RAYS & MOUNTAIN WAVES IN A WIND JET.

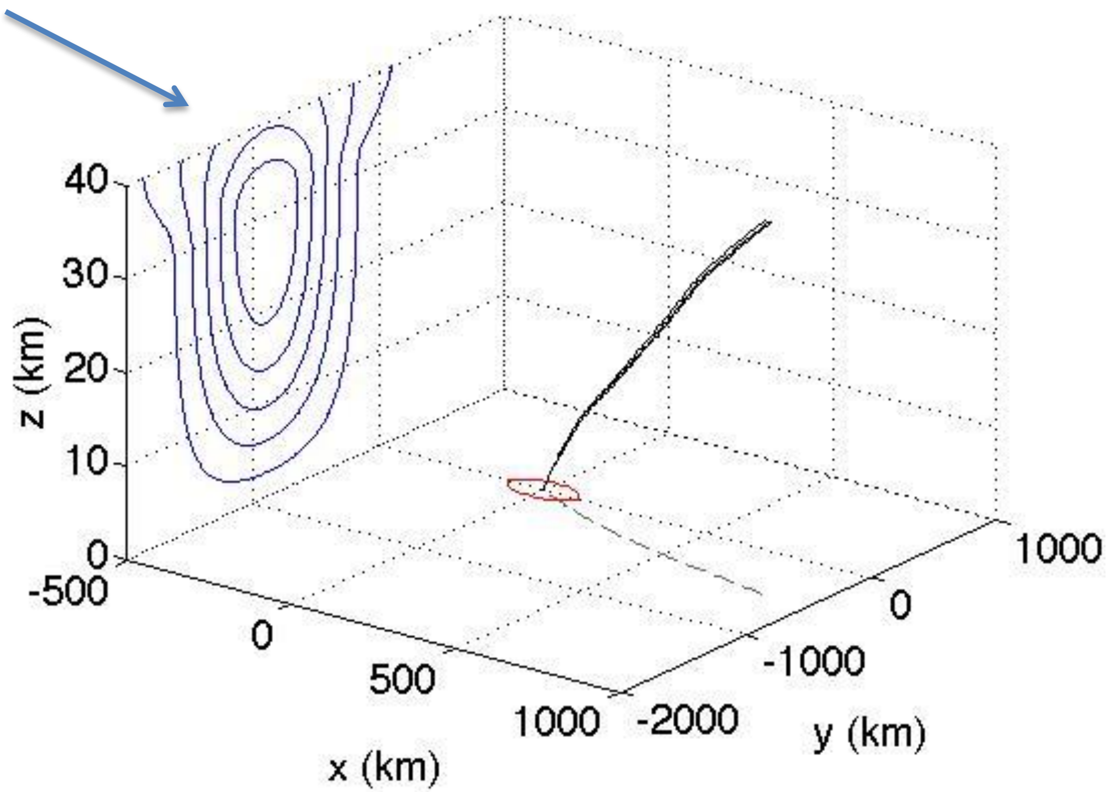
1. A comparison of
   an idealized COAMPS simulation
   (from Jim Doyle)
   and a very very simple ray calculation.

2. How we compute the ray solution
   in this case, relevant to Jim's simulation.

EXAMPLE: **Trace a mt-wave ray from a localized mountain into a wind jet U(y,z) (~ polar vortex.)**

