



**Mountain Wave Momentum Fluxes  
in the MLT during DEEPWAVE**

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# Quantification of GW momentum fluxes employing the AMTM

Dispersion relation for deep GWs (neglecting  $U_z$  and  $U_{zz}$ )

$$m^2 = k^2 (N^2/\omega_i^2 - 1) - 1/4H^2$$

for anelastic GWs

$$T'/T_0 = \theta'/\theta_0 + (\theta'/\theta_0 + \rho'/\rho_0)(1 - H_p/H)$$

$H_p \sim H \Rightarrow$

$$T'/T_0 \sim (\theta'/\theta_0) = -i(Nu'/g) (1 - \omega_i^2/N^2)^{-1/2}$$

simplified polarization relations, assuming  $\lambda_z^2 \ll (4\pi H)^2$  ( $\lambda_z < 30$  km)  $\Rightarrow$

$$\langle u'w' \rangle = (g^2 \omega_i / 2N^3) (1 - \omega_i^2/N^2)^{1/2} (T'/T_0)^2$$

the "cancellation" (phase averaging) factor for  $z_{FWHM} = 7$  km is

$$C = \langle T' \rangle / T'(z_0) = \exp(-\pi^2 z_{FWHM}^2 / 4 \ln 2 \lambda_z^2) = \exp(-1.78 z_{FWHM}^2 / \lambda_z^2)$$

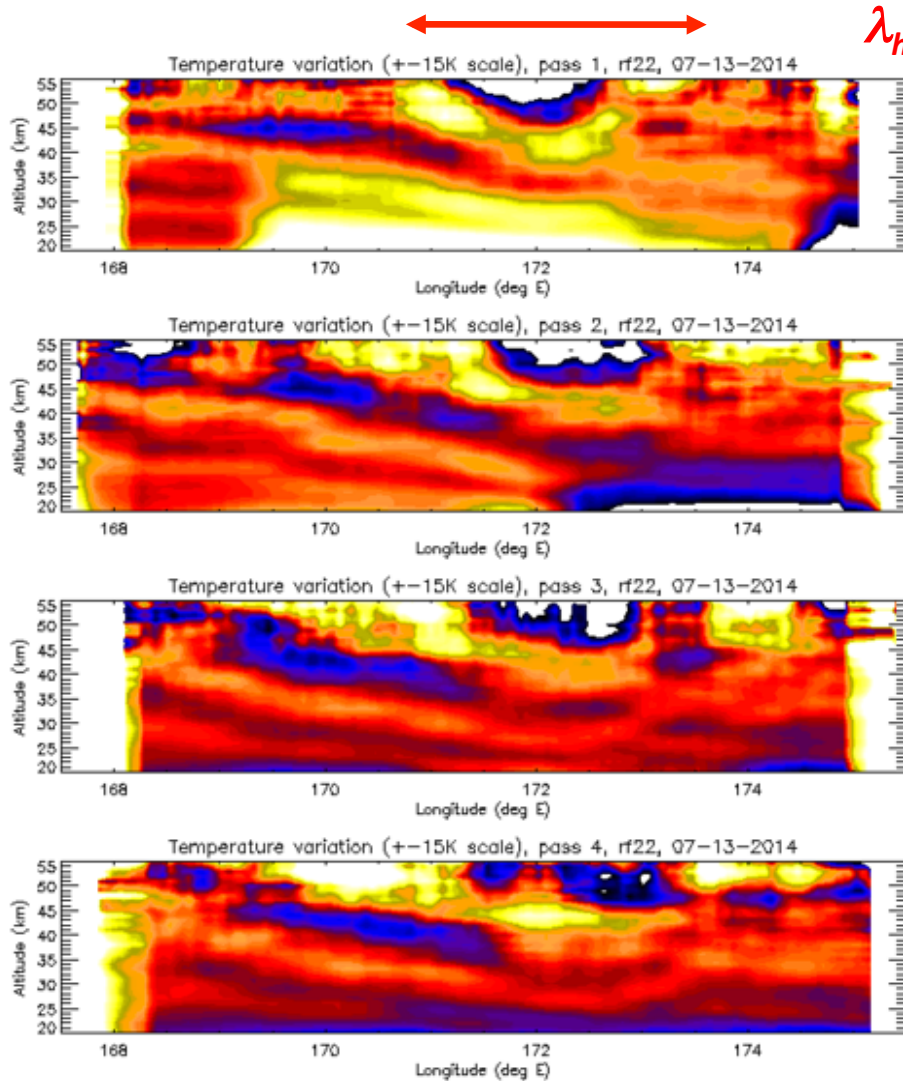
and the GW MF is given by

$$\langle u_h' w' \rangle = (g^2 \omega_i / 2N^3) (1 - \omega_i^2/N^2)^{1/2} (\langle T' \rangle / T_0)^2 / C^2$$

