Characterization of Crystal Size Distributions as Gamma Functions in High Ice Water Content Conditions

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[4]Environment Canada, Downsview, ON

OUTLINE

1. Review of technique to fit HIWC/HAIC SDs as volume of equally realizable solutions
    1. What determines uncertainty in SDs?
2. Stratification of HIWC/HAIC cases
    - According to correlation of IWC & D_{mm}
3. Surfaces in (N_0, λ, μ) phase space for HIWC conditions
4. Occurrence of multiple modes during HIWC/ HAIC
5. Future

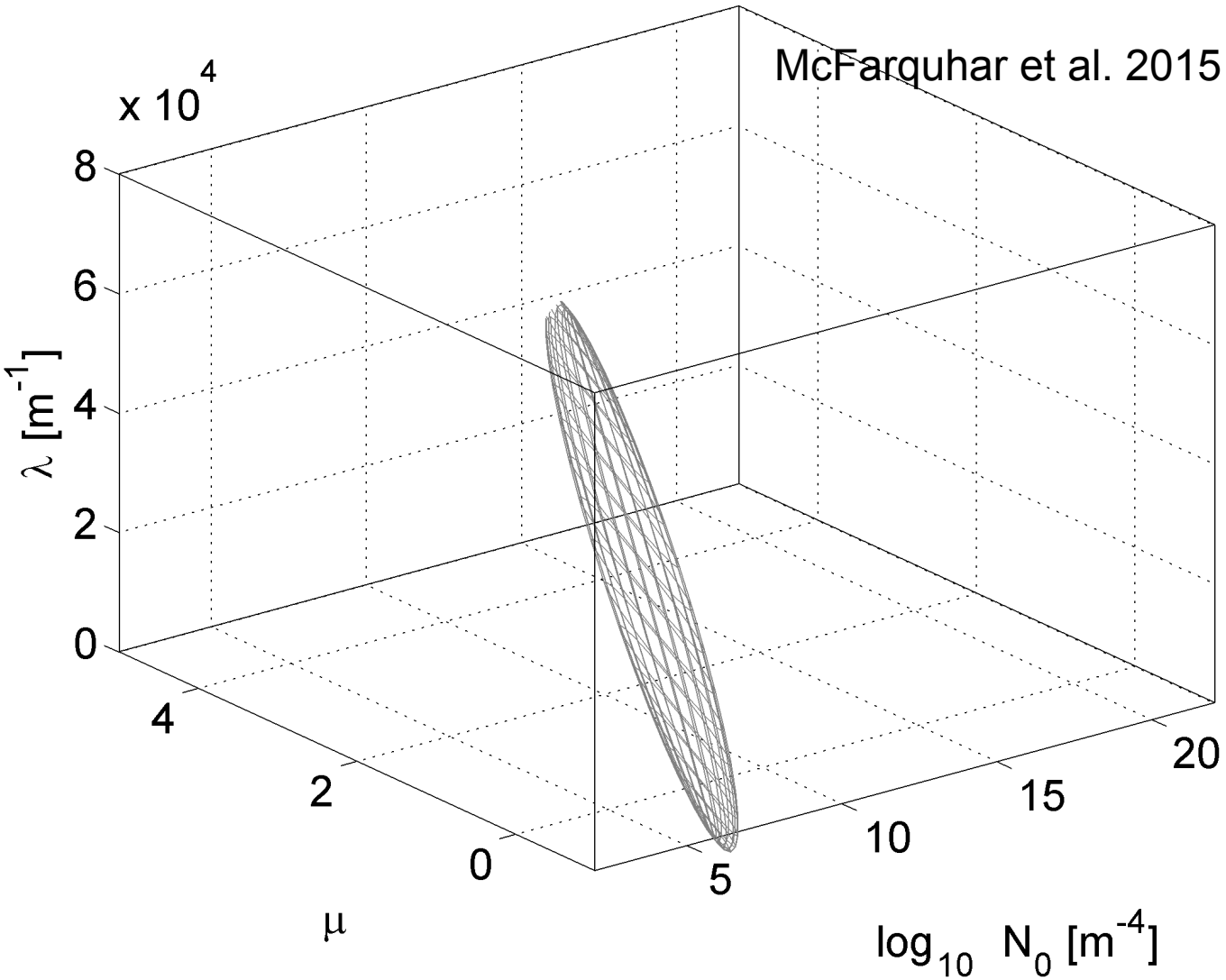
Gamma Functions

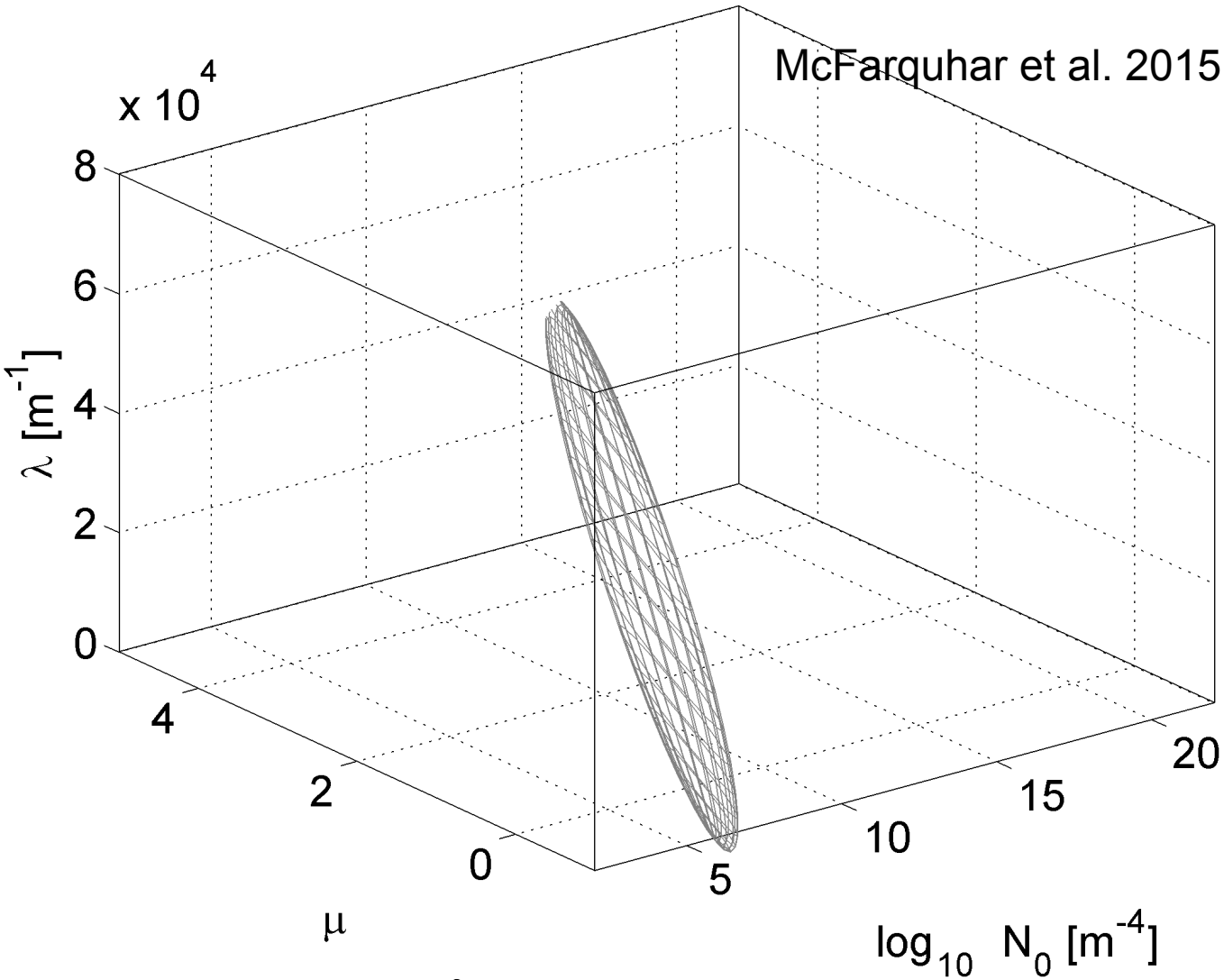
- Gamma functions used to characterize N(D)

$$N(D) = N_0 \; D^{\mu} \exp(-\lambda D)$$

with N_0 intercept, λ slope and μ shape

- N_0, μ, and λ calculated from Incomplete Gamma Fit (IGF) that minimizes χ^2 difference between fit and observed moments
- Any (N_0,μ,λ) within $\Delta\chi^2$ of minimum χ^2 regarded as equally realizable solution
- $\Delta\chi^2$ determined from statistical uncertainty on measured moments on which fit based
- Uncertainty in family of SDs also originates from variability of SDs


McFarquhar et al. 2015
x 10^4
8
6
4
2
0
λ [m^-1]
4
2
0
μ
5
10
15
20
log10 N0 [m^-4]

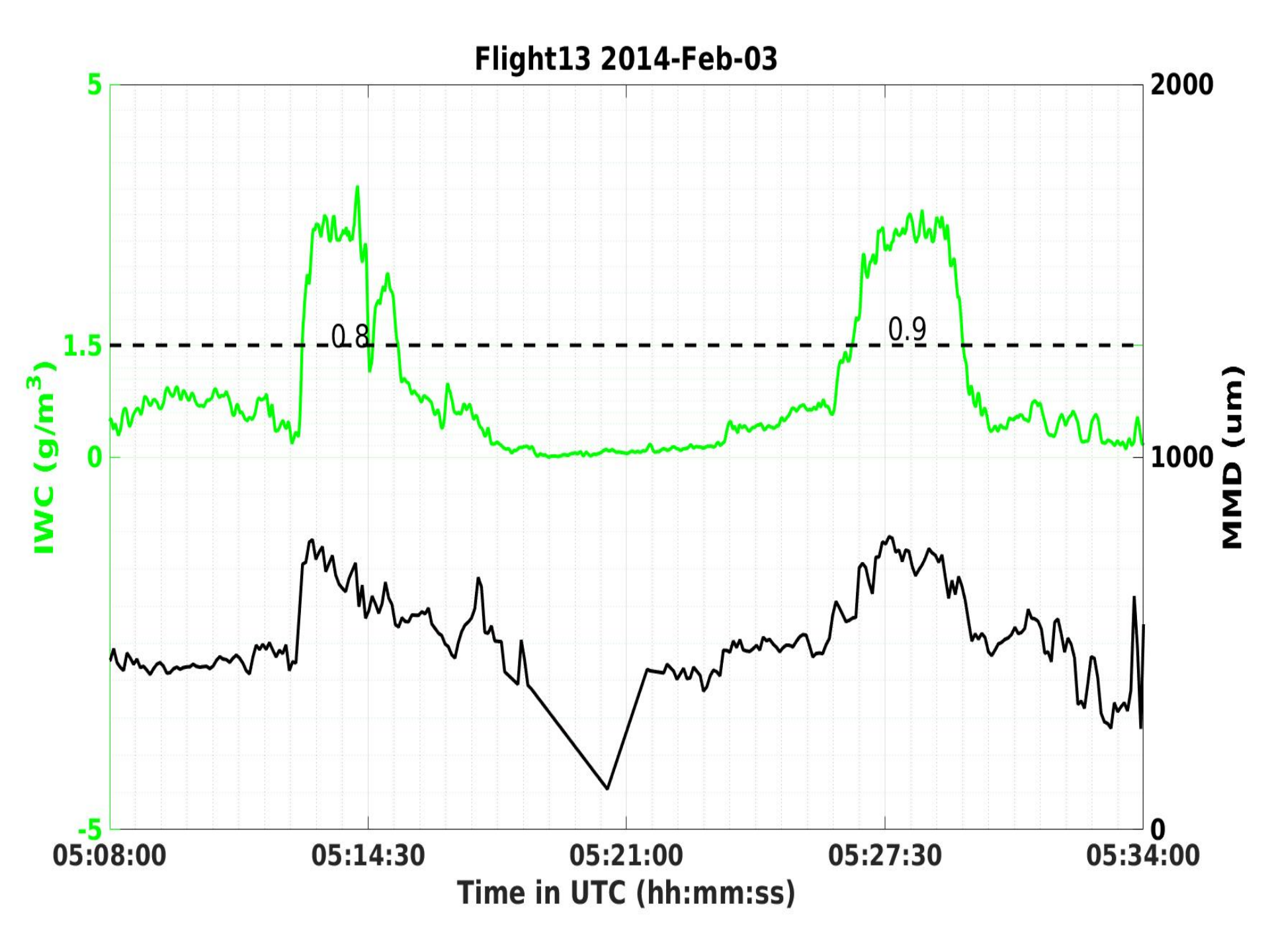


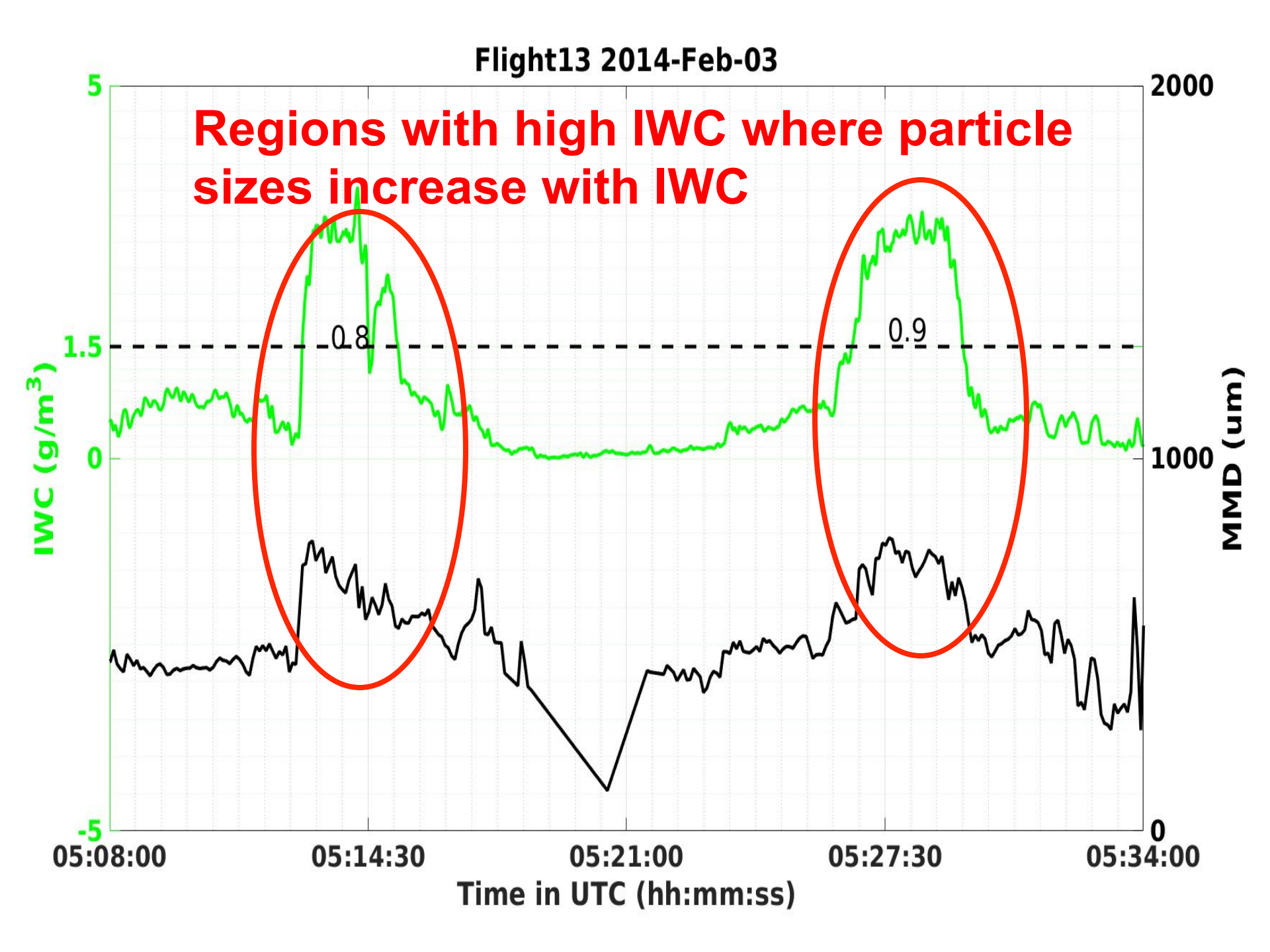
Broad range of $N_0/\mu/\lambda$ that fit family of SDs

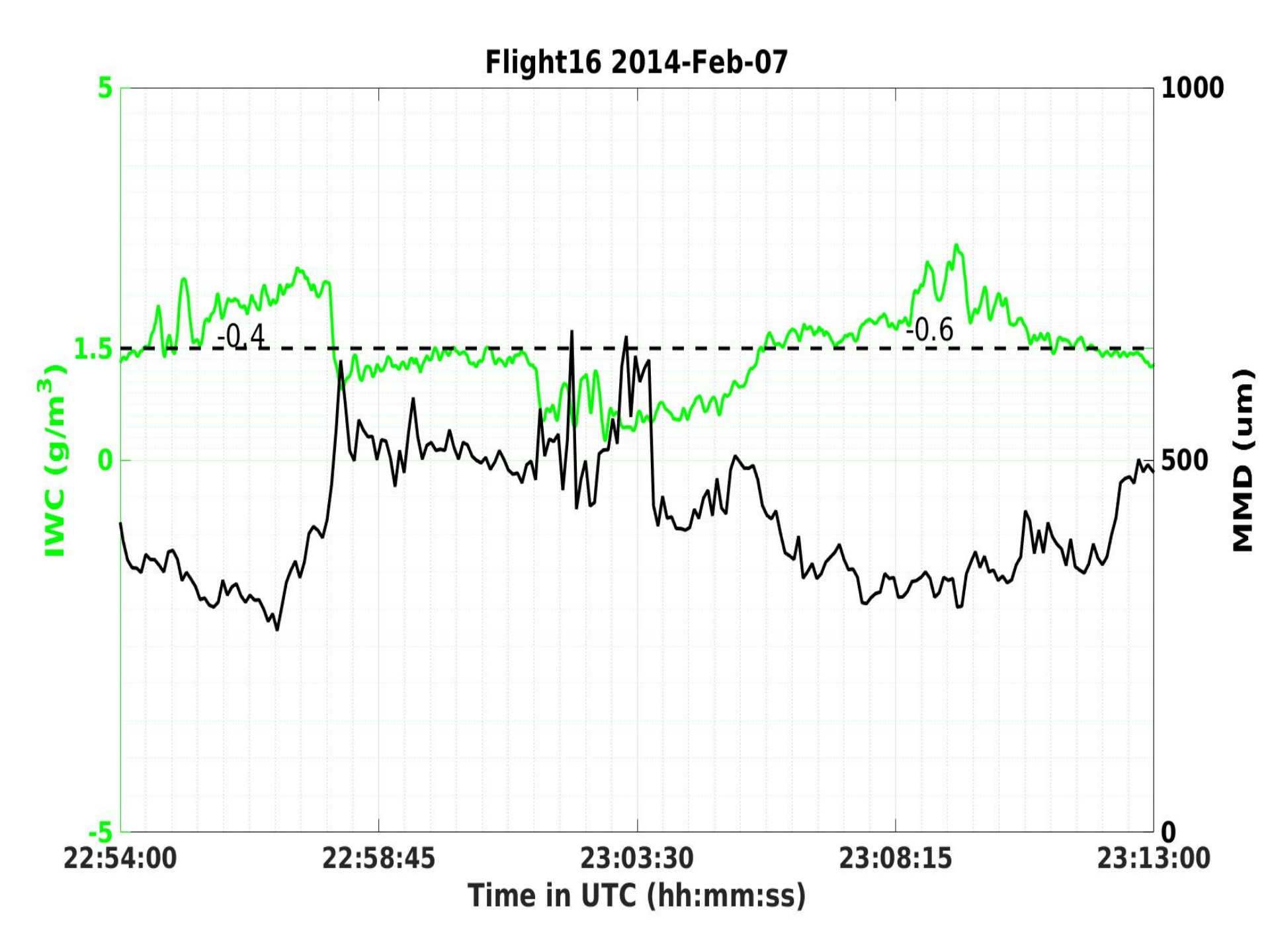
→ How do these volumes vary with environmental parameters