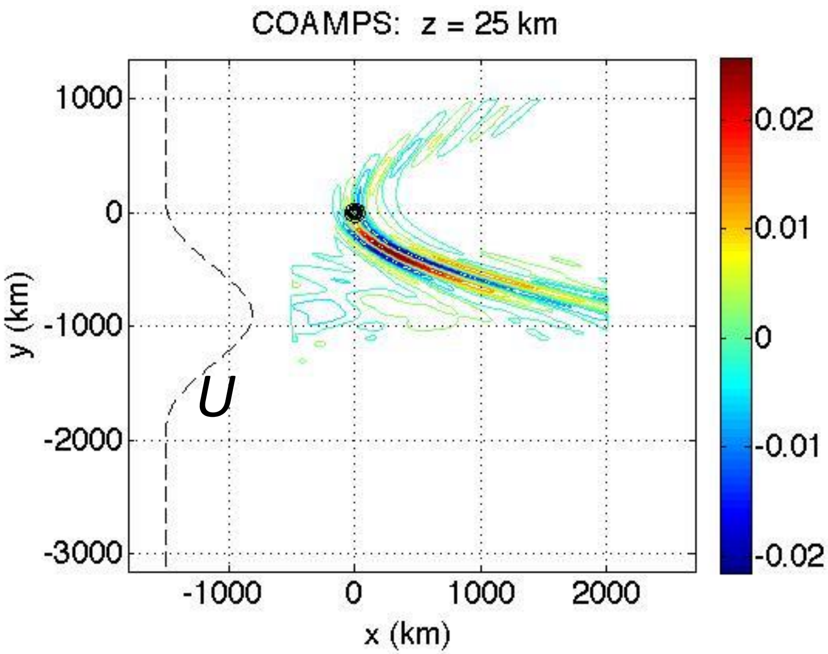
U(y,z): 13 – 75 m/s.

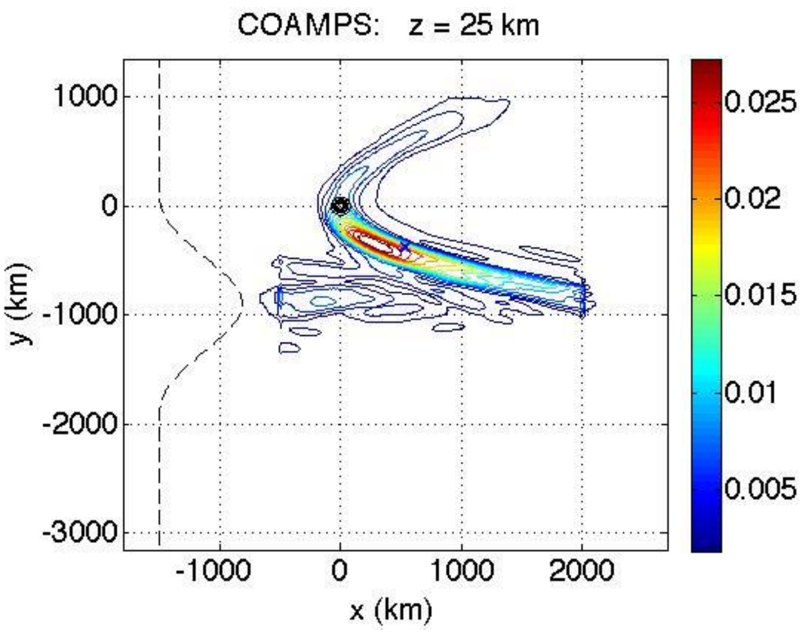
COAMPS sponge layer z > 30km.