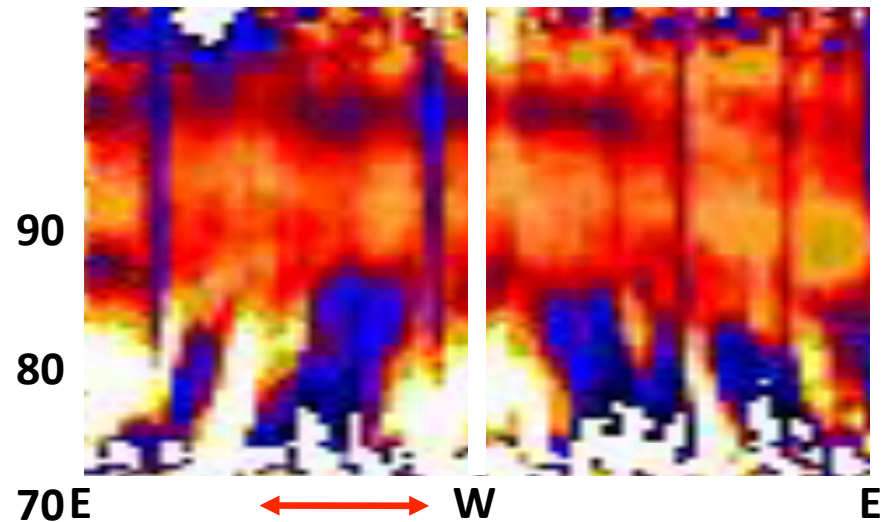
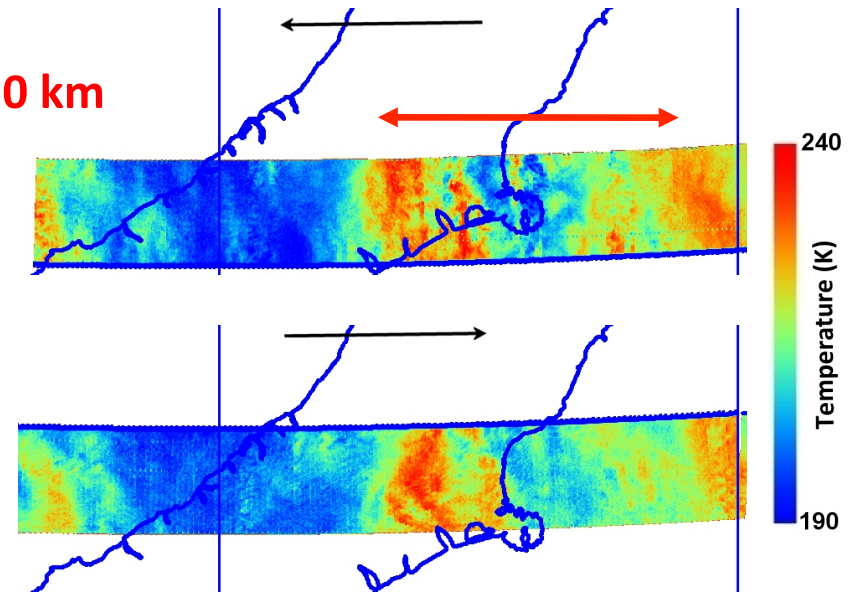
# MWs on RF22 (13 July)

