963 A + 5.30, \tag{15}
\]

where \( A \) is the aerosol concentration in the range \( 36 \leq A \leq 280 \text{ cm}^{-3} \) and

(ii) in continental air masses:

\[
 k = 0.67 \pm 0.07 \quad (1 \text{ standard deviation})
\]

\[
 N_{\text{TOT}} = -2.10 \times 10^{-4} A^2 + 0.568 A - 27.9 \tag{16}
\]

Chen et al. Global climate response to anthropogenic aerosol indirect effects: Present day and year 2100. J. Geophys. Res.
<table>
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<tr>
<th>Project</th>
<th>Location</th>
<th>Aircraft</th>
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The FSSP-100 provides accurate measurements of the mean droplet diameter, but it significantly overestimates the width of the spectrum, hence underestimates the k coefficient. This is especially true when droplets are smaller, i.e. in high concentration clouds.
Instrumental Biases

- **Fast 5.2-38.4µm**
- **FSSP 2.6-52µm**
- **SCMS-1995**
  - C130-FSSP & Fast-FSSP
- **FSSP 5.2-52µm**

- **Fast 5.9-43.8µm**
- **SPP 1-47µm**
- **DYCOMS-2001**
  - C130-SPP/Fast-FSSP
- **SPP 5.5-43.5µm**

**k ratio** vs **Fast-FSSP MVD (µm)**
Results
During the course of this work it was found that the most suitable parameterization for effective radius of droplets in layer clouds is

\[ r_e \, [\mu m] = 10^3 \left( \frac{3L \, [g \, m^{-3}]}{4\pi \rho_w k N_{TOT} \, [cm^{-3}]} \right)^{1/3}, \]  

(14)

where the values of \( k \) and \( N_{TOT} \) are

(i) in maritime airmasses:

\[ k = 0.80 \pm 0.07 \] (1 standard deviation)

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(15)

where \( A \) is the aerosol concentration in the range \( (36 \leq A \leq 280 \, cm^{-3}) \) and

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\[ N_{TOT} = -2.10 \times 10^{-4} A^2 + 0.568 A - 27.9 \]  

(16)

When entrainment effects become important, the relationship between \( r_e \) and \( r_v \) breaks down and such data have been ignored in the analysis.
K coefficient and sub-adiabaticity

\[ k' = 0.872, \quad <k> = 0.858 \pm 0.052 \]
\[ <N> = 107.3 \pm 67.91, \quad N_{act} (p_{98^{th}}) = 240.0 \text{ cm}^{-3} \]
\[ <q_l/q_{lAd}> = 0.363 \pm 0.260, \quad N_{data} = 252.0 \]
Vertical Integration

\[ W = \frac{4}{3} \pi \rho_w \int_0^H N(h) r_3^3(h) dh = \frac{4}{3} \pi \rho_w \int_0^H M_3(h) dh \]

\[ \tau = \int_0^H \pi Q_{\text{ext}}(\bar{x}) N(h) r_2^2(h) dh = \int_0^H \pi Q_{\text{ext}}(\bar{x}) M_2(h) dh \]

\[ \tau = A(kNH)^{\frac{1}{3}} W^{\frac{2}{3}} \quad \text{with} \quad k = \frac{M_3}{N M_3} \]

Is true only in vertically uniform clouds!
Vertical Integration

In convective clouds, that are vertically stratified, with LWC increasing from cloud base to top: $q_c = C_w h$

$$k^* = \left| M_2 \right|^3 / \left| N \right| | M_3 |^2,$$ where $|x| = \int_0^H x(h) dh$.

Assuming $k$ is constant throughout the cloud, $r_2^2 = k^{\frac{1}{3}} \alpha^{\frac{2}{3}} h^{\frac{2}{3}}$, where $\alpha = C_w / (4/3 \pi \rho_w)$.

It follows that $|M_2| = 3/5 k^{\frac{1}{3}} \alpha^{\frac{2}{3}} NH^{\frac{5}{3}}$, and $|M_3| = 1/2 \alpha N H^2$, and finally:

$$k^* = \left( \frac{3}{5} \right)^3 \left( \frac{1}{2} \right)^{-2} k = 0.864 k$$
Results

\[ k^* = \frac{|M_2|^3}{|N||M_3|^2} \]

\( \langle N \rangle \) (cm\(^{-3}\))
Parameterization of Cloud Radiative Transfer in GCM

Cloud Module $\Rightarrow$ LWP
Aerosol Module $\Rightarrow$ $N_{\text{act}}$

$\Rightarrow$
Radiative transfer module
COT & N

But $N \neq N_{\text{act}}$
because of entrainment-mixing
Results

\[ \frac{N}{N_{\text{act}}} = 0.81 \pm 0.09 \text{ in Sc} \]

\[ \frac{N}{N_{\text{act}}} = 0.46 \pm 0.08 \text{ in shallow Cu} \]
Entrainment and mixing: Conceptual Model

**Inhomogeneous**

Key Parameters (Baker et al., 1979)
- droplet life time: $\tau_d = -(d^2 / AS)$

**Homogeneous**

- turbulent homogeneity: $\tau_T = (X^2/\varepsilon)^{1/3}$

Key Parameters:
- droplet life time: $\tau_d = -(d^2 / AS)$
- turbulent homogeneity: $\tau_T = (X^2/\varepsilon)^{1/3}$

Graphs showing the relationship between normalized mean volume diameter ($d_v$) and normalized concentration for both inhomogeneous and homogeneous cases.
Entrainment and mixing: Case Studies

\[
\frac{\tau_d}{\tau_T} = 6.6
\]

SCMS Cu

\[
\frac{\tau_d}{\tau_T} = 1.9
\]

SCMS Cu

\[
\frac{\tau_d}{\tau_T} = 0.05
\]

DYCOMS-II Sc

**Burnet & Brenguier, JAS 2006**

J. L. Brenguier, Météo-France CNRS

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VOCALS - Miami - 21/03/2011
Conclusion

\[ \tau = A (k_{act}^* N_{act} H)^{1/3} W^{2/3} \]

where \( k_{act}^* = k^* \frac{N}{N_{act}} \)

\( \frac{N}{N_{act}} = 0.81 \pm 0.09 \) in stratocumulus clouds

\( \frac{N}{N_{act}} = 0.46 \pm 0.08 \) in shallow cumuli

\( k_{act}^* = 0.73 \times 0.81 = 0.59 \) in stratocumulus clouds

\( k_{act}^* = 0.73 \times 0.46 = 0.34 \) in shallow cumuli
Thank you for your attention.
### Key Parameters:

- **Droplet life time**
  \[ \tau_d = - \left( \frac{d^2}{AS} \right) \]

- **Turbulent homogenisation**
  \[ \tau_T = \left( \frac{X^2}{\varepsilon} \right)^{1/3} \]

### Table: Entrainment and mixing: Case Studies

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