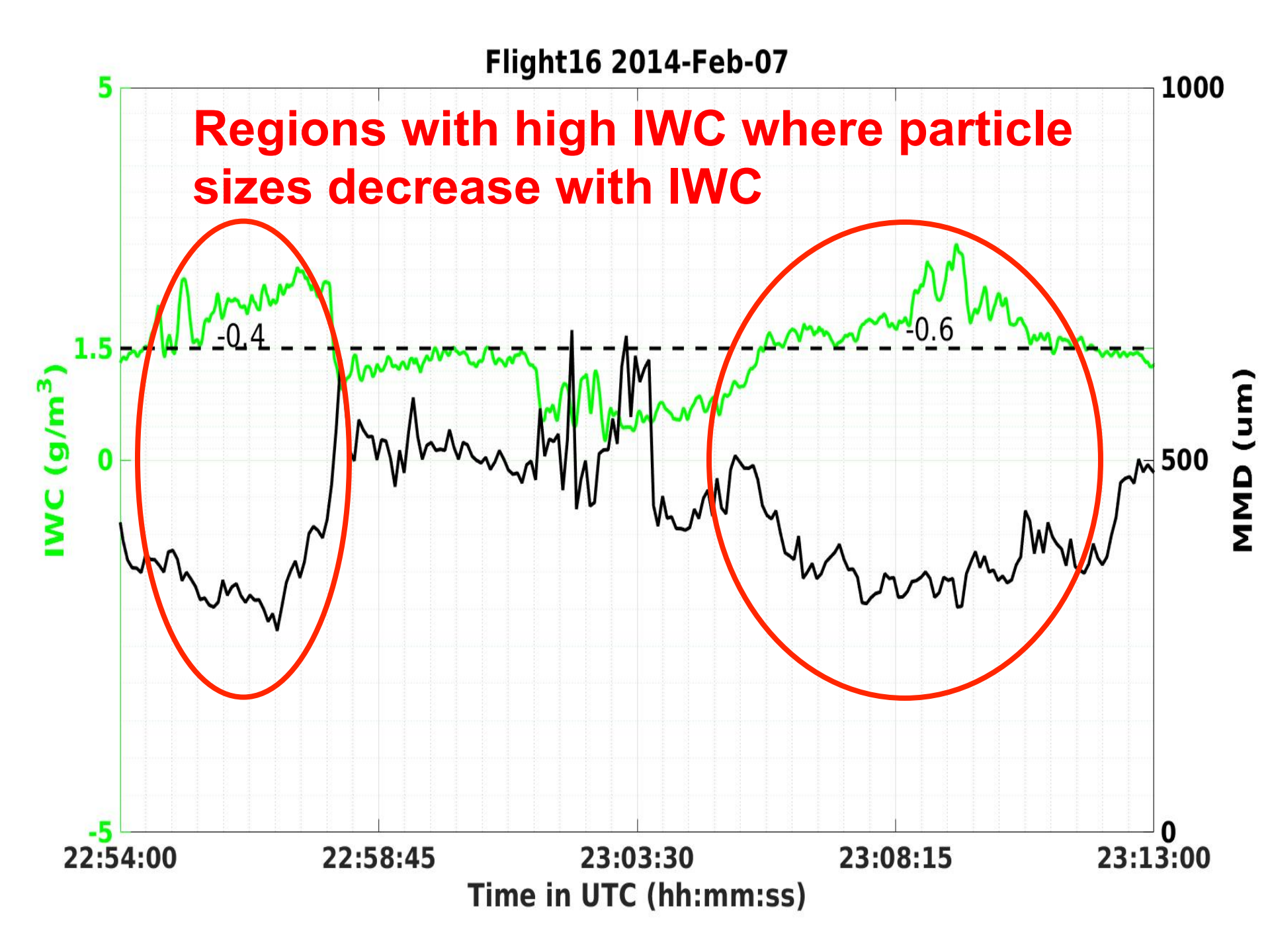
Phase I of HIWC/HAIC

- Phase I measured high IWCs (> 1.5 g m^{-3}) on 19 flight days, representing 12,352 s of data (~ 2280 km)
  - Are SDs in these regions similar to SDs in non high-IWC conditions?
- Leroy et al. (2016) showed D_{mm} decreased in high IWC regions for young convective systems, but D_{mm} could increase for longer-lived cases
  - Do we have to represent SDs differently in these different regions?


Flight13 2014-Feb-03
5
1.5
0
-5
IWC (g/m3)
2000
1000
0
MMD (um)
0.8
0.9
05:08:00
05:14:30
05:21:00
05:27:30
05:34:00
Time in UTC (hh:mm:ss)


Flight13 2014-Feb-03
Regions with high IWC where particle sizes increase with IWC
0.8
0.9
IWC (g/m3)
MMD (um)
5
1.5
0
-5
2000
1000
0
05:08:00
05:14:30
05:21:00
05:27:30
05:34:00
Time in UTC (hh:mm:ss)


Flight16 2014-Feb-07
5
1.5
0
-5
1000
500
0
IWC (g/m3)
MMD (um)
-0.4
-0.6
22:54:00
22:58:45
23:03:30
23:08:15
23:13:00
Time in UTC (hh:mm:ss)


Flight16 2014-Feb-07
Regions with high IWC where particle sizes decrease with IWC
-0.4
-0.6
IWC (g/m3)
MMD (um)
5
1.5
0
-5
1000
500
0
22:54:00
22:58:45
23:03:30
23:08:15
23:13:00
Time in UTC (hh:mm:ss)

HIWC Cases

Flight #	# HIWC Cases	R (IWC,D_{mm})	Age & Lifetime (h)
2	1	-0.65	15 – 21
4	2	-0.4	9 – 14
6	16	-0.31	3 – 10
8	12	-0.42	4 – 14
9	1	0.12	5 – 9
10	12	-0.35	5 – 13
12	11	0.1	15 – 41
13	6	0.55	21 – 41
14	3	-0.29	11 – 24
15	10	-0.37	8 – 19
16	13	-0.41	10 – 17
18	9	0.06	18 – 30
19	3	0.14	8 – 20
22	9	-0.36	10 – 17
23	13	0.45	7 – 11

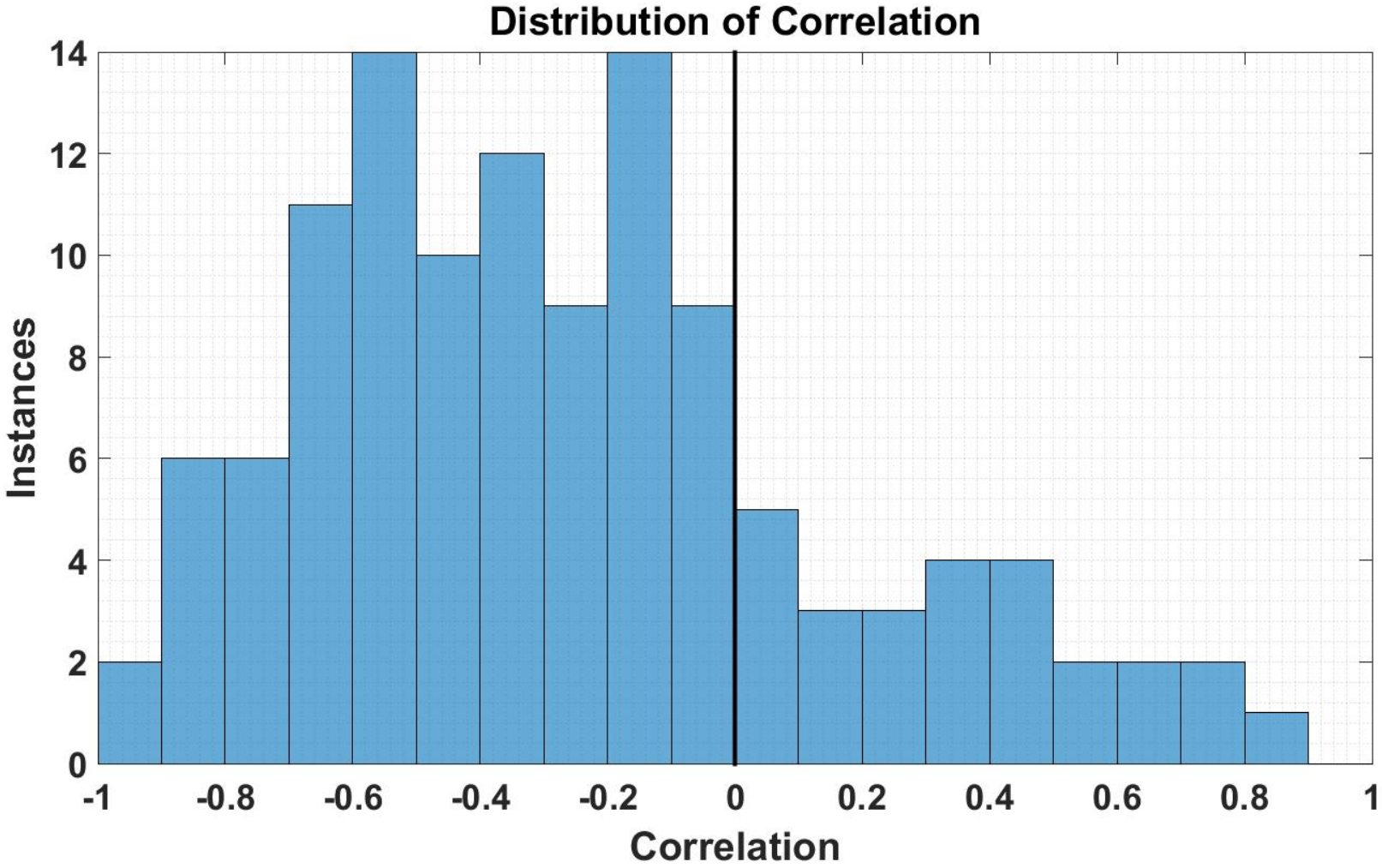
HIWC Cases

Flight #	# HIWC Cases	R (IWC,D_{mm})	Age & Lifetime (h)
2	1	-0.65	15 – 21

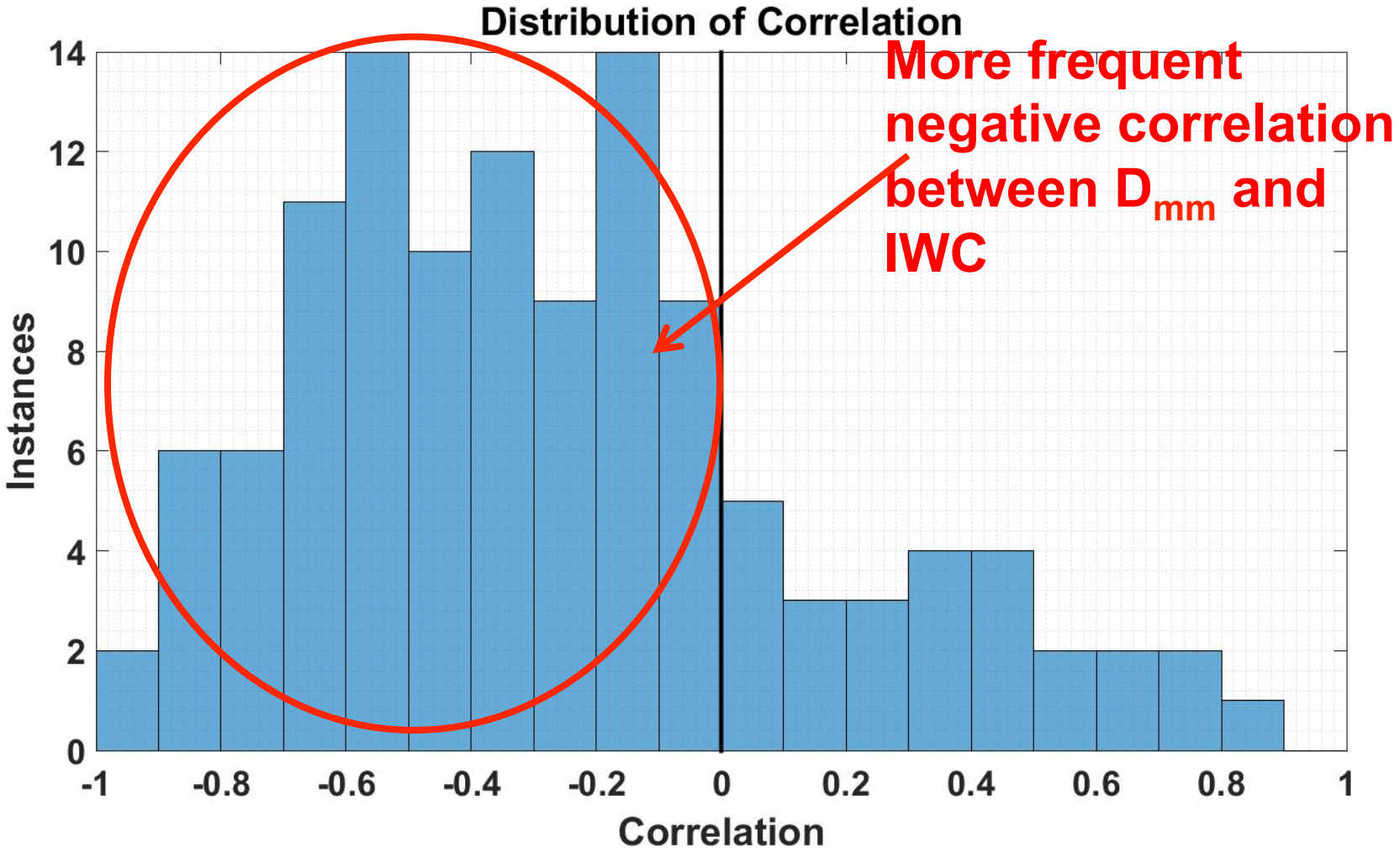
More than just age is causing different behavior, but we haven't figured out yet what causes the difference!

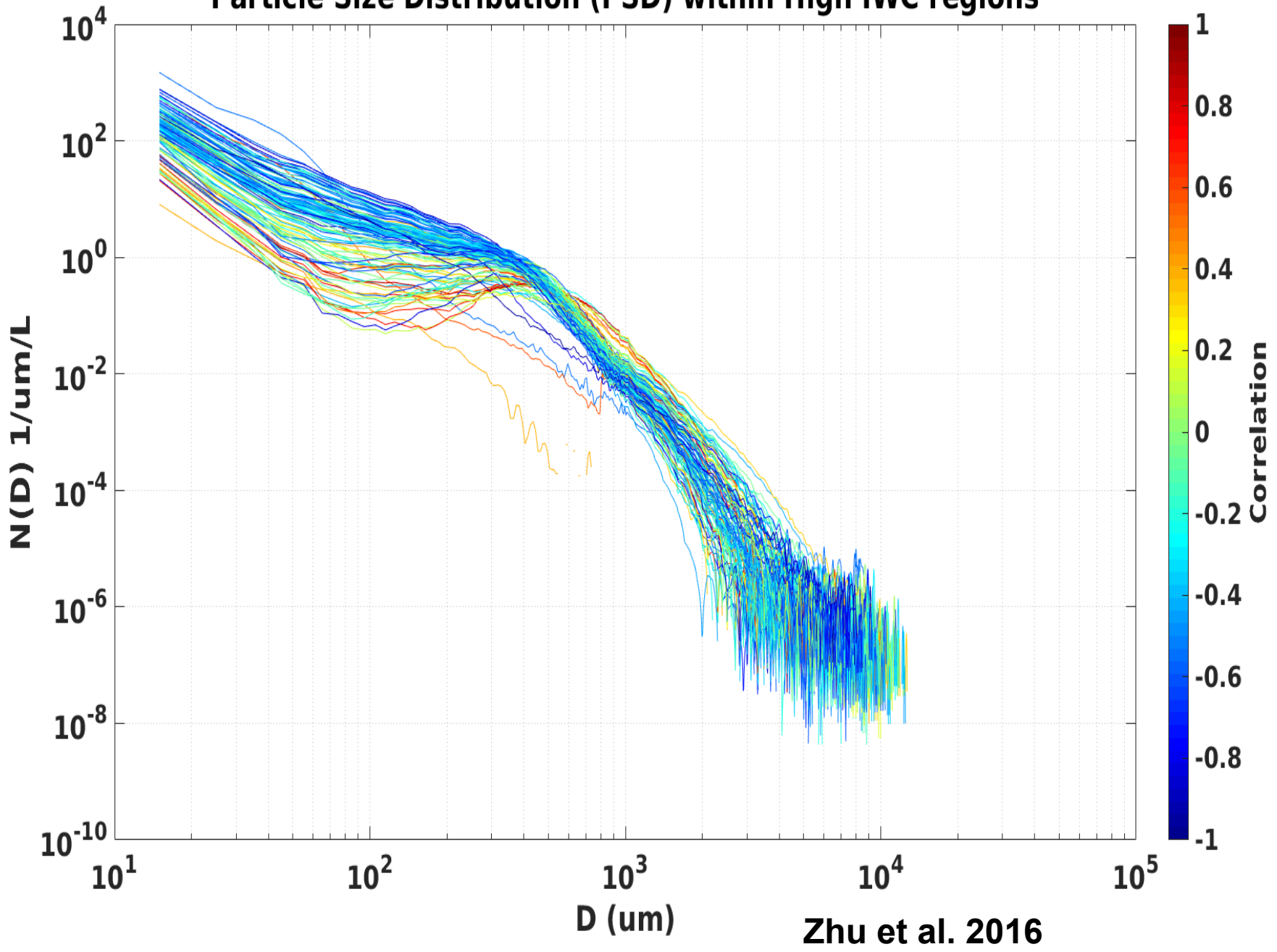
12	11	0.1	15 – 41
13	6	0.55	21 – 41
14	3	-0.29	11 – 24
15	10	-0.37	8 – 19
16	13	-0.41	10 – 17
18	9	0.06	18 – 30
19	3	0.14	8 – 20
22	9	-0.36	10 – 17
23	13	0.45	7 – 11