## large-amplitude MW events superposed

### 1. large-scale MW at 20-55 and ~85 km



$\lambda_h \sim 250$  km



**MWs on RF22 (13 July)**  
**large-amplitude MW events superposed**  
**1. large-scale MW at 20-55 and ~85 km**

**Observed parameters:  $\lambda_h \sim 250$  km**

**at 55 km:**

**Rayleigh lidar:  $\lambda_z \sim 25$  km,  $T' \sim 15-20$  K,**

**$\Rightarrow \langle u'w' \rangle \sim 100 \text{ m}^2/\text{s}^2$  at 55 km**

**at ~87 km:**

**AMTM  $T' \sim 20$  K variations,**

**Na lidar  $\lambda_z \sim 25$  km,  $\delta z \sim 3-5$  km, possible overturning (needs refinement)**

**$\Rightarrow T' \sim 20-40$  K**

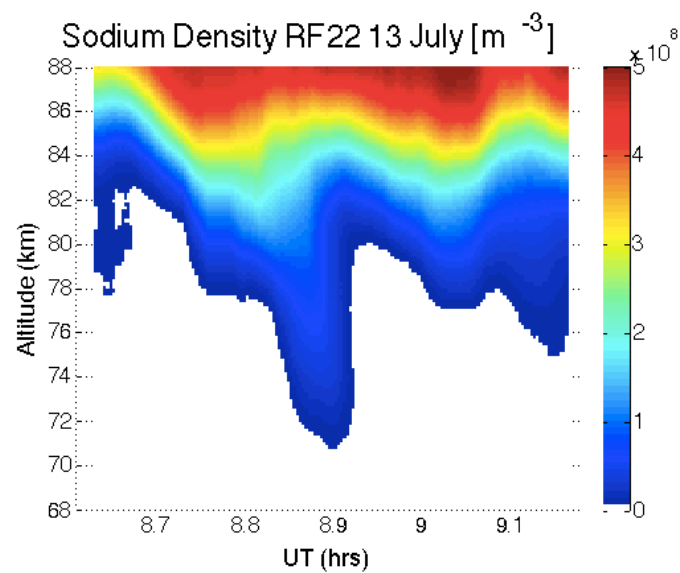
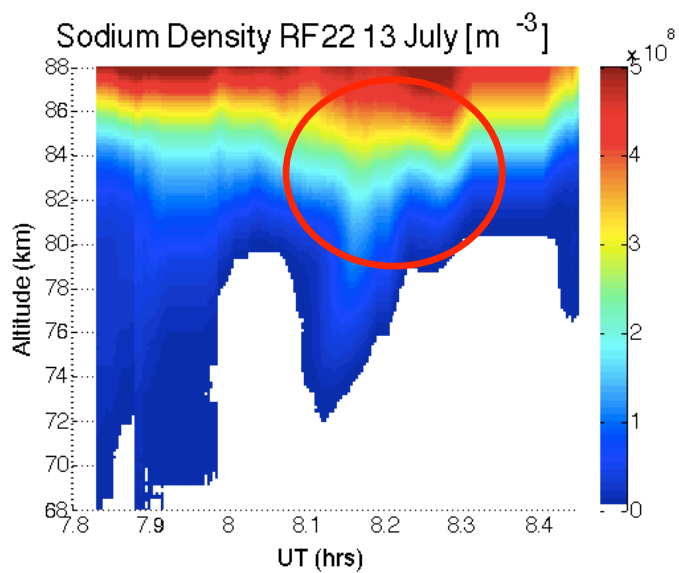
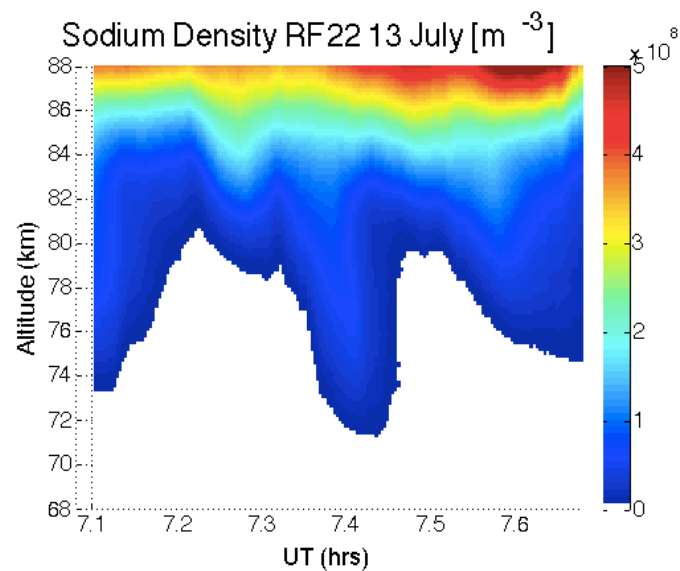
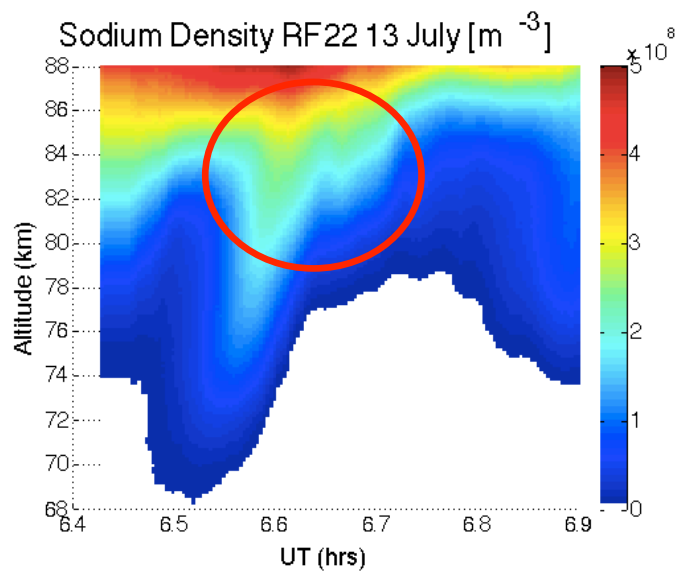
**$\Rightarrow \langle u'w' \rangle \sim 300-1000 \text{ m}^2/\text{s}^2$  at 85 km**