COAMPS SOLUTION

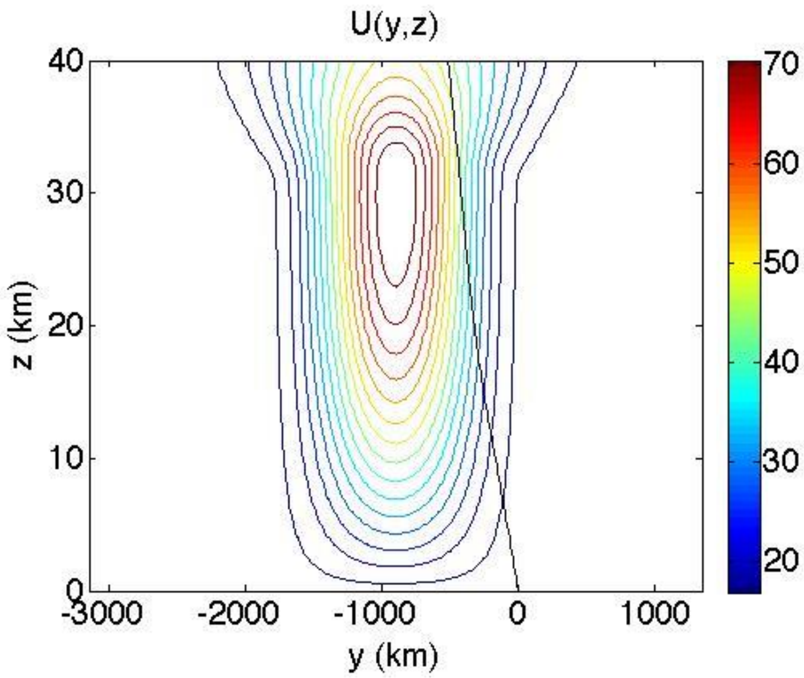
for w(x,y) at z = 25km $(* \rho(z)/\rho_0)^{1/2}$

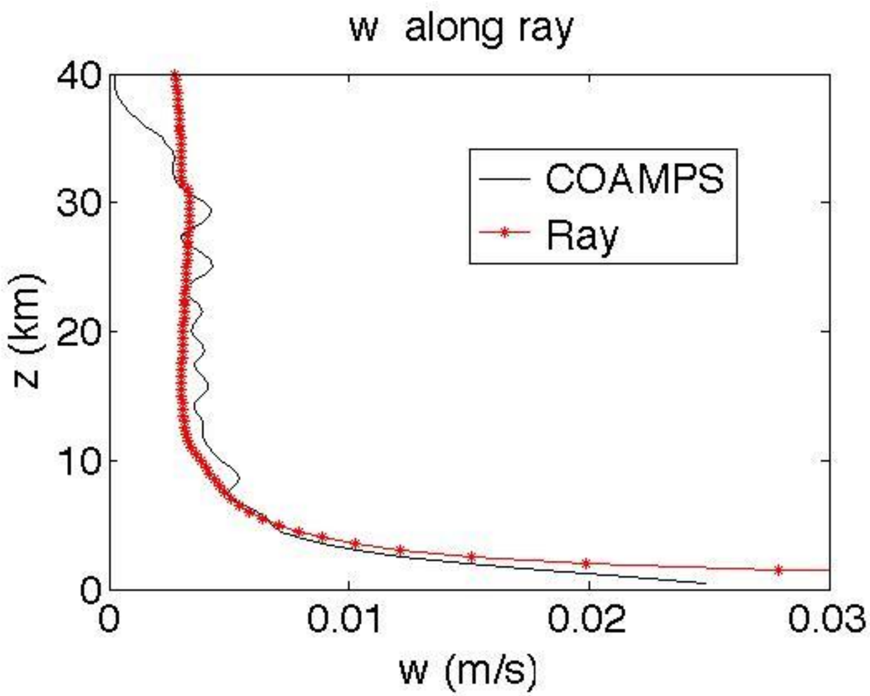




Left: w(x,y) = vertical velocity, larger on jet side.

Right: Take a Hilbert transform of w(x,y)|, take the magn., smooth out the phase for comparison with ray amplitude.