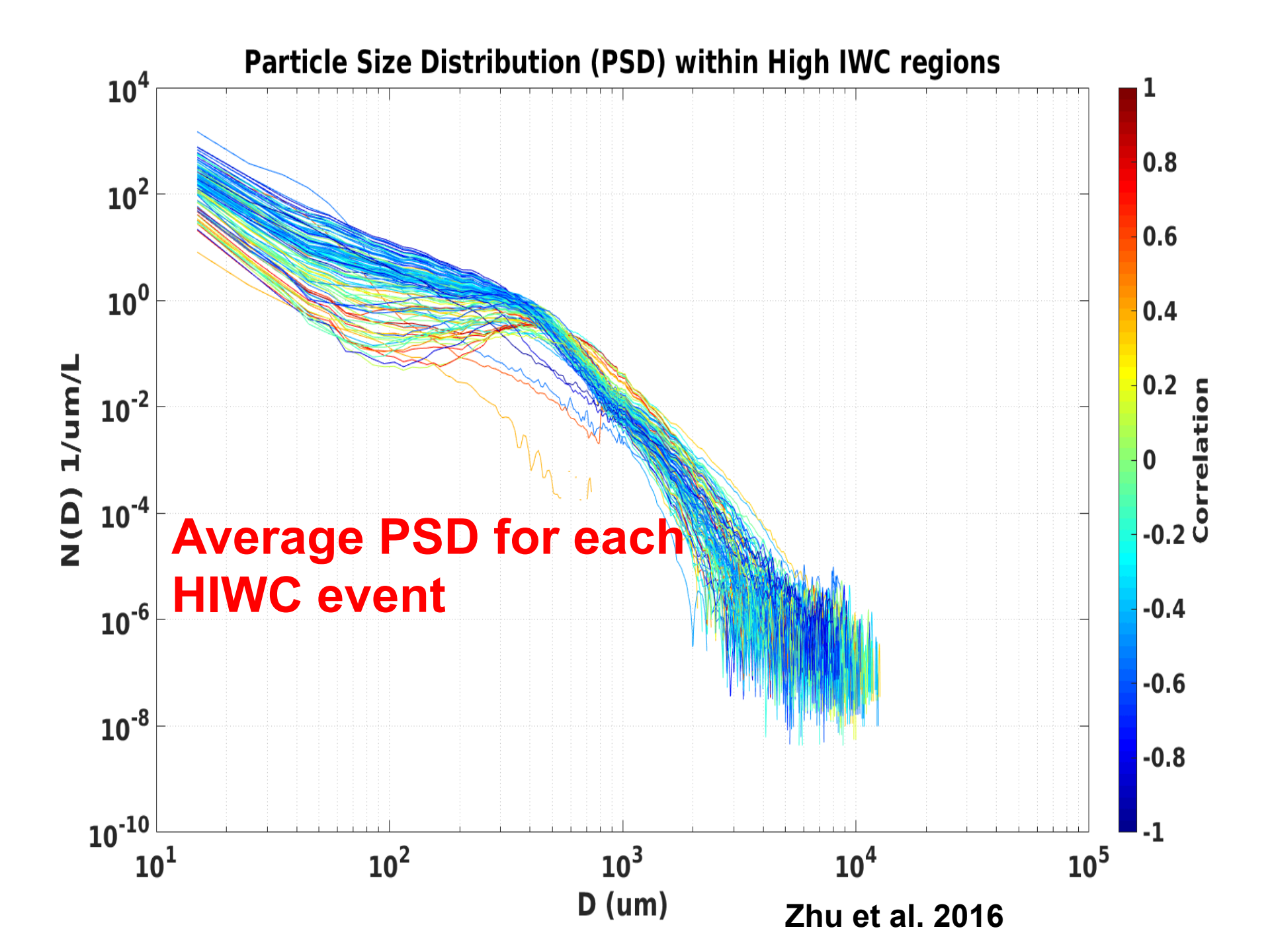
HIWC Cases

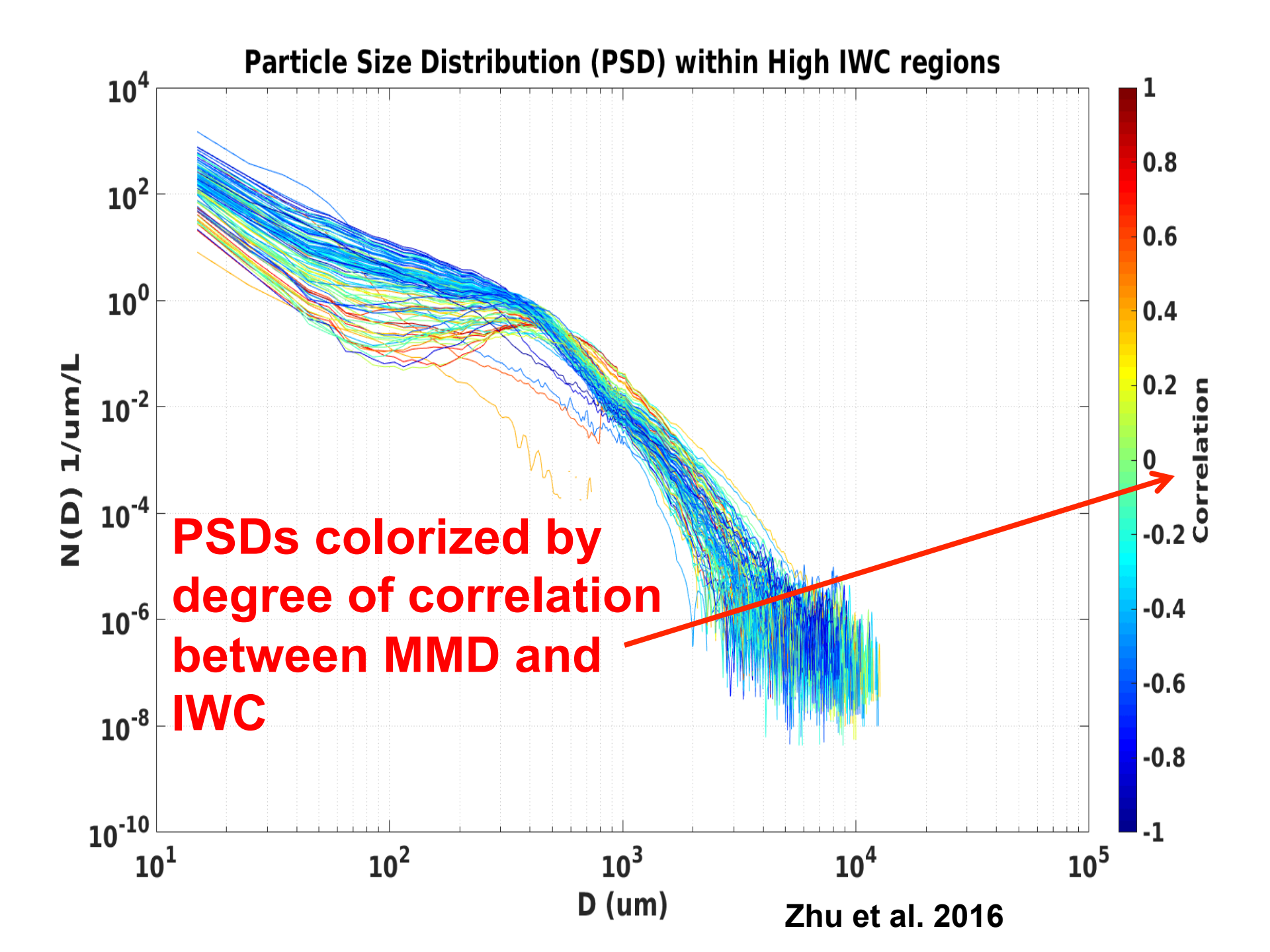

Distribution of Correlation
Instances
Correlation
0
2
4
6
8
10
12
14
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

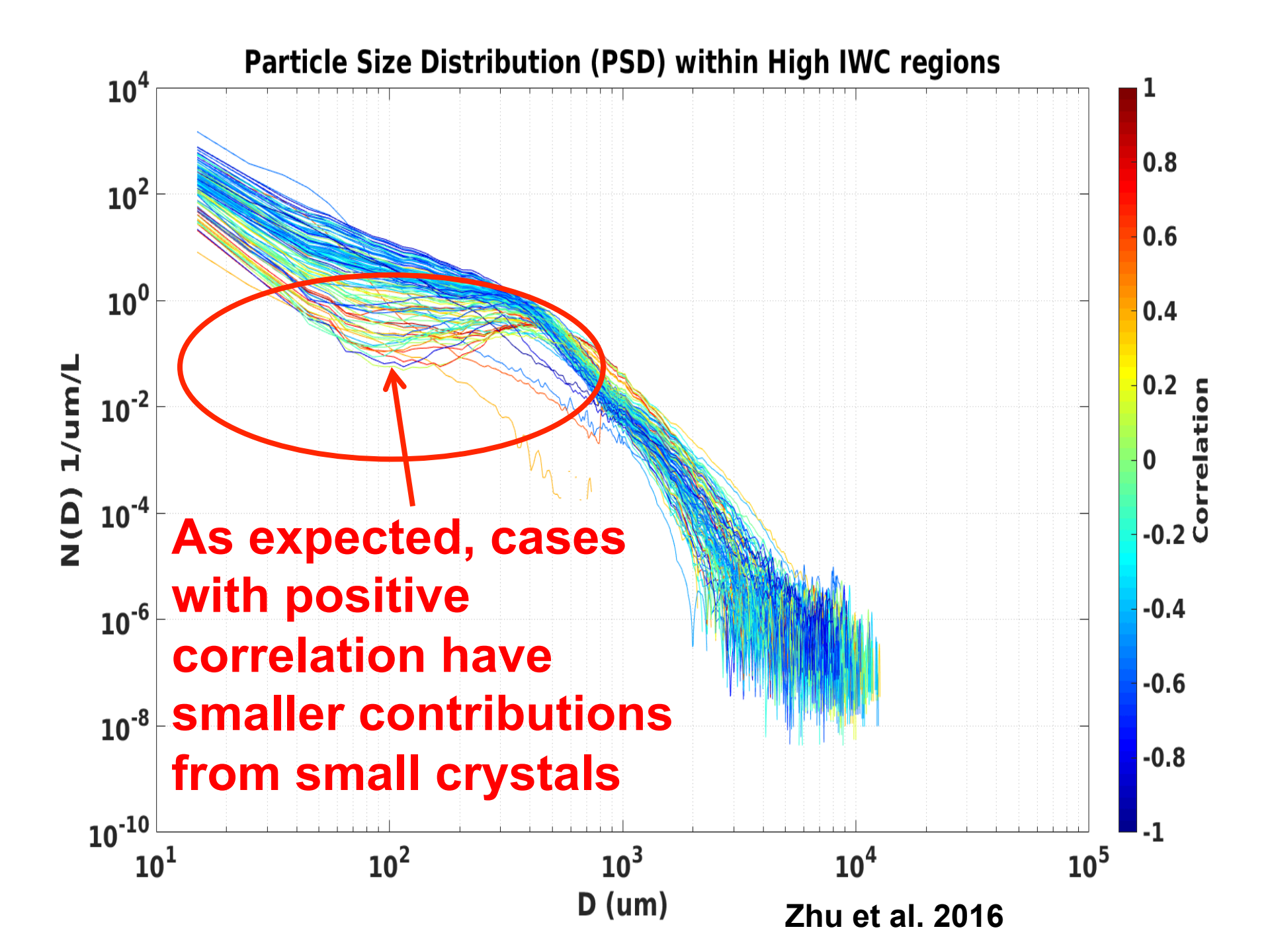
HIWC Cases

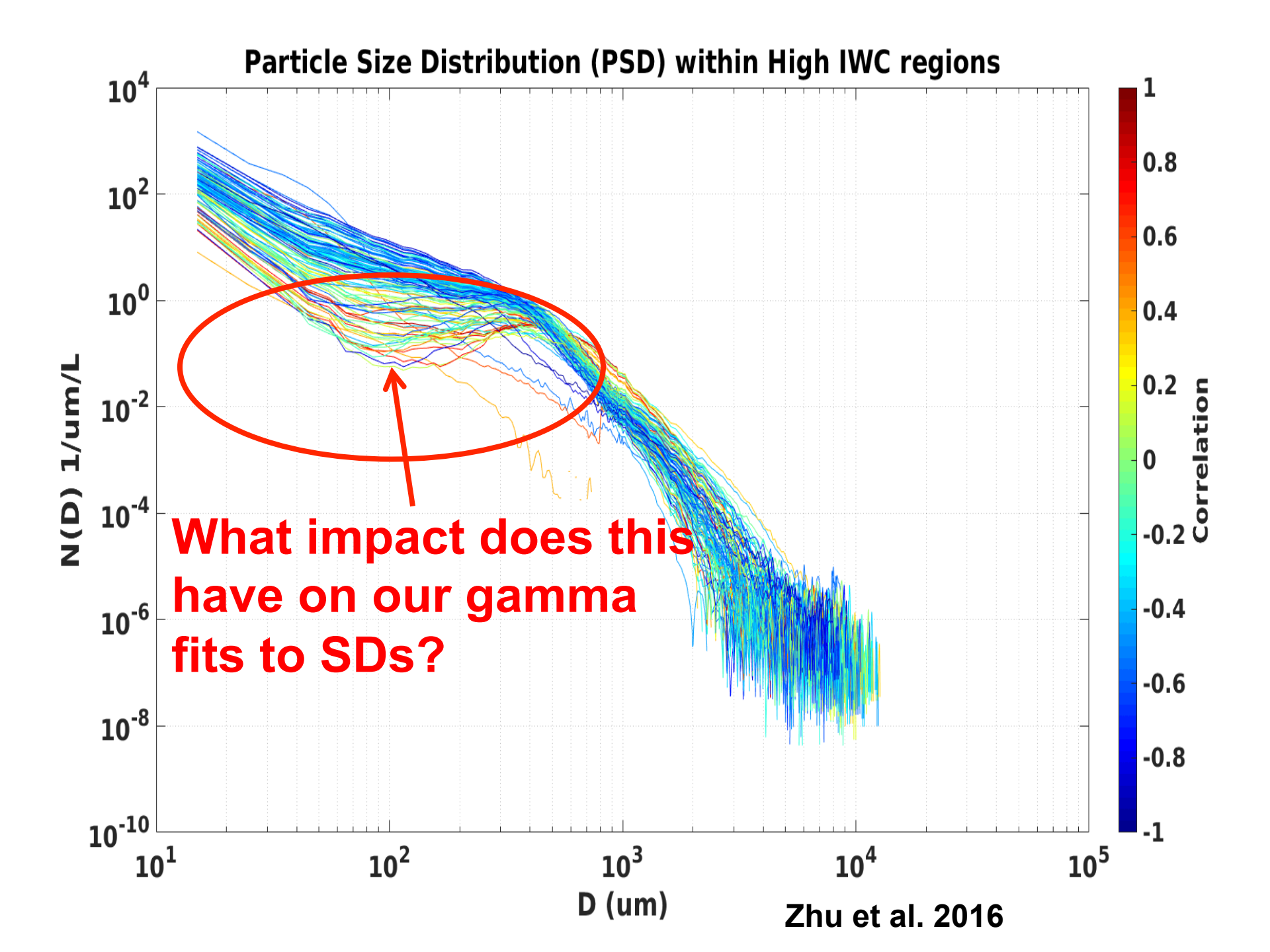

Distribution of Correlation
Instances
Correlation
0
2
4
6
8
10
12
14
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
More frequent negative correlation between D_{mm} and IWC


Particle Size Distribution (PSD) within High IWC regions
N(D) 1/um/L
D (um)
Correlation
Zhu et al. 2016


Particle Size Distribution (PSD) within High IWC regions
N(D) 1/um/L
D (um)
Correlation
Average PSD for each HIWC event
Zhu et al. 2016

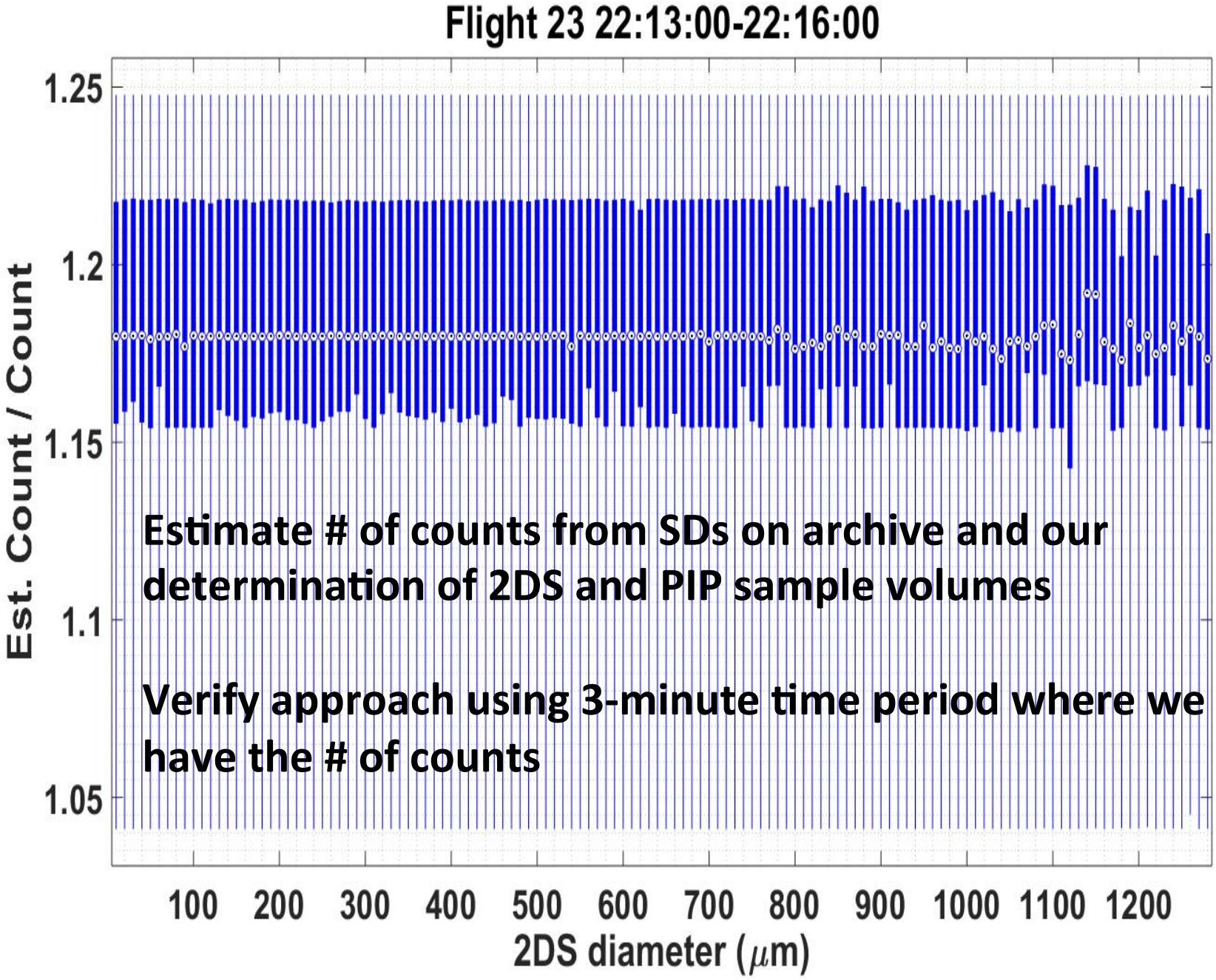

Particle Size Distribution (PSD) within High IWC regions
N(D) 1/um/L
D (um)
Correlation
PSDs colorized by degree of correlation between MMD and IWC
Zhu et al. 2016