# MWs on RF22 (13 July)

## large-amplitude MW events superposed

### 2. smaller-scale MWs at 20-55 and ~85 km



**MWs on RF22 (13 July)**  
**large-amplitude MW events superposed**  
**2. smaller-scale MWs at 20-55 and ~85 km**

**Observed parameters:  $\lambda_h \sim 60-80$  km**

**at 85 km:**

**AMTM  $T' \sim 10$  K variations,**

**Na lidar  $\lambda_z \sim 25$  km,  $dz \sim 1-2$  km**

**=>  $T' \sim 10-15$  K (evidence of overturning would increase this)**

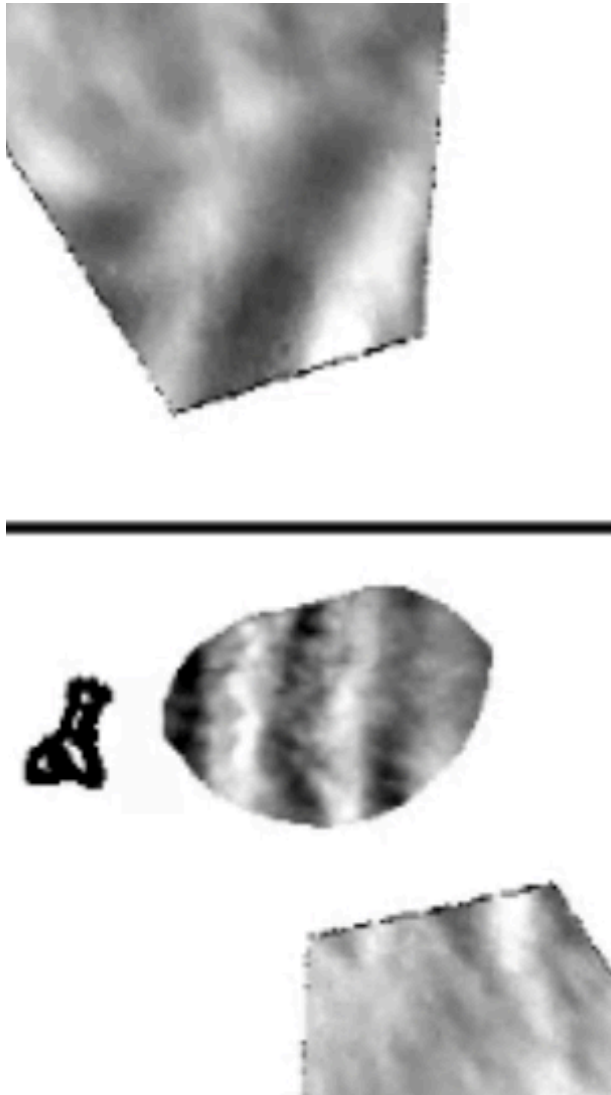
**=>  $\langle u'w' \rangle \sim 600-1000$  m<sup>2</sup>/s<sup>2</sup> at 85 km**

**note that  $\langle u'w' \rangle \sim 1000$  m<sup>2</sup>/s<sup>2</sup>**

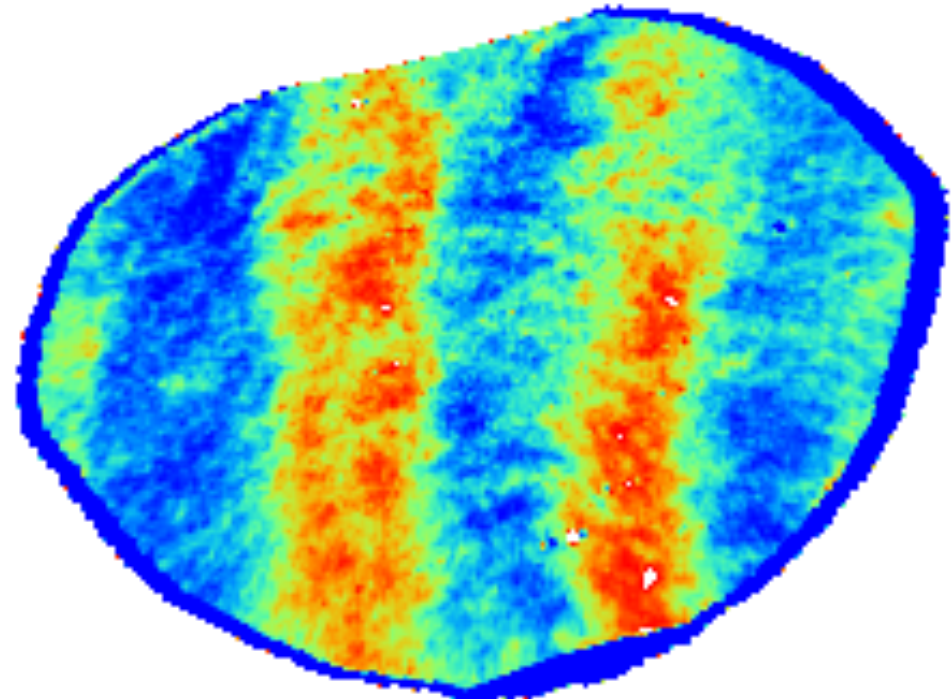
**=> local acceleration of  $\sim 30$  m/s/hr, rapid local wind reductions,  
and possible explanation for strong temporal modulation**