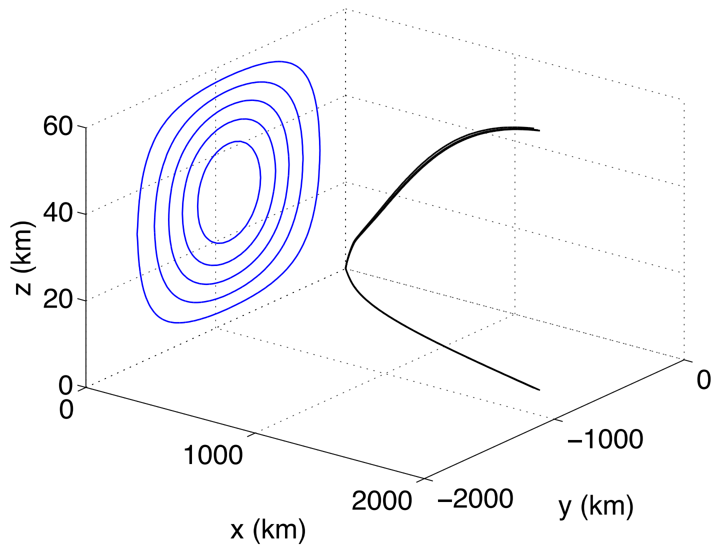


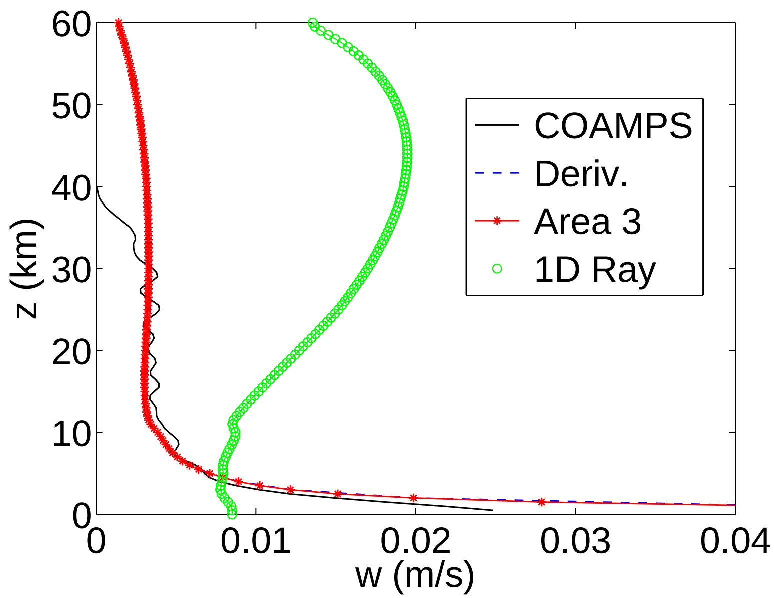
Ray path proj. on y,z, and wind jet U(y,z)

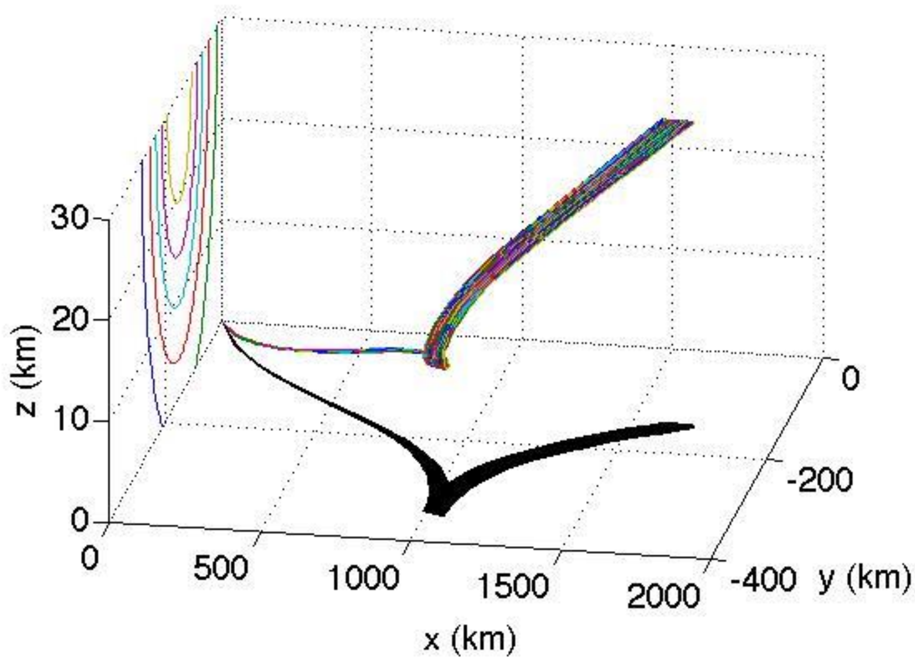
z < 10km, geometrical spreading dominant.

z > 10km. amplitude more constant with altitude.