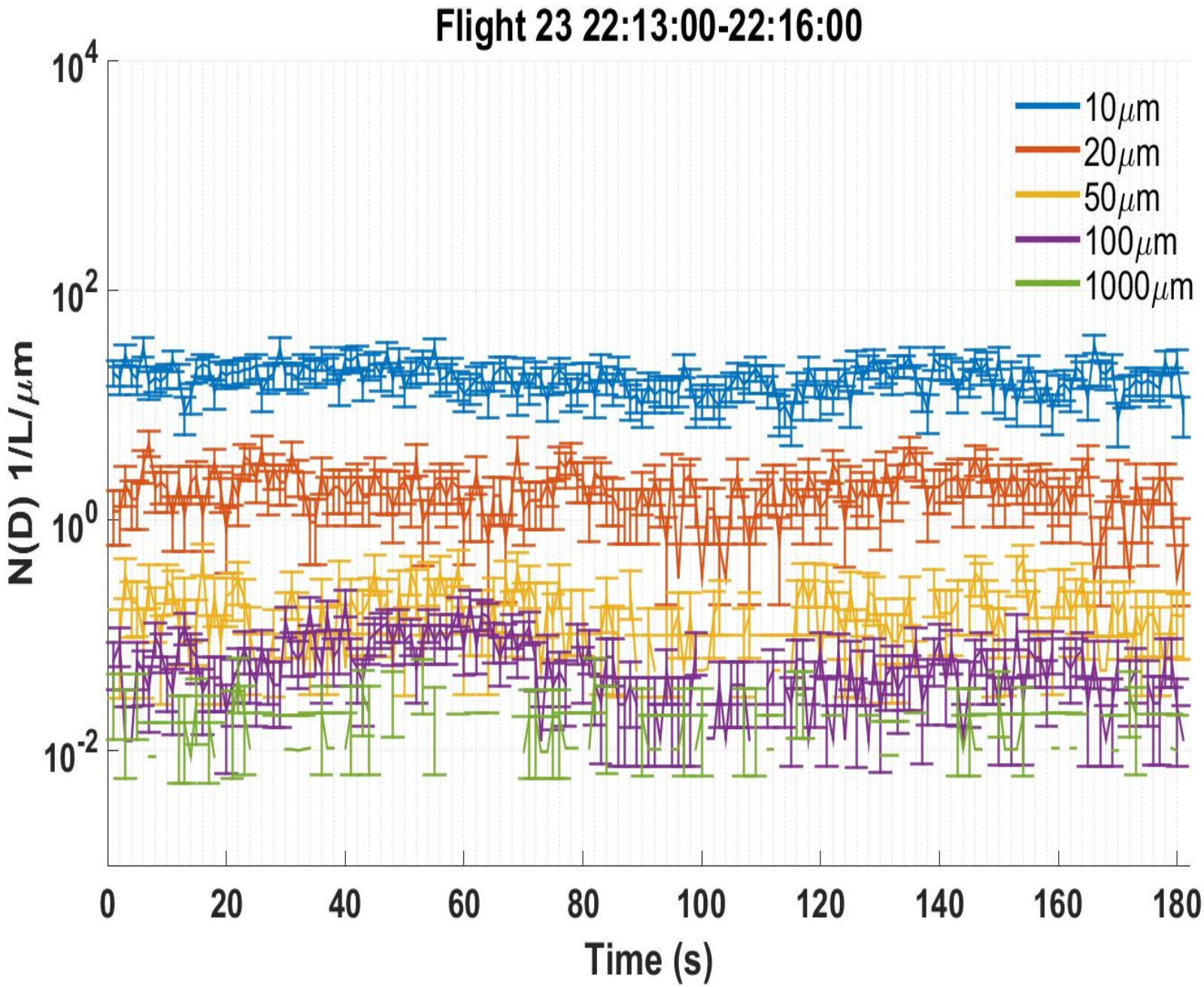

Particle Size Distribution (PSD) within High IWC regions
N(D) 1/um/L
D (um)
Correlation
As expected, cases with positive correlation have smaller contributions from small crystals
Zhu et al. 2016

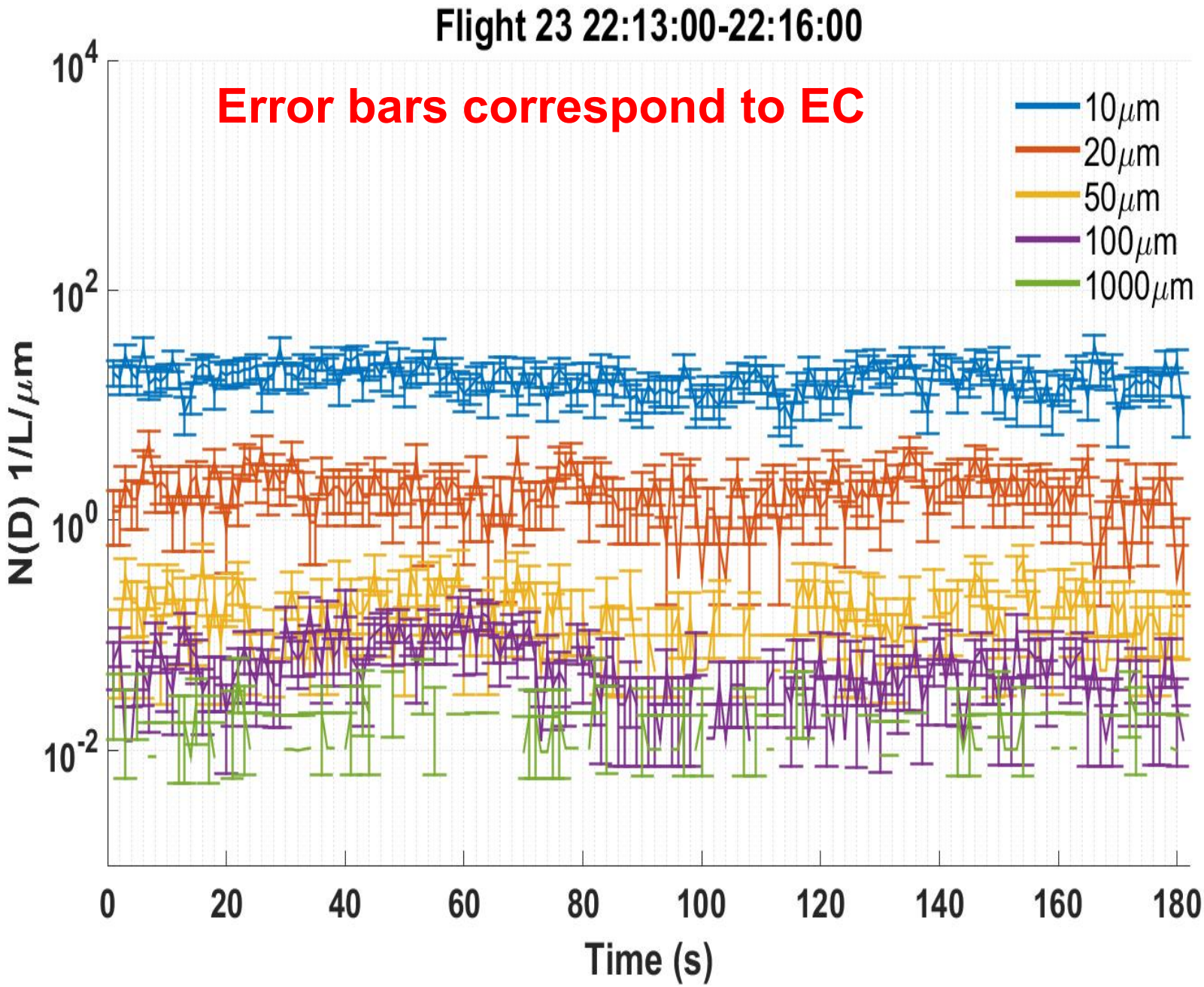

Particle Size Distribution (PSD) within High IWC regions
N(D) 1/um/L
D (um)
Correlation
What impact does this have on our gamma fits to SDs?
Zhu et al. 2016

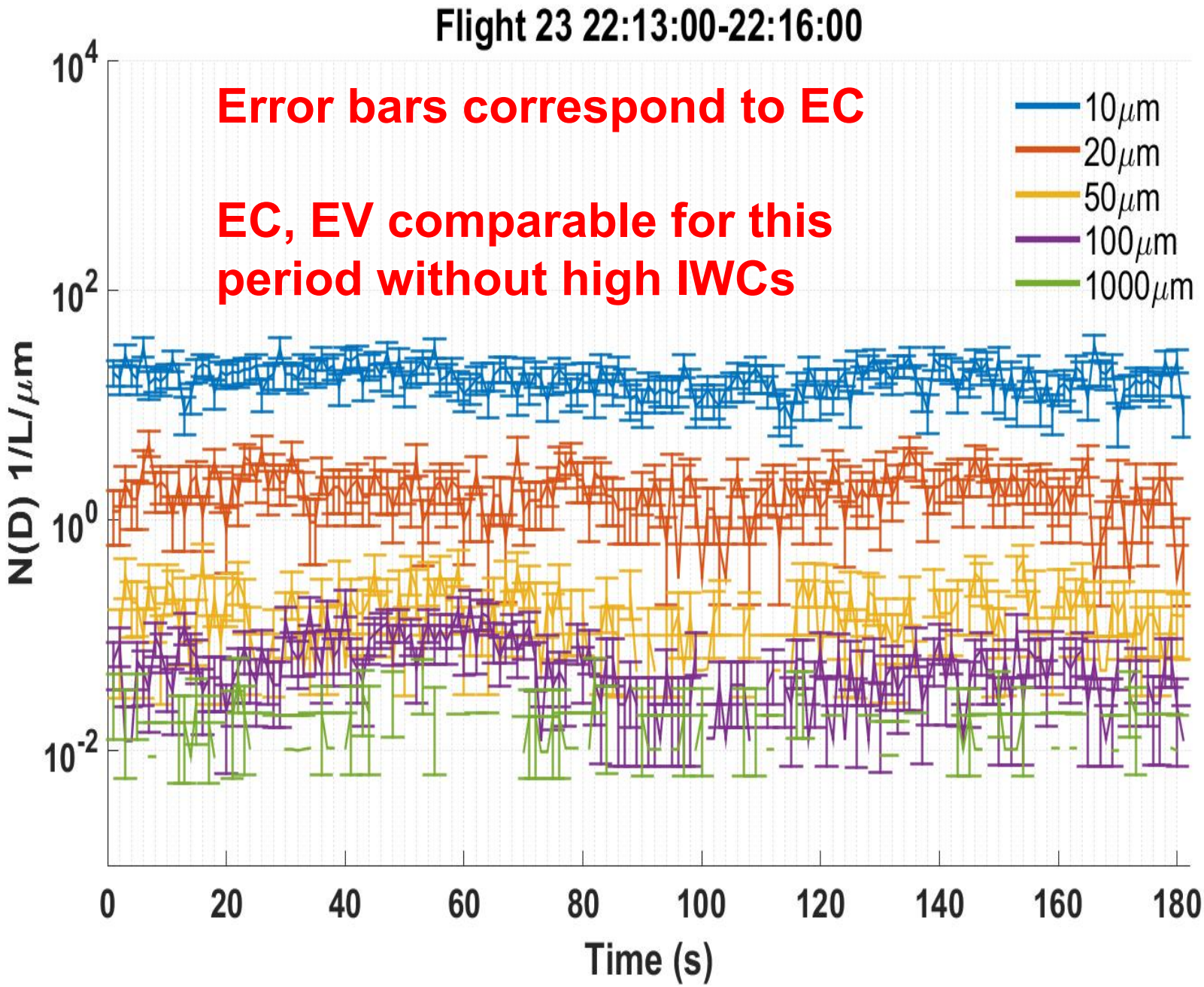
Two Sources of Uncertainty

- EC: Uncertainty in counting statistics proportional to number of counts in each bin ($N^{1/2}$, where N # of counts gives minimum & maximum moments to use in fits)
- EV: Variability in SDs in given conditions (e.g., how much SDs can vary in high IWC conditions)
- McFarquhar et al. (2015) treated both uncertainties in determining volume of solutions in (N_0, λ, μ) phase space
  - Need # of counts to calculate EC

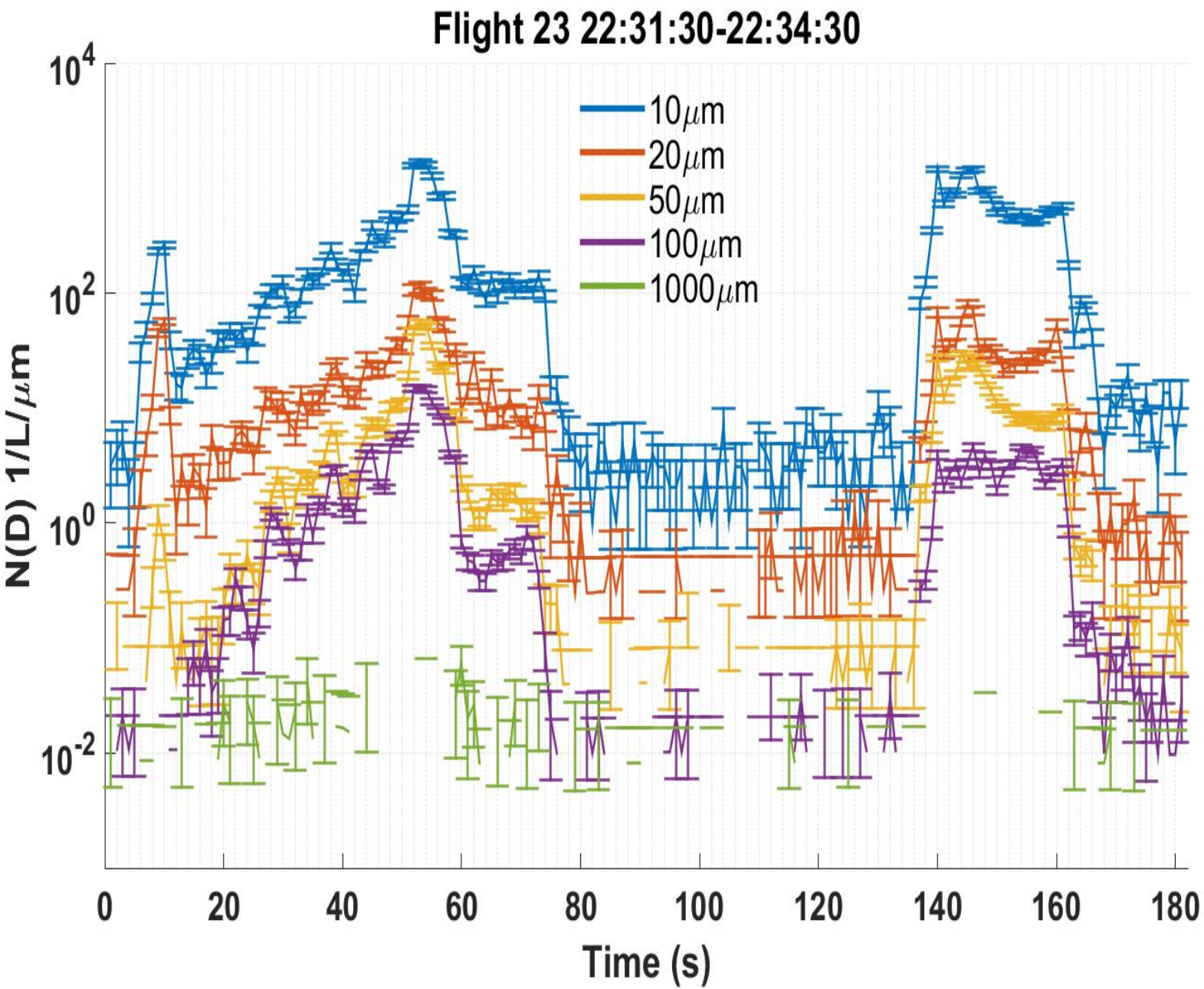

Flight 23 22:13:00-22:16:00
Est. Count / Count
1.25
1.2
1.15
1.1
1.05
100
200
300
400
500
600
700
800
900
1000
1100
1200
2DS diameter (μm)
Estimate # of counts from SDs on archive and our determination of 2DS and PIP sample volumes
Verify approach using 3-minute time period where we have the # of counts

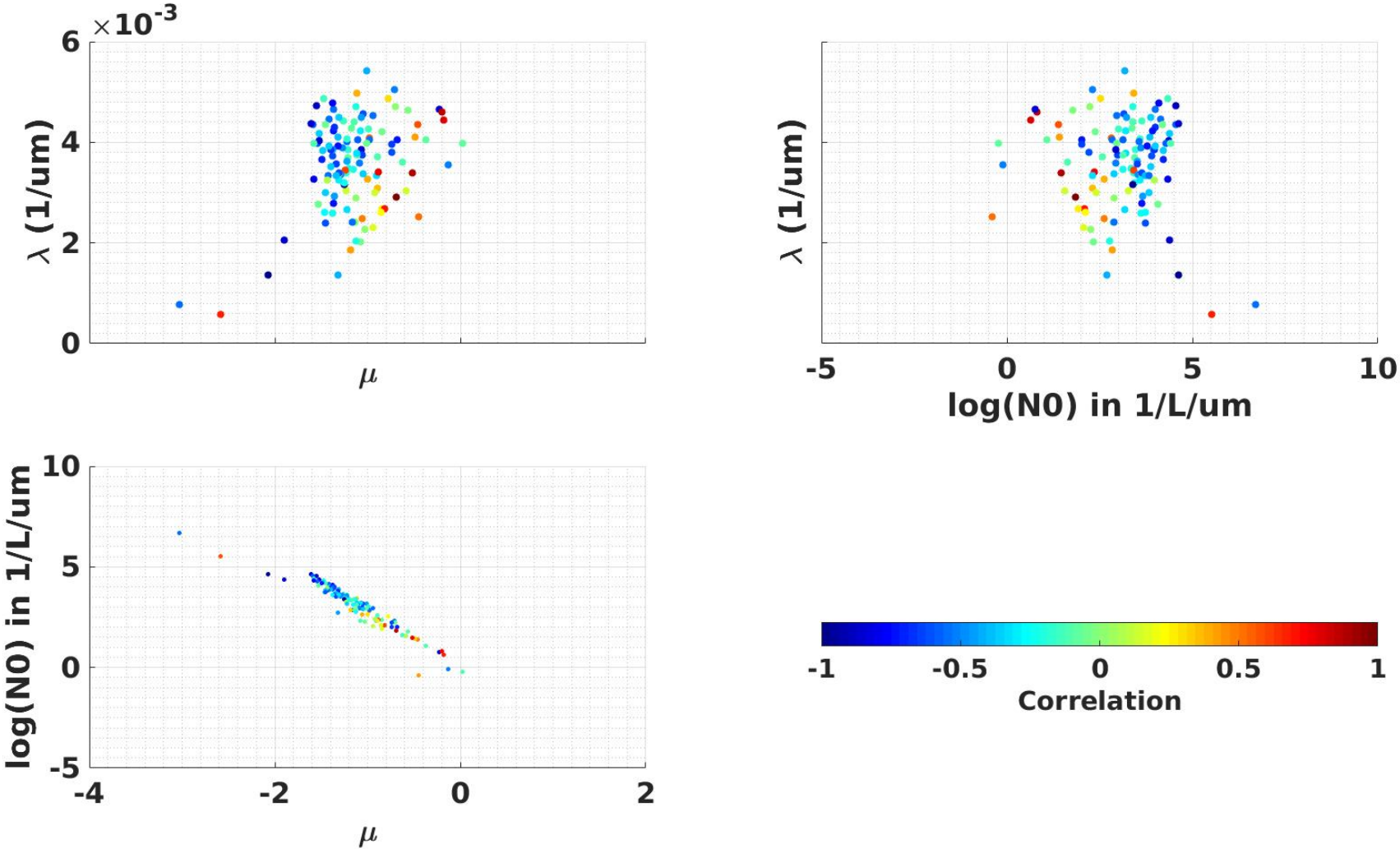

Flight 23 22:13:00-22:16:00
10μm
20μm
50μm
100μm
1000μm
N(D) 1/L/μm
Time (s)
10^4
10^2
10^0
10^-2
0
20
40
60
80
100
120
140
160
180


Flight 23 22:13:00-22:16:00
Error bars correspond to EC
10μm
20μm
50μm
100μm
1000μm
N(D) 1/L/μm
10^4
10^2
10^0
10^-2
0
20
40
60
80
100
120
140
160
180
Time (s)