## Auckland Island MWs on RF23 (14 July)

- clear "ship-wave" response at ~87 km



$\lambda_z \sim 40 \text{ km}, T' \sim 10 \text{ K}$



## Auckland Island MWs on RF23 (14 July)

- clear "ship-wave" response at ~87 km

Observed parameters at ~87 km:

AMTM:  $\lambda_h \sim 40$  km,  $T' \sim 10$  K

=>  $\delta z \sim 1-2$  km

=>  $\lambda_z > 2\pi \delta z > 10$  km (larger more likely due OH phase cancellation)

- need large  $\lambda_z$  to yield observed AMTM  $T'$

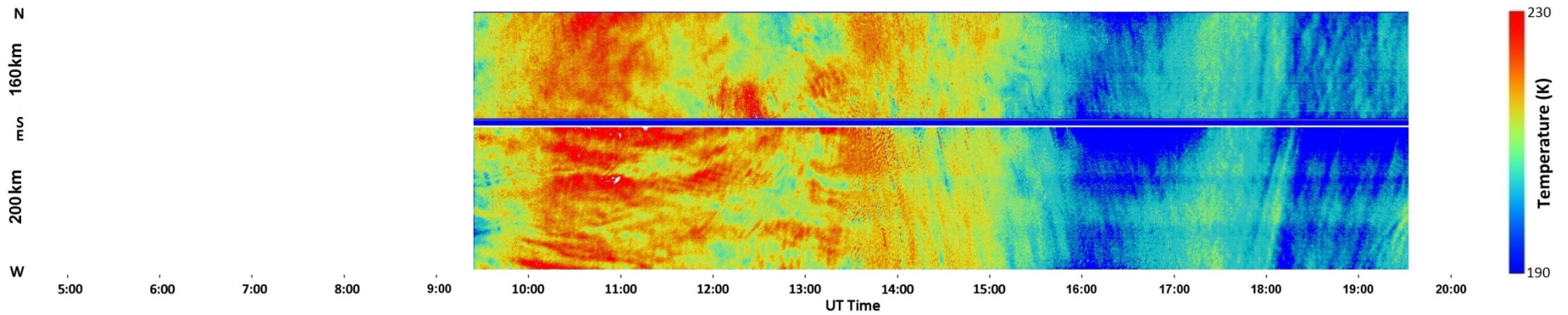
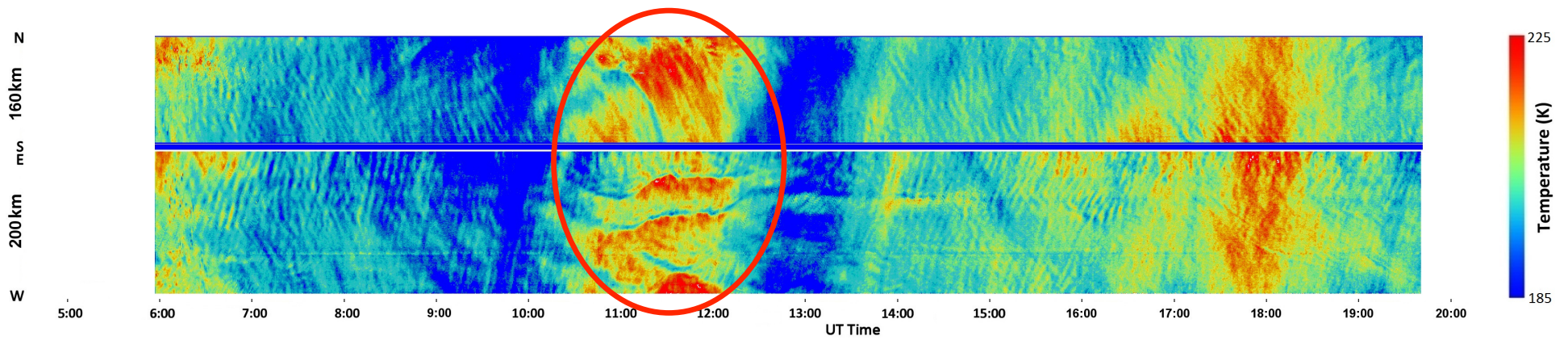
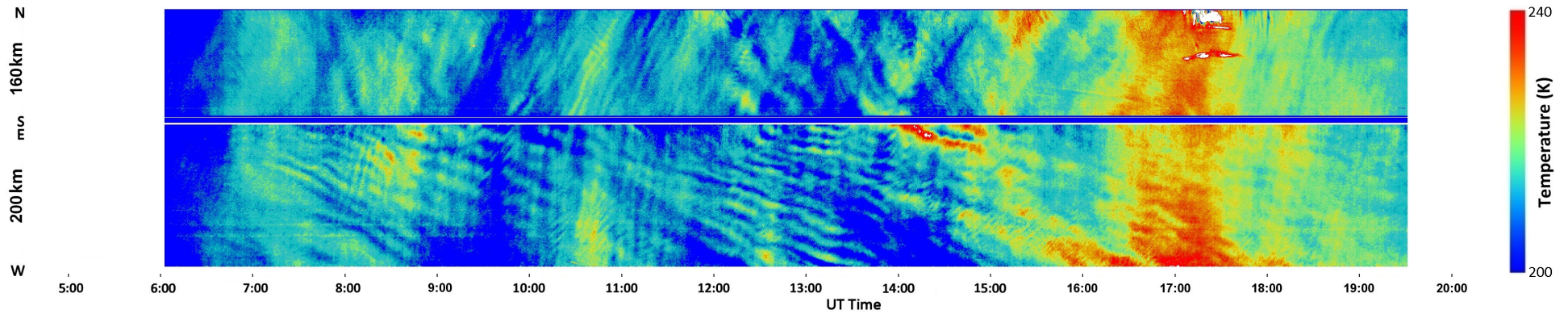
- winds of ~30-50 m/s =>  $\lambda_z \sim 14-20$  km