Trace ray to higher altitude than COAMPS.







By the way

also find horizontally trapped rays in wind jet.

But these waves may not be energetically important.

How to get the ray solution.

Consider a thin tube of rays (e.g. previous slide) . Use

C_g * Area * $|\eta|^2 / \omega$ = *constant along ray*.

So

$|\eta|$ ~ (*constant along ray* / Area) $^{1/2}$

Two significant issues:

(i) Area, which can vanish, meaning $|\eta| \to \infty$

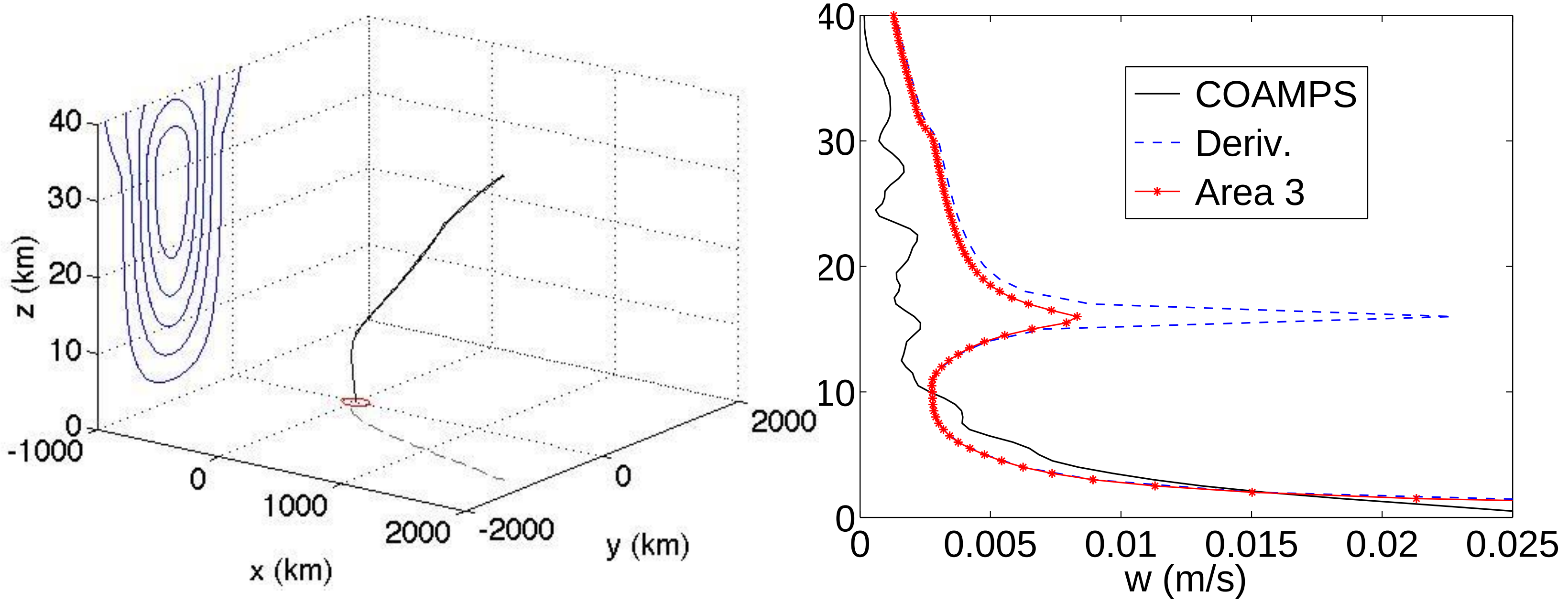
(ii) setting constant along the ray (ray initialization.)