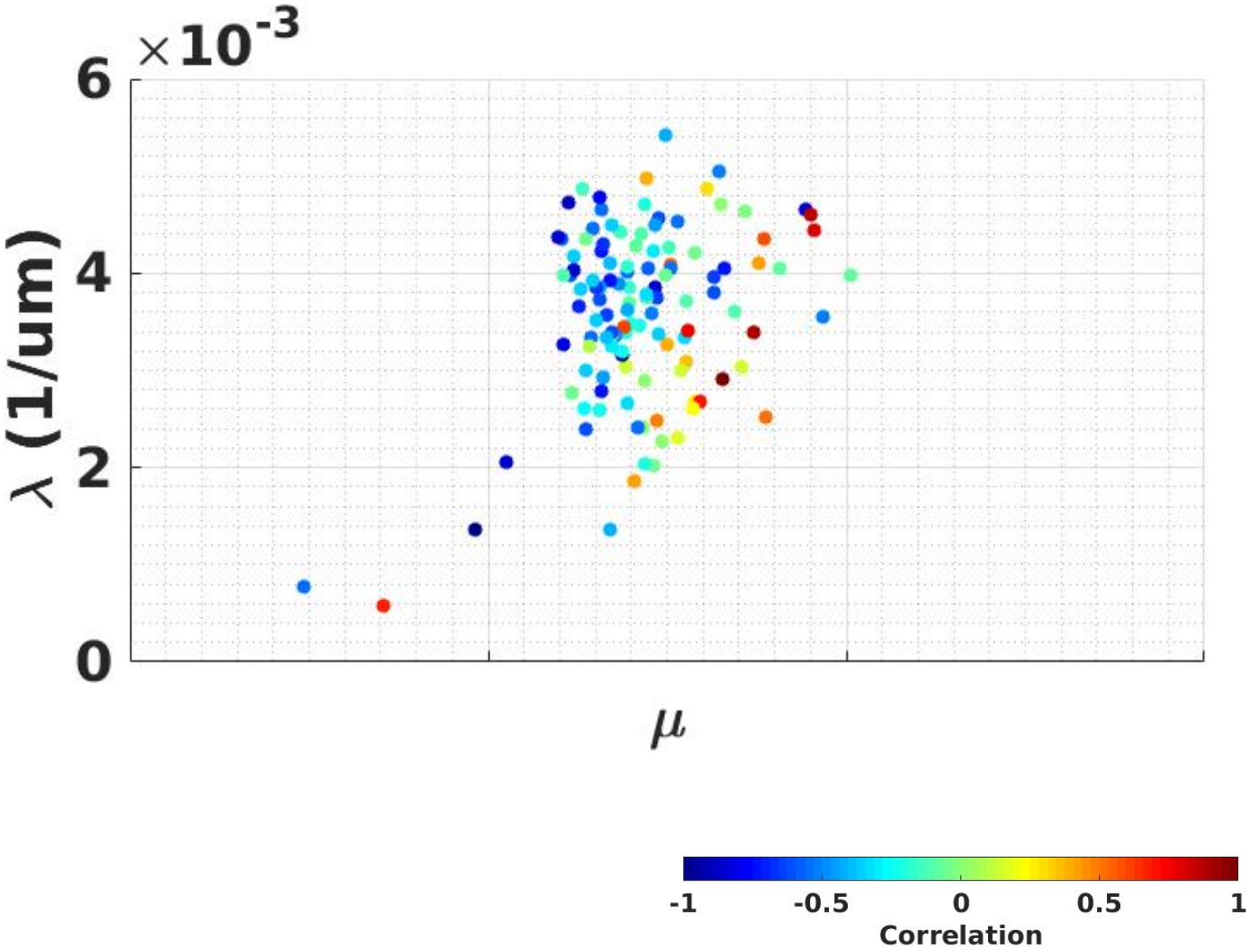

Flight 23 22:13:00-22:16:00
Error bars correspond to EC
EC, EV comparable for this period without high IWCs
10μm
20μm
50μm
100μm
1000μm
N(D) 1/L/μm
10^4
10^2
10^0
10^-2
0
20
40
60
80
100
120
140
160
180
Time (s)