=>  $\langle u'w' \rangle \sim 100-300$  m<sup>2</sup>/s<sup>2</sup> at 87 k



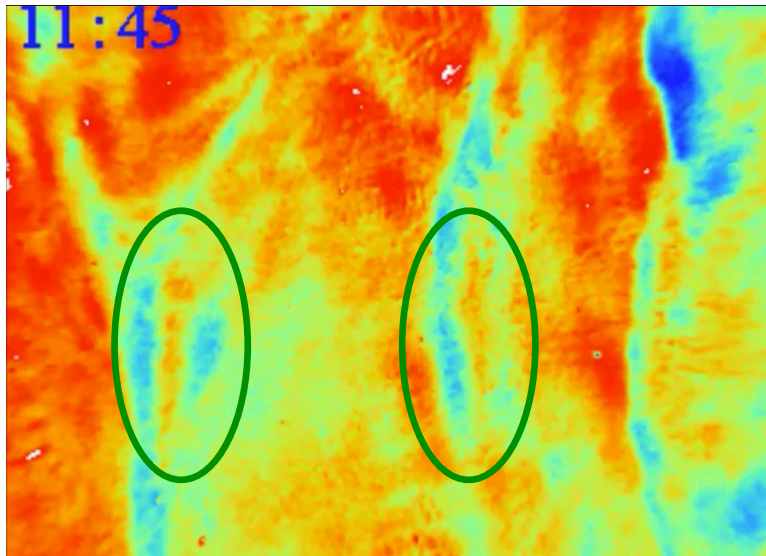
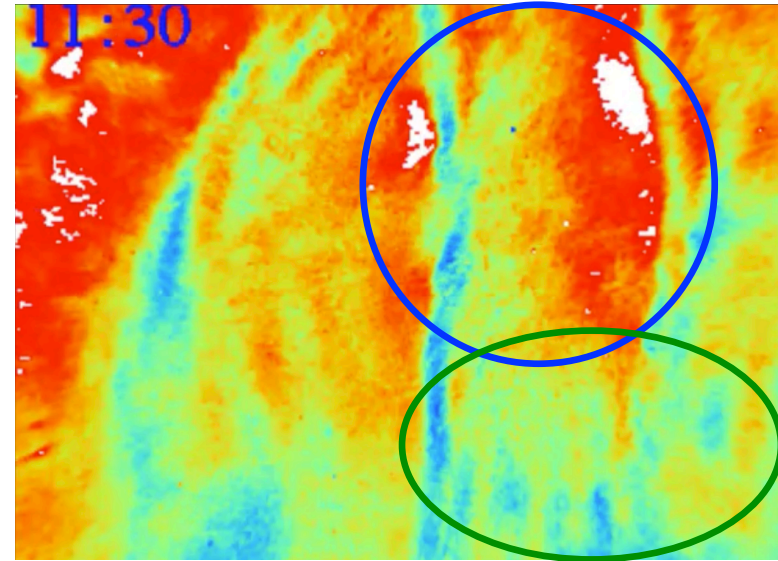
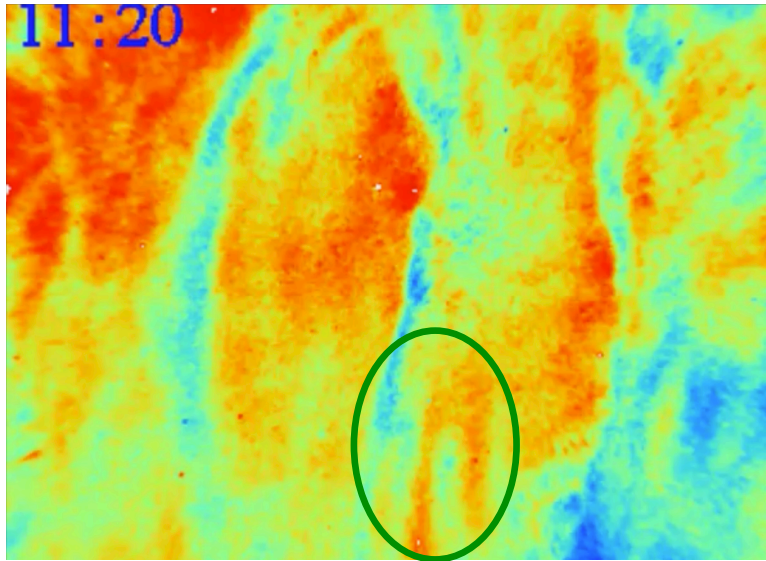
# GW momentum fluxes in the MLT during DEEPWAVE