But, EC smaller than EV for period with high IWC

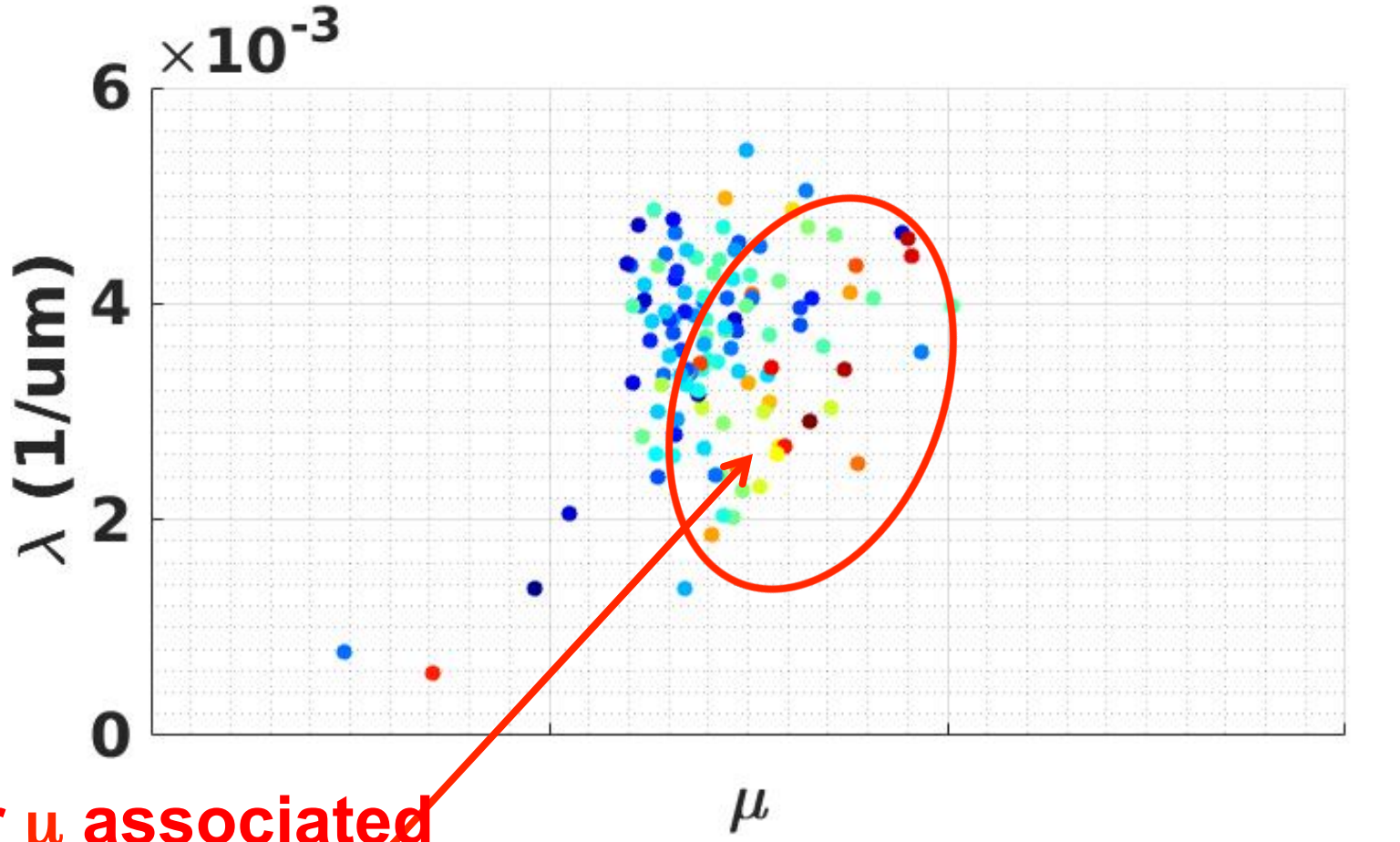




Zhu et al. 2016

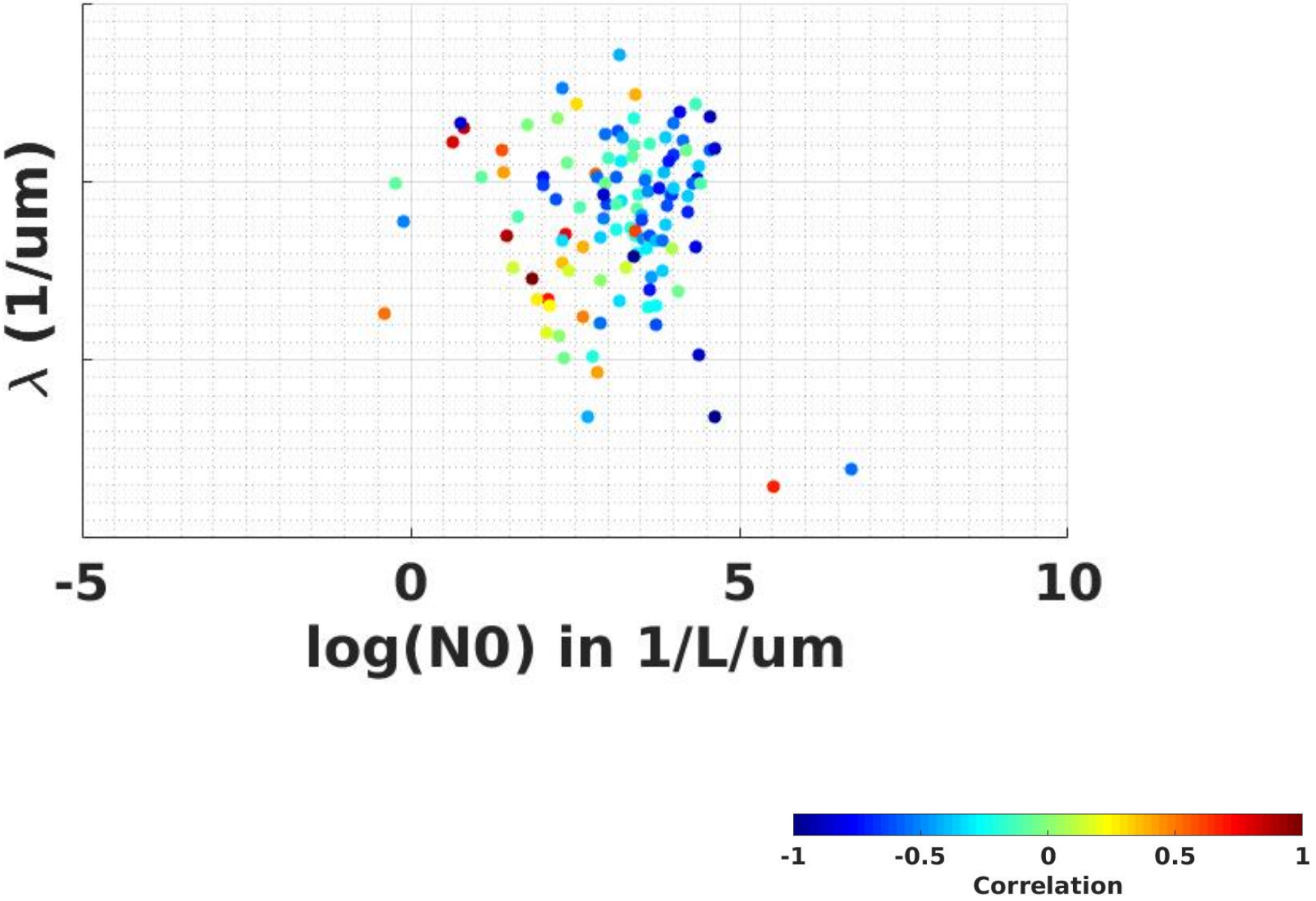


Zhu et al. 2016

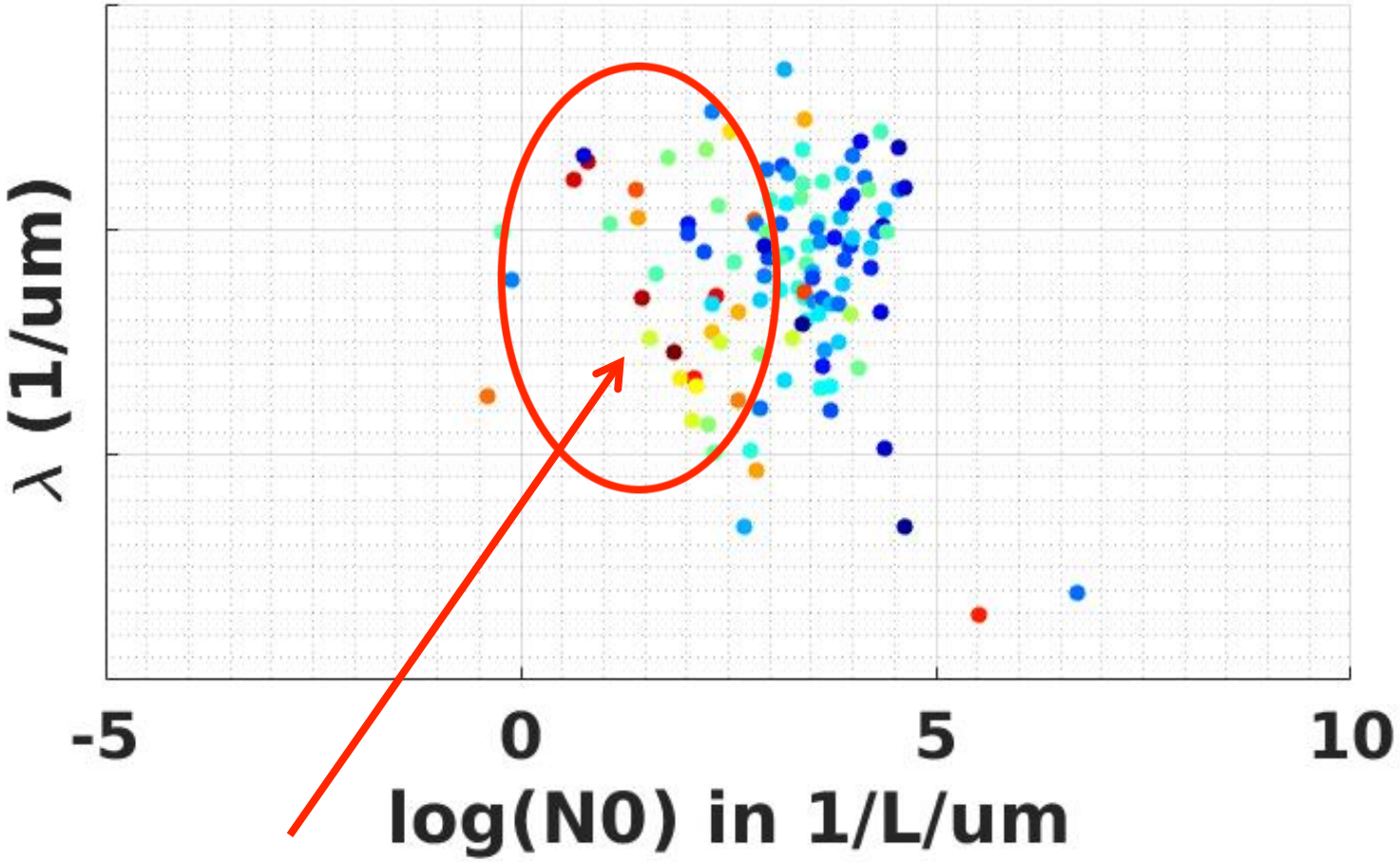


Larger μ associated with points with positive correlation between IWC and MMD

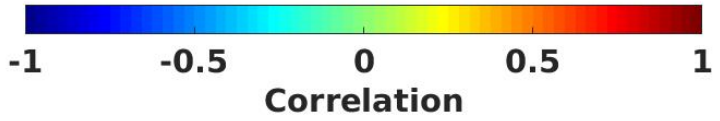
Zhu et al. 2016



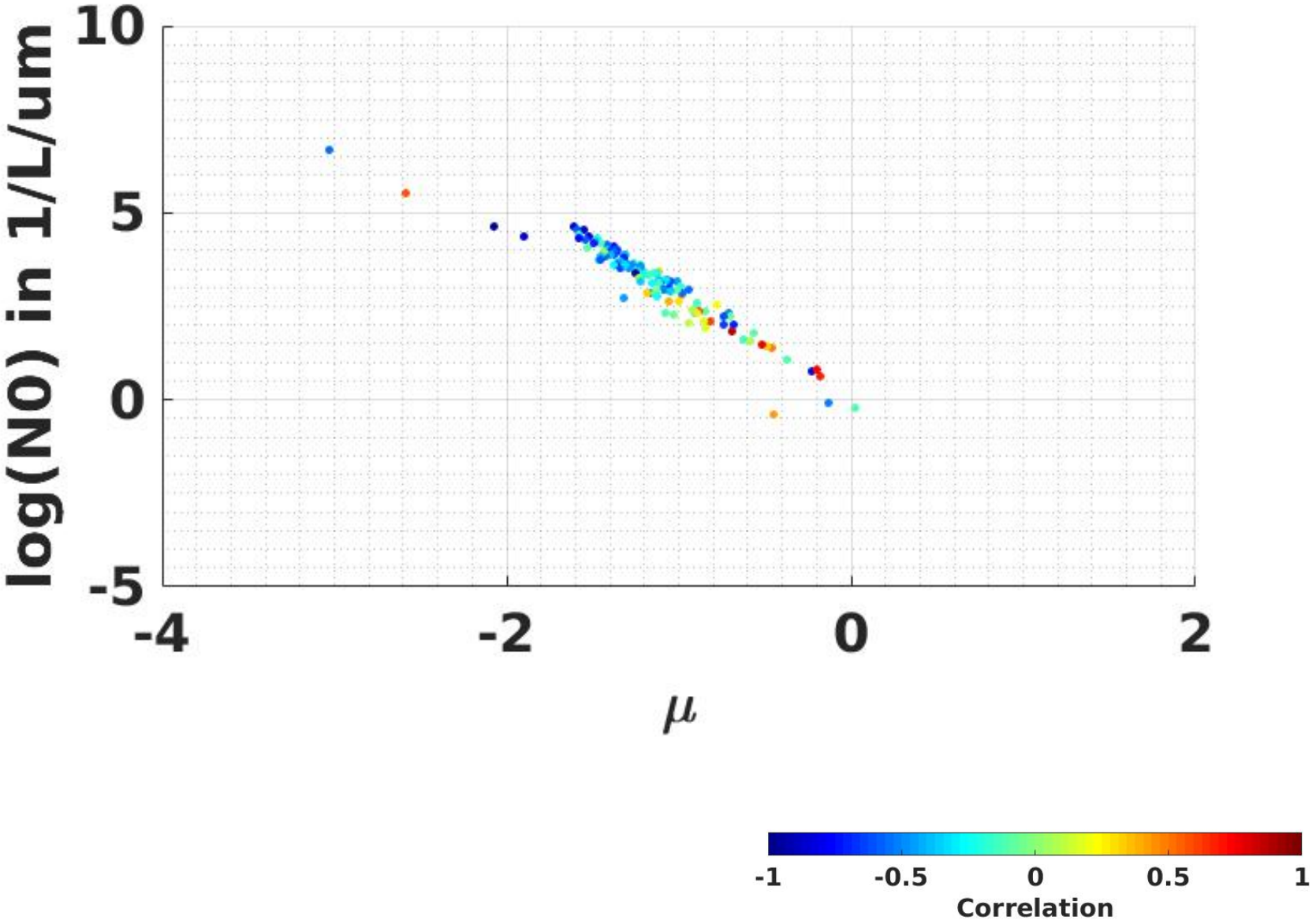
Zhu et al. 2016



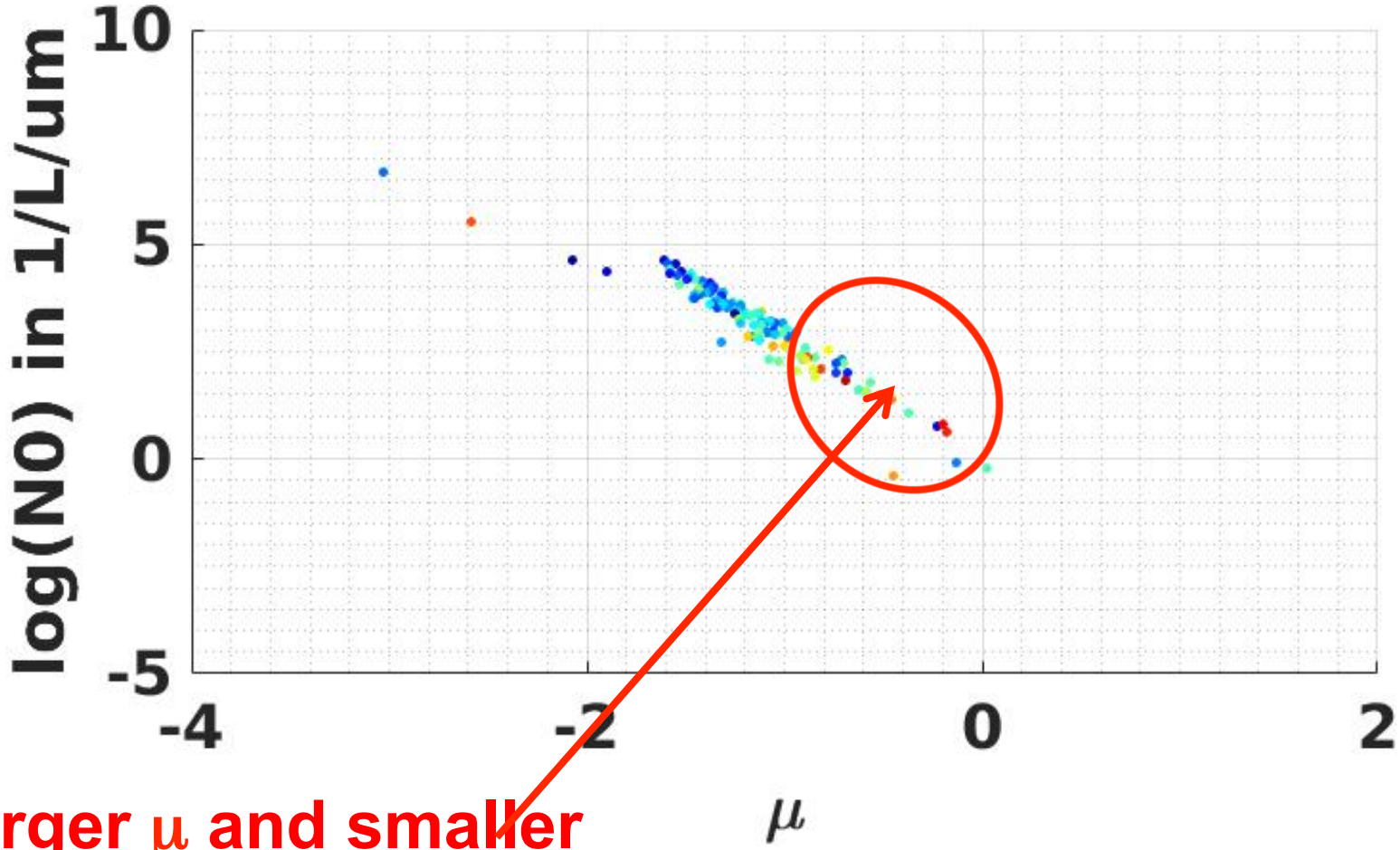
Smaller N_0 associated with points with positive correlation between IWC and MMD



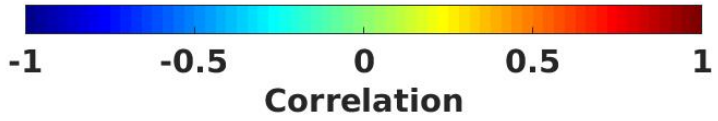
Zhu et al. 2016



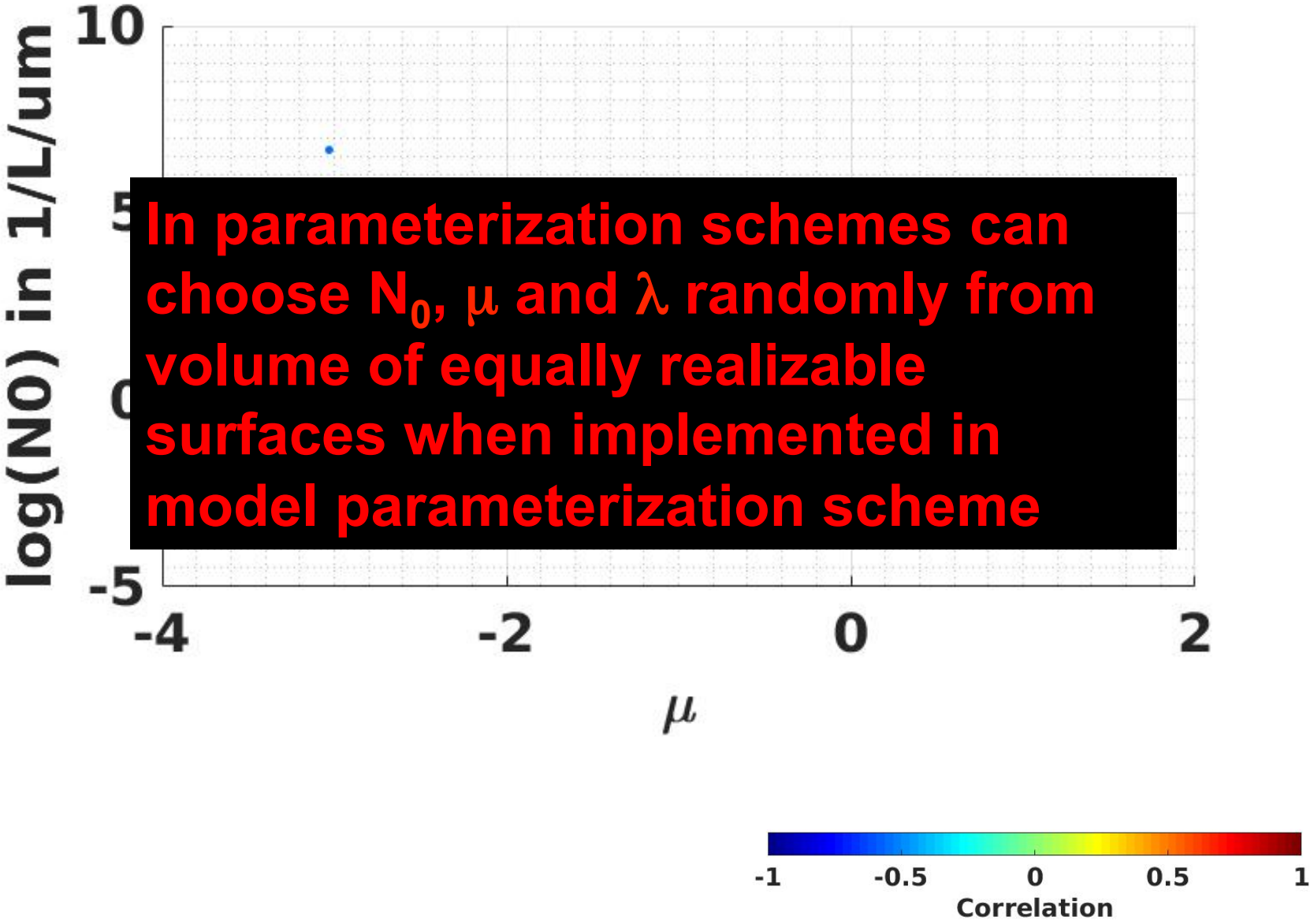
Zhu et al. 2016



Larger μ and smaller N_0 associated with points with positive correlation between IWC and MMD

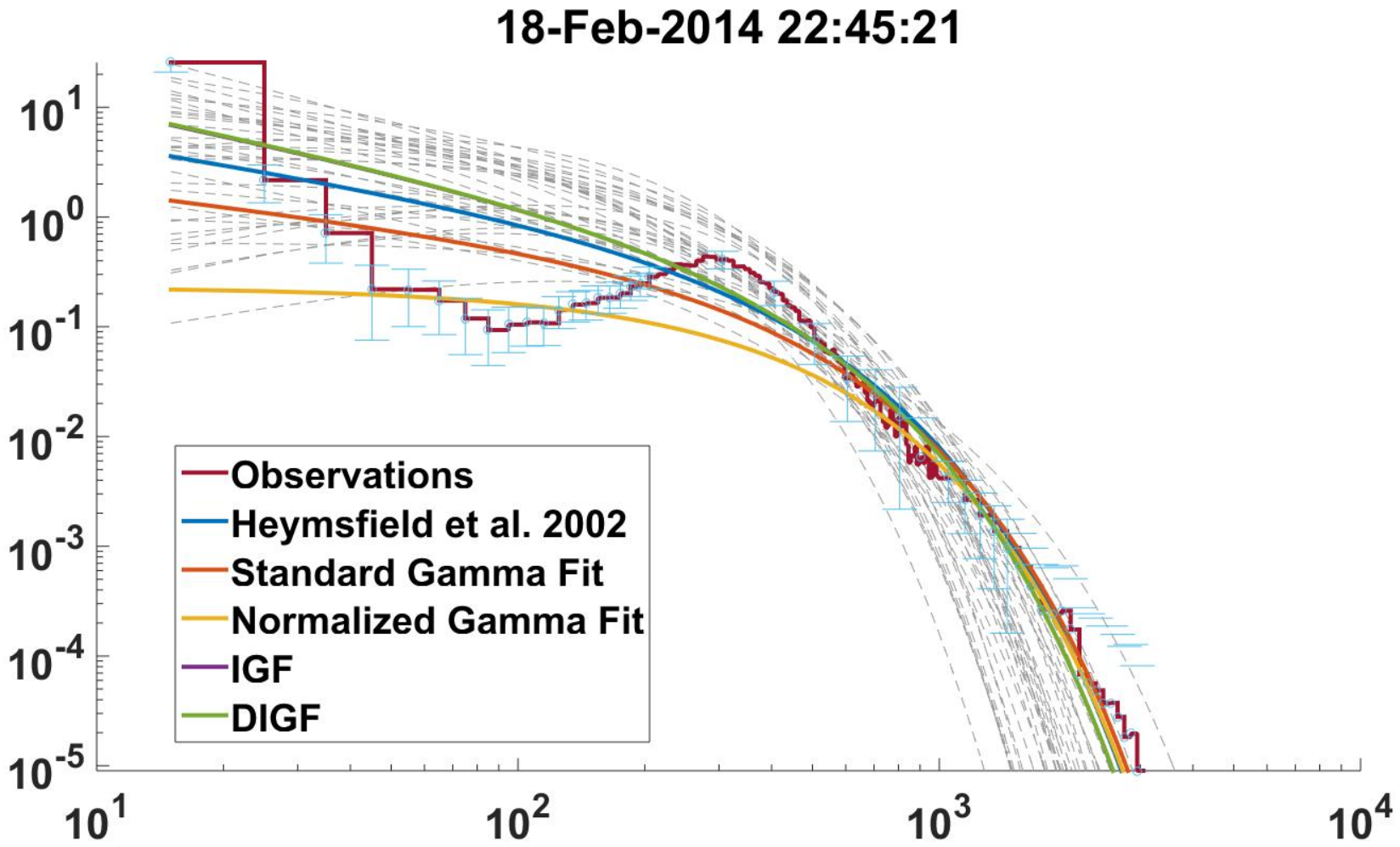


Zhu et al. 2016



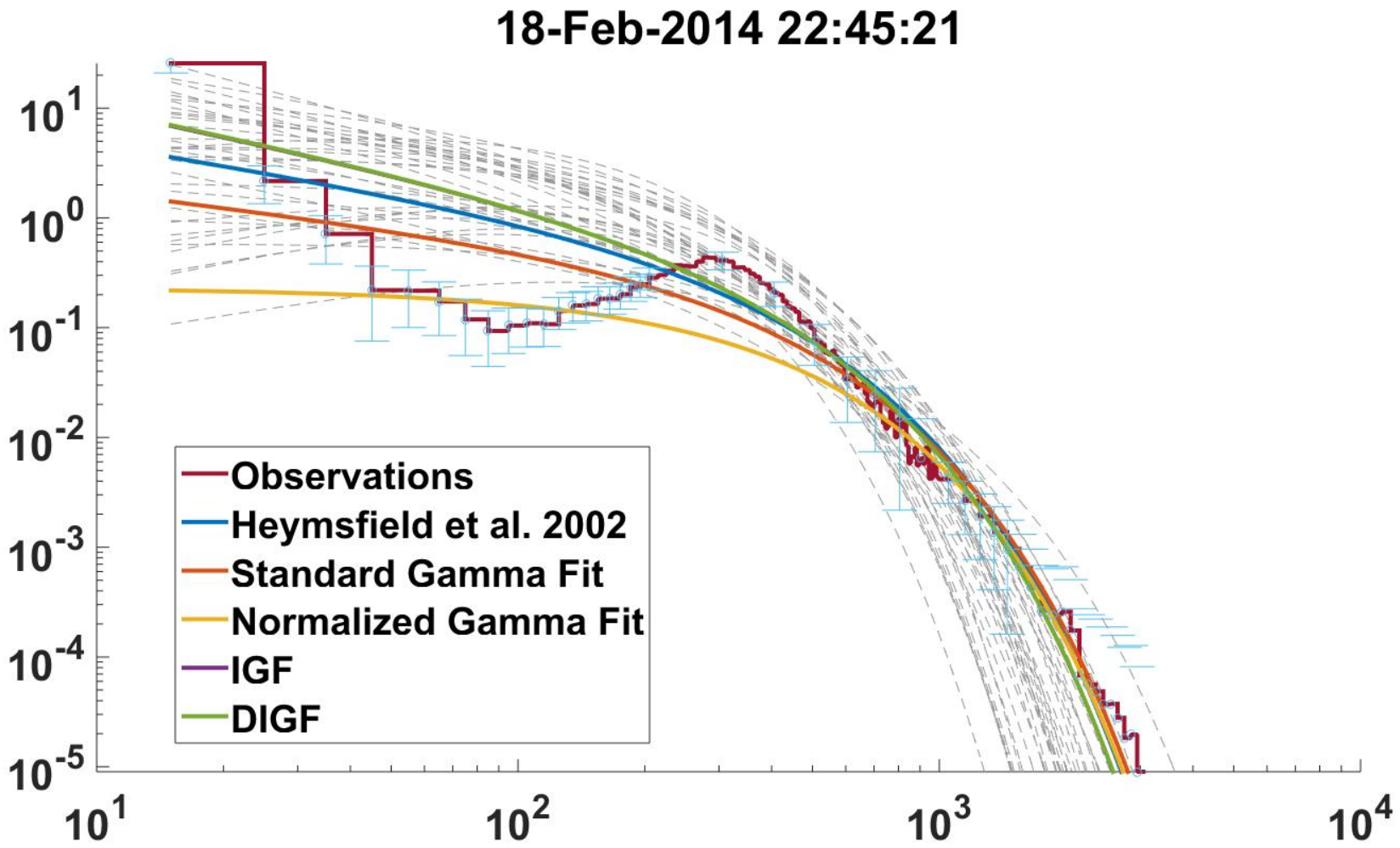
Zhu et al. 2016

HIWC SDs



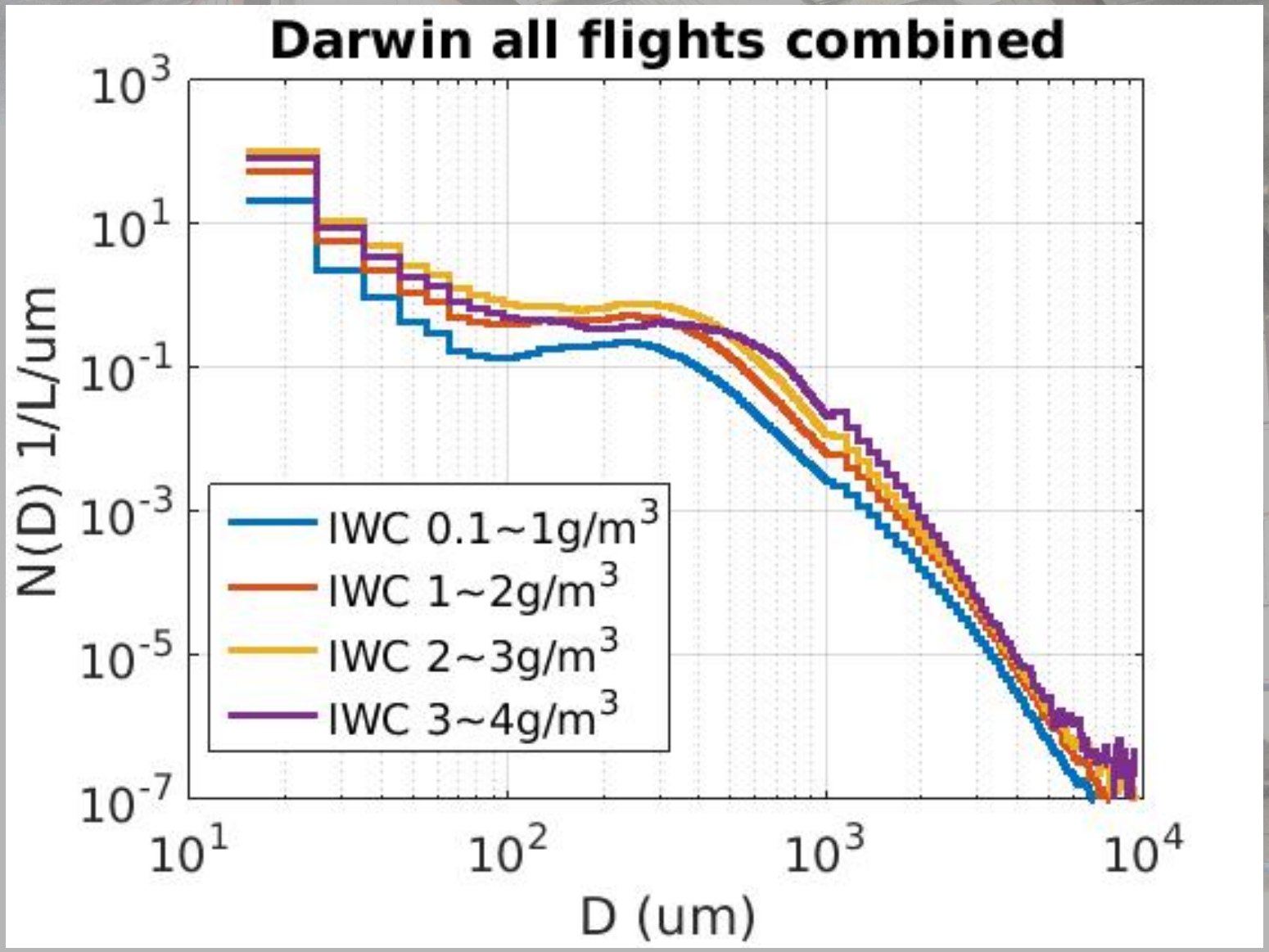
- But, many of the HIWC SDs have multiple modes!