- multiple large-amplitude MW events (during weak forcing)





**GWs observed by the Lauder AMTM on 21 June 2014  
- several large-amplitude MW events superposed**



**Observed parameters:**

$\lambda_h \sim 12-15$  and  $\sim 65$  km

AMTM  $T'$  and sawtooth GW  $T(x)$  variations

=>

$T' \sim 20$  K,  $T'/T \sim 0.1$  or greater for  $\sim 65$  km GW

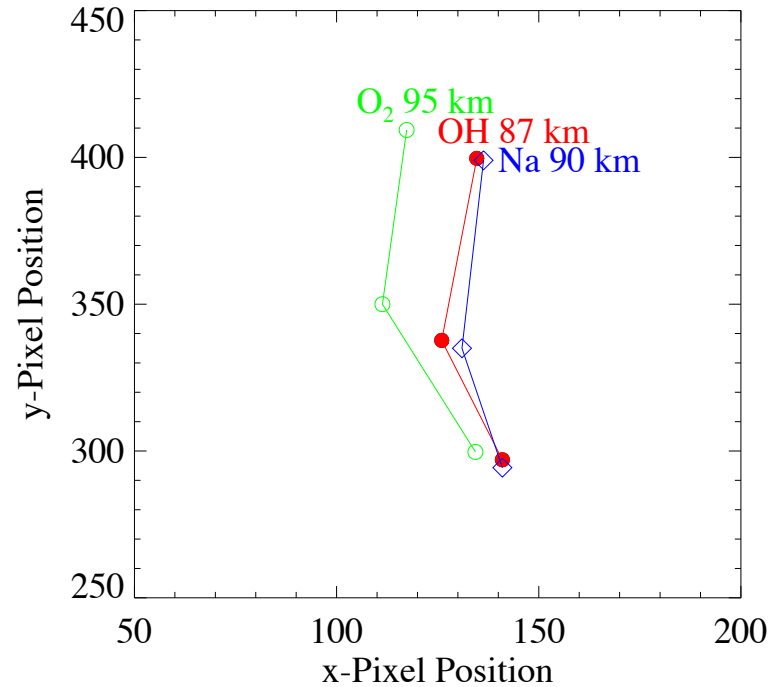
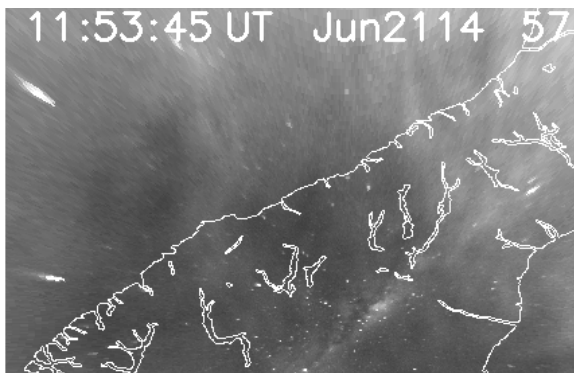
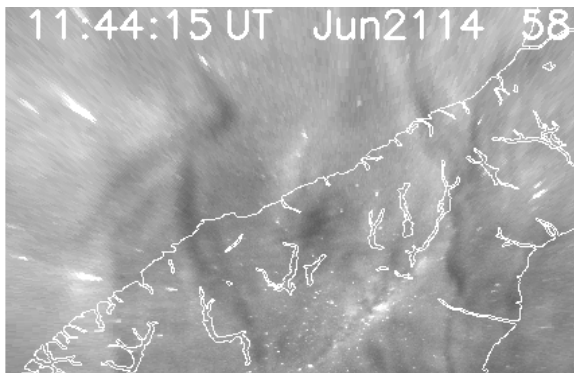
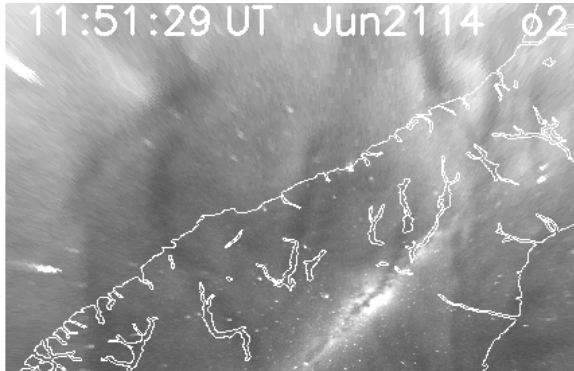
$\delta z \sim 2$  km, overturning