HIWC SDs

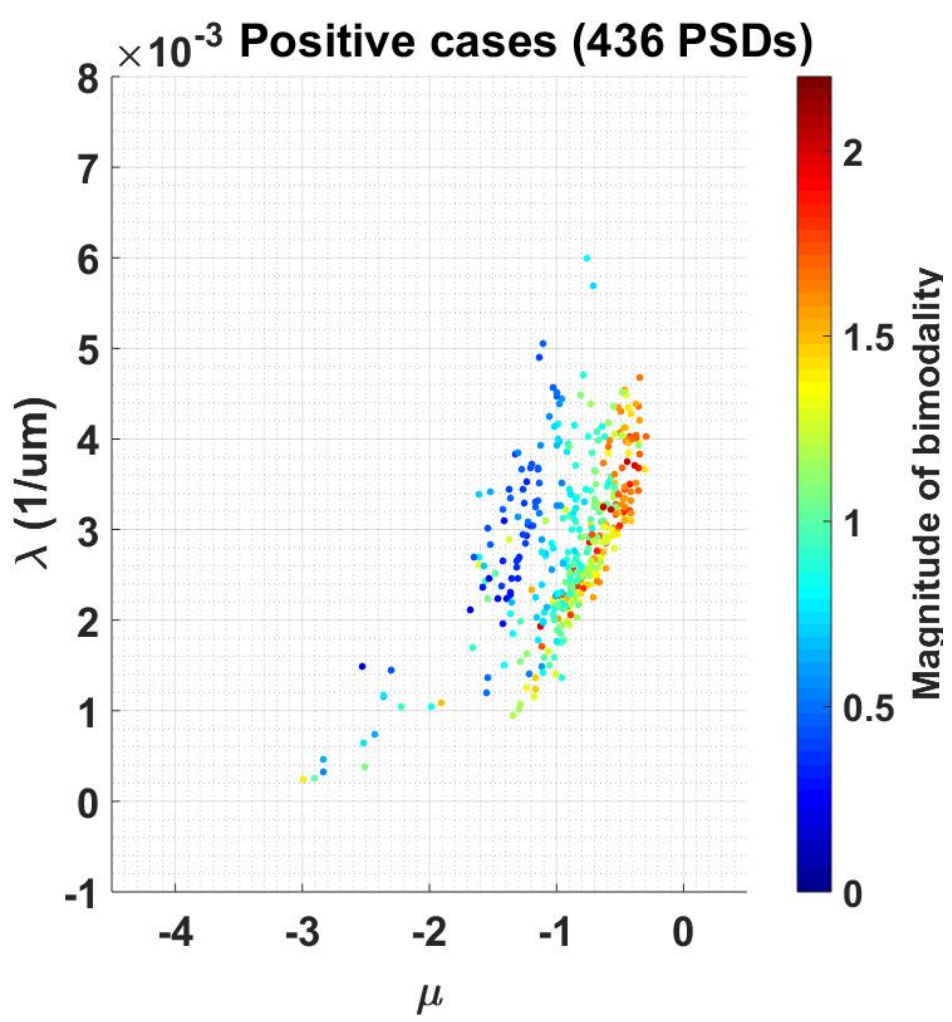


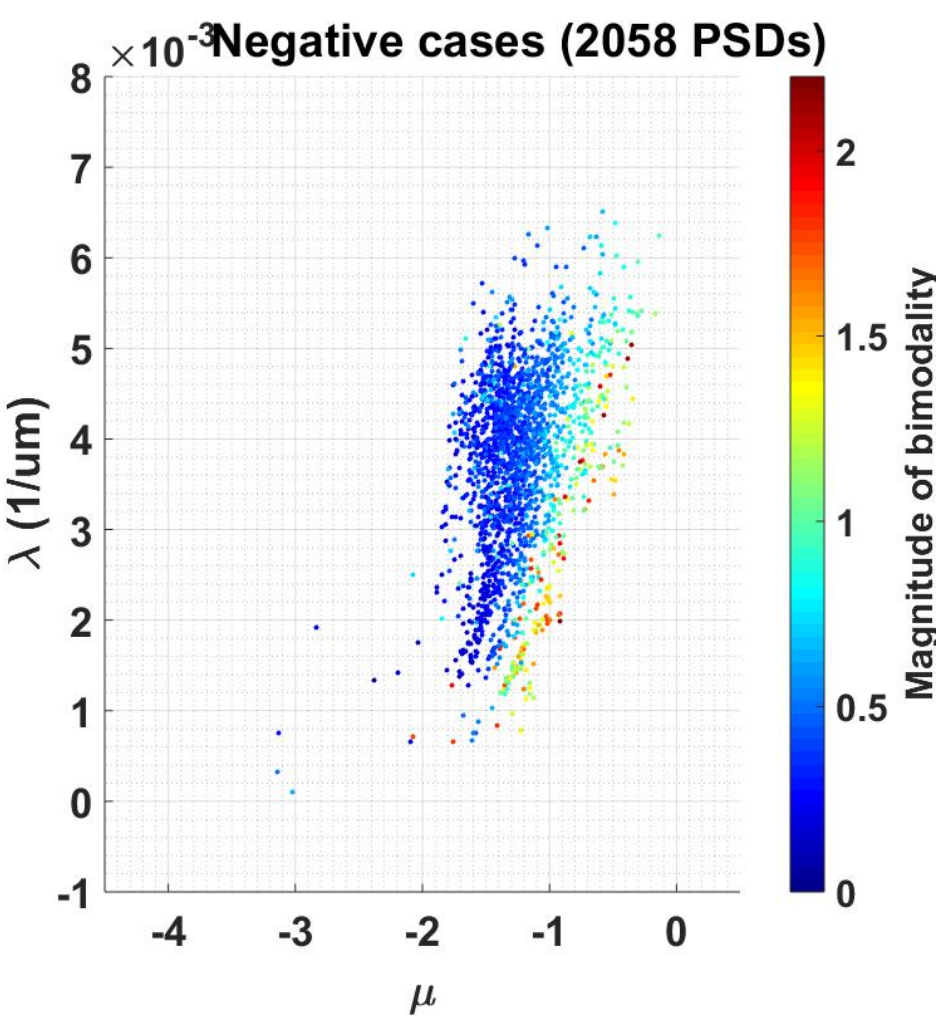
- But, many of the HIWC SDs have multiple modes!
- Gamma fit does not fit data well

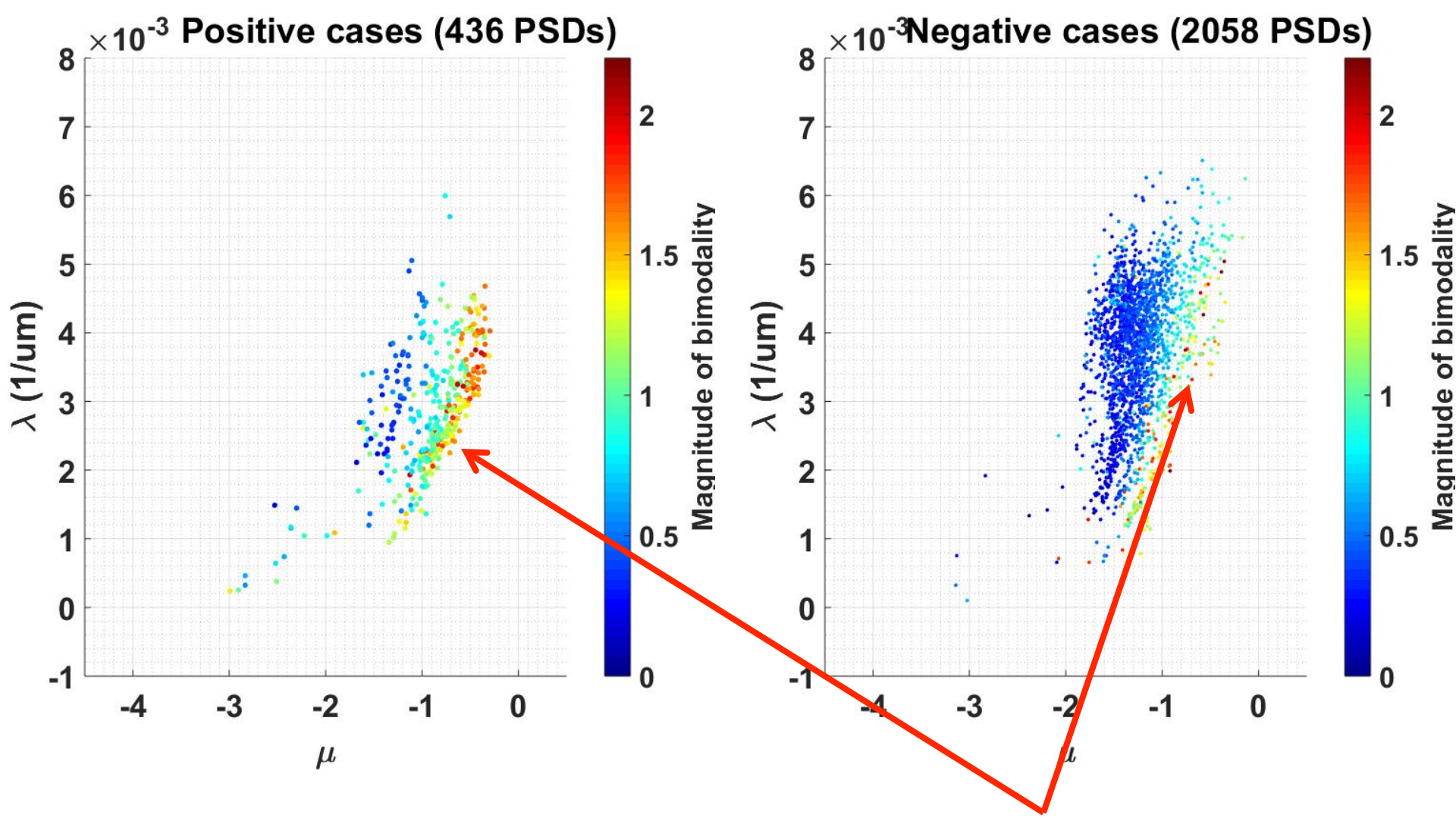
HIWC SDs



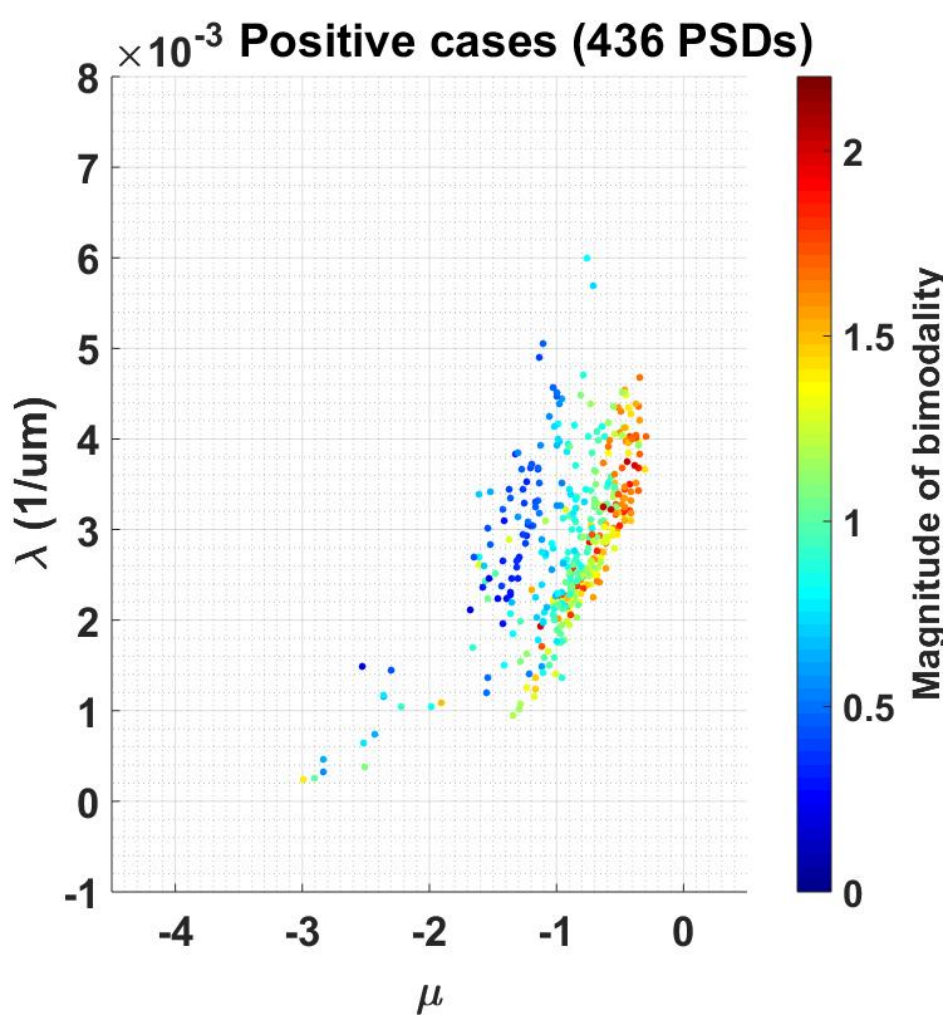
- SDs from HIWC have frequent multiple modes
  → application of IGF difficult

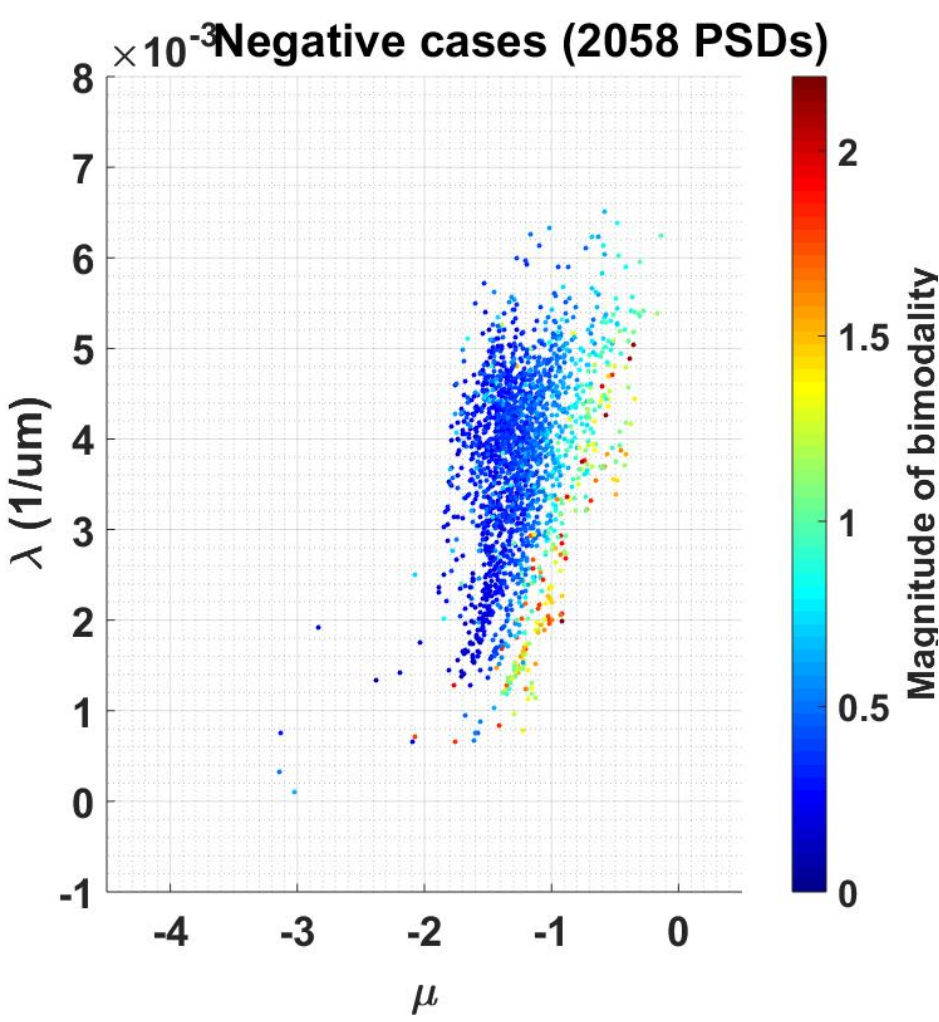

×10-3 Positive cases (436 PSDs)
λ (1/um)
μ
Magnitude of bimodality
8
7
6
5
4
3
2
1
0
-1
-4
-3
-2
-1
0
2
1.5
1
0.5
0


×10-3 Negative cases (2058 PSDs)
λ (1/um)
μ
Magnitude of bimodality
8
7
6
5
4
3
2
1
0
-1
-4
-3
-2
-1
0
2
1.5
1
0.5
0



Bimodality more frequent when IWC & D_{mm} positively correlated



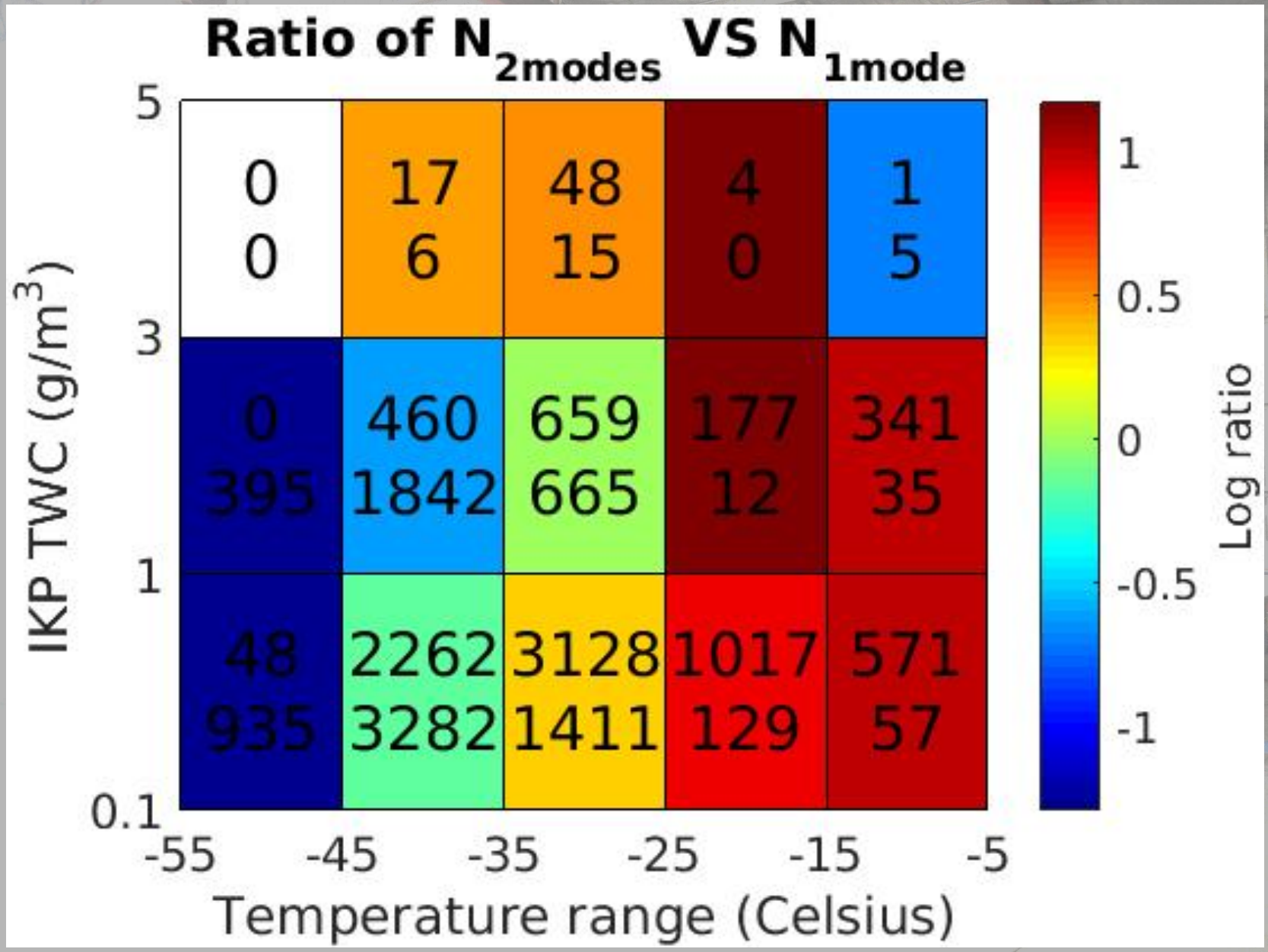


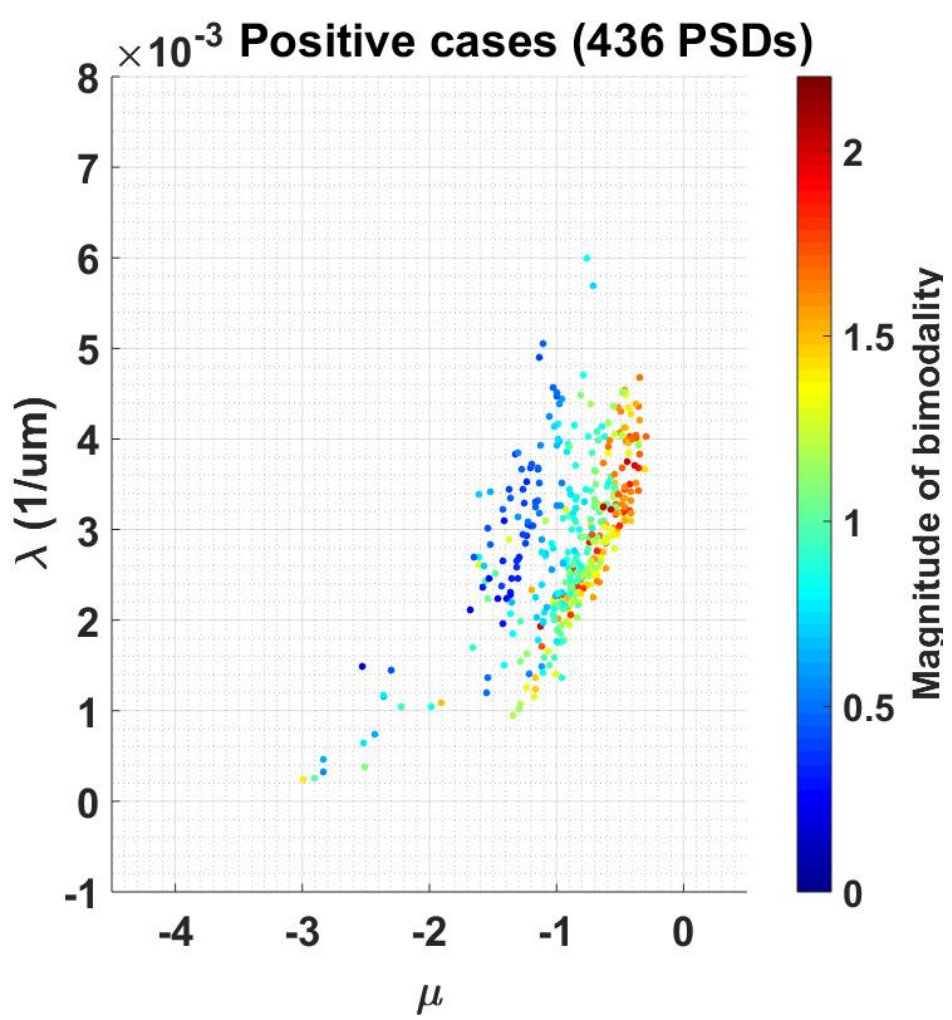
Larger μ when multiple modes present

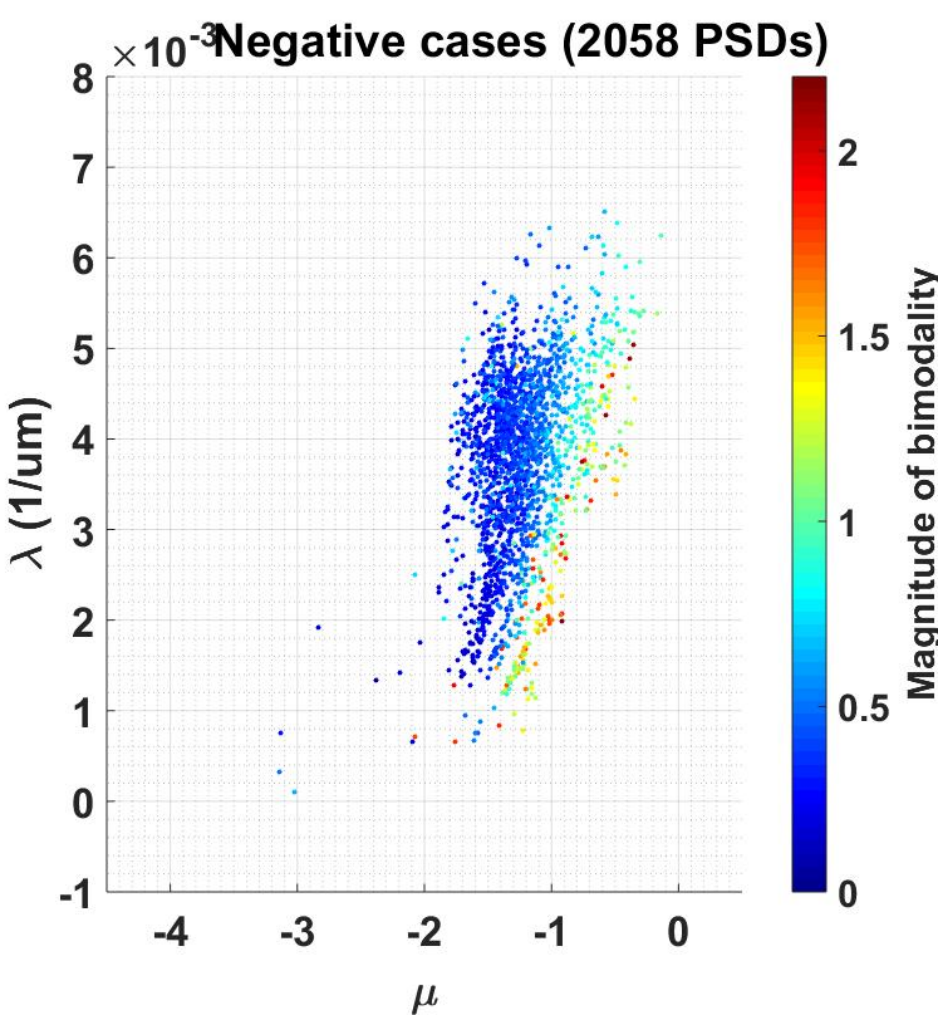
Summary

- Applied IGF technique to determine SD parameters as volume of equally realizable solutions in N_0, λ and μ phase space
  - Separate solutions for HIWC cases depending on how IWC/D_{mm} correlated; bimodality more common when IWC/D_{mm} correlated
- Separate parameterizations, applicable for Monte Carlo schemes, required for different regions
- In future, determine which parameterization should be applied depending on meteorology/age of convection

Frequency of Multi-mode distributions







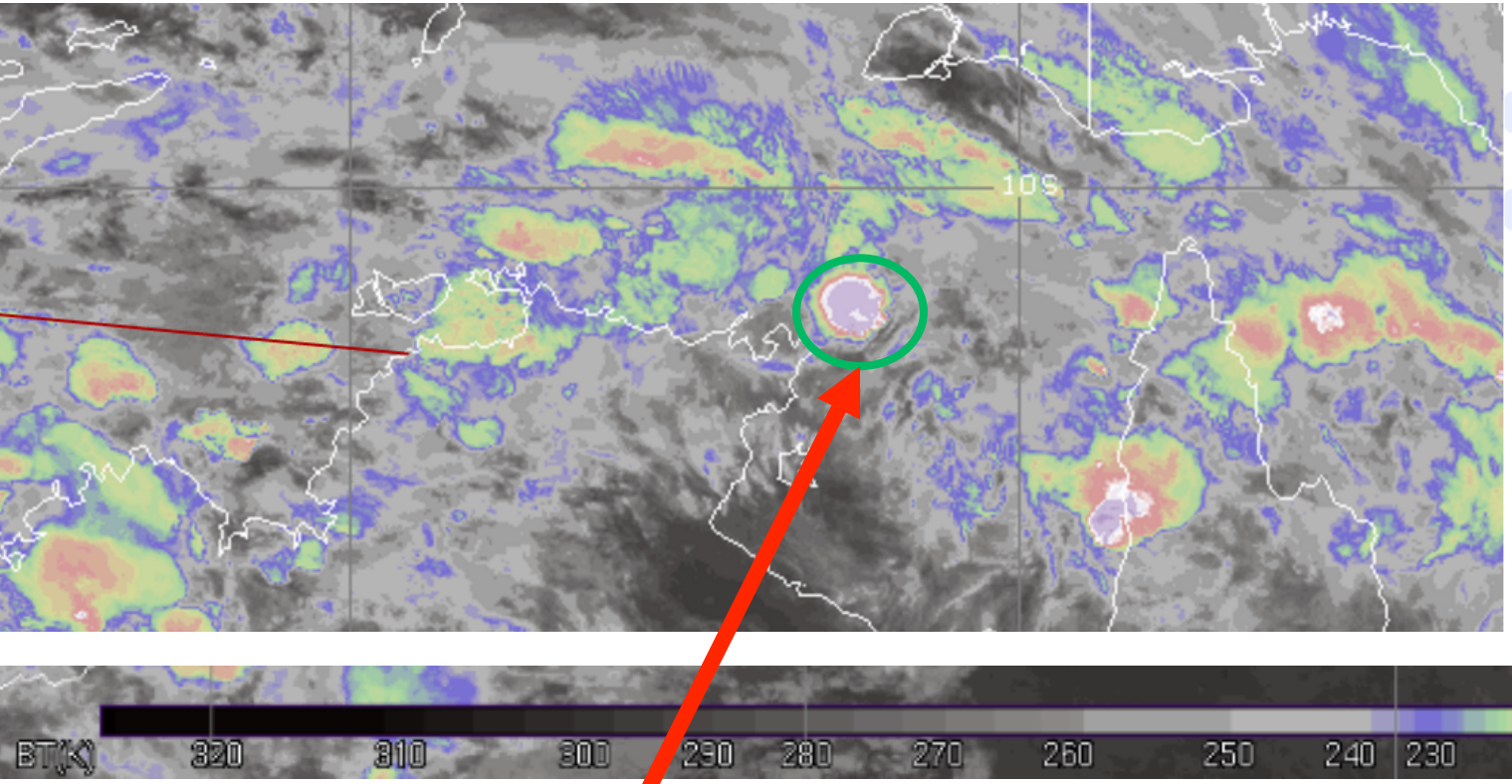
Magnitude of bimodality is originally log(BmArea).
Magnitude of bimodality is deviation of the PSD from the best moment-fitted gamma distribution.
To be specific, Log(BmArea) = $\log(\sum i\uparrow \log(D\downarrow i+1 / D\downarrow i)(\log(PSD\downarrow obs,i / PSD\downarrow fit,i))\uparrow 2)$

For age / lifetime

- Take flight 10 for example: age/lifetime: 5/13

- 0h: 2014-1-29 15:00 ------ Time system starting to form
- 5h: 2014-1-29 20:00 ------ Time taking off:
- 13h: 2014-1-30 04:00 ------ System dissipates

- Figure source: NCAR field catalog
- Overlay: MTSAT-1R Channel 2 Enhanced



Map Time: 2014-01-29 16:00 UTC

Reset to Latest

Time Step

back | 1 hour | forward

Date / Time Select

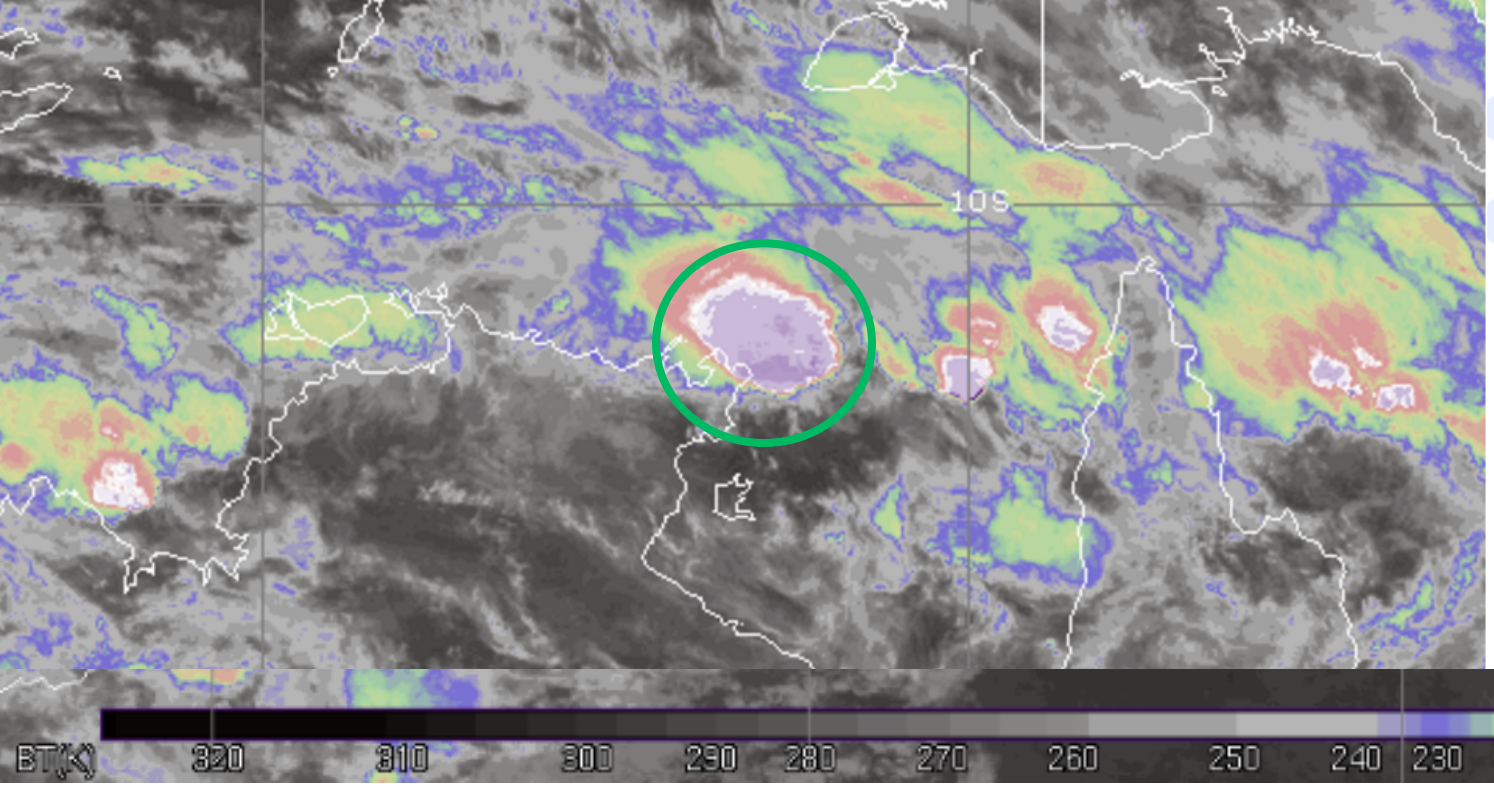
January 2014

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

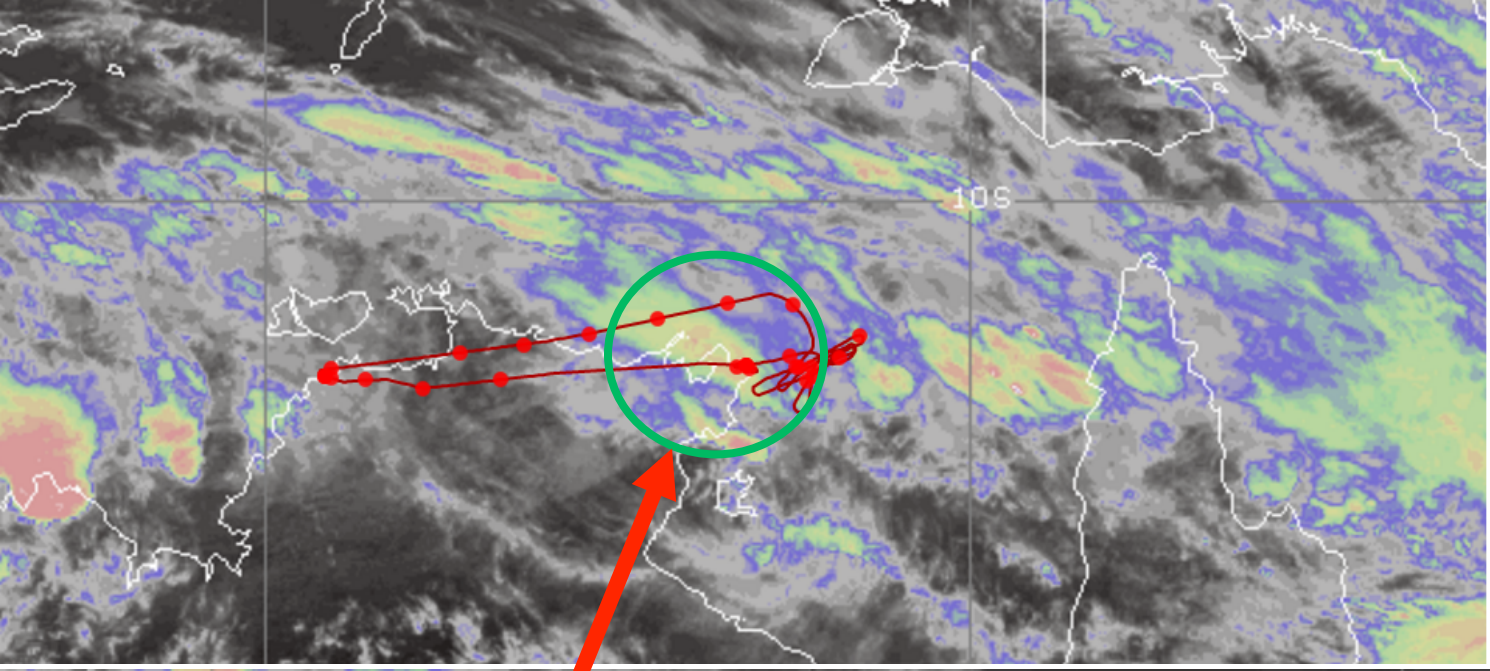
Hour: 16 Minute: 0

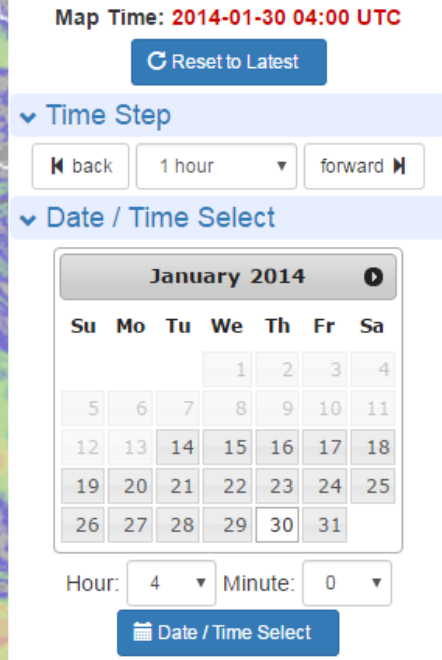
Date / Time Select

2014-1-29 16:00 ------ The first hour where I track white color appearing on the system area (about BT(K)<195)



2014-1-29 20:00 ------ Time taking off:
(exact taking off time is 19:34, rounding to the nearest hour)





2014-1-30 04:00 ------ System dissipate judgement:

1) System falling apart
2) No longer red color appearing on the system (about BT(K)>200)