=>

$\lambda_z \sim 2\pi \delta z \sim 12.5$  km or greater

## Other estimates or constraints on $\lambda_z$

- Mt. John airglow phase differences among OH, Na, and O<sub>2</sub> layers



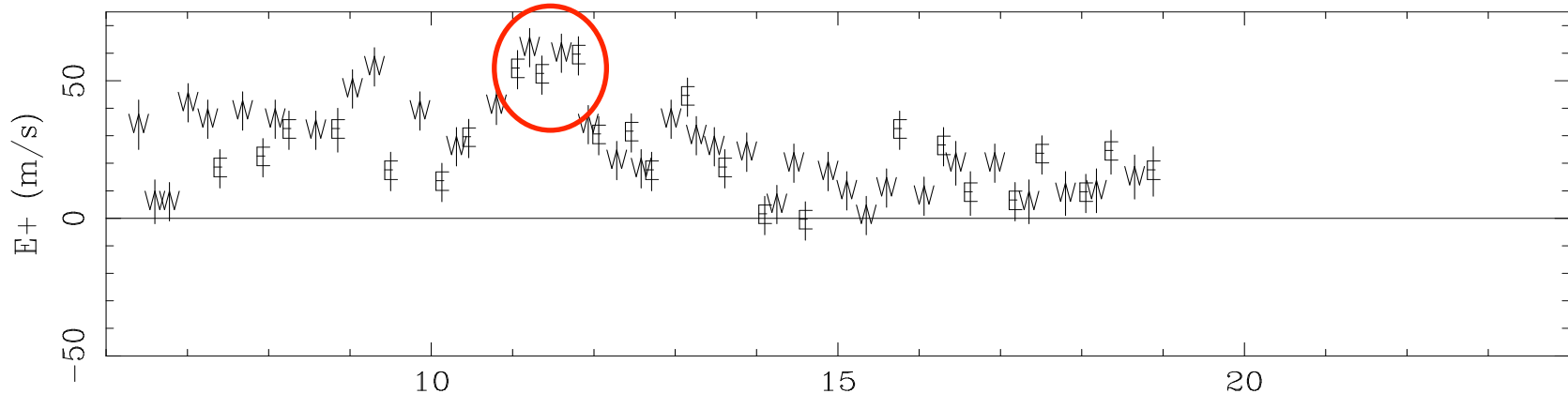
**Observed and inferred wavelengths  
(from Steve Smith)**

$\lambda_h \sim 65$  km

$\lambda_z \sim 33$  km (from OH and O<sub>2</sub> airglow)

## Other estimates or constraints on $\lambda_z$ - Mt. John FPI E-W winds

11902K MJO 140621 Day ( 21 )



**$U \sim 50$  m/s during this event**

**$\Rightarrow$**

**$\lambda_z \sim 2\pi U/N \sim 15$  or  $30$  km for  $N \sim 0.02$  or  $0.01$  s $^{-1}$**

**Summary of options:**

**$\lambda_z \sim 33$  km  $\Rightarrow \omega'/N \sim 0.45$ ,  $\langle u'w' \rangle \sim 530$  m $^2$ /s $^2$**

**$\lambda_z \sim 15$  km  $\Rightarrow \omega'/N \sim 0.22$ ,  $\langle u'w' \rangle \sim 540$  m $^2$ /s $^2$**

**$\lambda_h \sim 15$  km,  $T'/T \sim 0.03 \Rightarrow \omega'/N \sim 0.71$ ,  $\langle u'w' \rangle \sim 110$  m $^2$ /s $^2$**



## Summary

- larger MLT momentum fluxes preferentially accompany large-amplitude MWs
- dominant horizontal wavelengths,  $\lambda_h \sim 20\text{-}200$  km
- largest coherent MWs in the stratosphere and MLT occur for weak tropospheric forcing