Area → 0 => $|\eta| \to \infty$. Caustic

Caustic: ray tube focused (pinched) to zero area.
Singularity of ray theory.
Airy function correction (in textbook case).
Expect locally high amplitudes near caustics.

Swimming pool caustics. Bright spots on bottom are caustics for the light..

Example with a caustic



This case has a caustic near z = 18km.

Dashed curve – ray solution.

Red curve – smoothed ray solution, cf. Vadas & Fritts (2009).

BUT NO hint in COAMPS of any caustic amplification.

If generally true, can smooth the caustics away.

Jacobian matrix

In limit of infinitesimally thin ray tube

$$\text{Area} \rightarrow \text{Jacobian} = S \begin{vmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{vmatrix} S^{-1}$$

At a caustic, either $\lambda_1 = 0$ or $\lambda_2 = 0$.

Previous example: $\lambda_1 = 0$ and $\lambda_2 >> 1$.
Caustic amplification in one direction
balanced by GS in the perpendicular direction.

(Very tentative conclusion in many ways.)

Some general questions:

* **Is there a preferred direction for geometrical spreading (e.g. parallel to the wind)? Are eigenvalues/vectors of J useful?**

* **Are caustics common or important?**

* **What is the the net effect of GS for waves that emerge from the top of the wind jet, compared to what you'd expect from U(z only).**

COMMENTS

There is no all purpose ray-code, robust for all applications. Issues of geom. spr., caustics and ray initialization.

Sometimes have to look at each ray.

J assumed to be 1, GROGRAT – caustic free.

J = 1 (in wavenumber space) Fourier-ray.

Best for mt waves directly over the mt.

For non-constant Jacobian:

what can be learned (wind jet case) and Is it worth the effort.

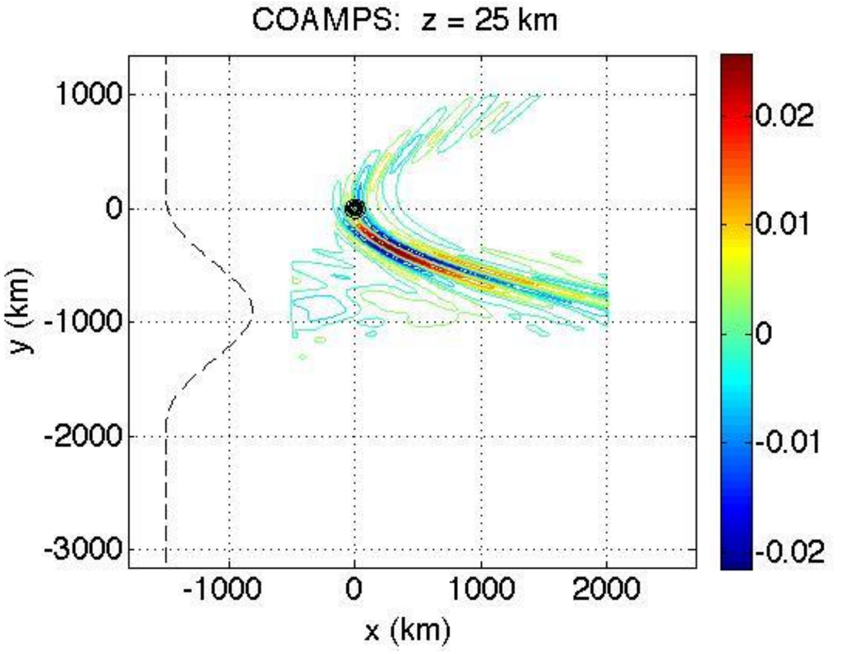
Initialization of the rays

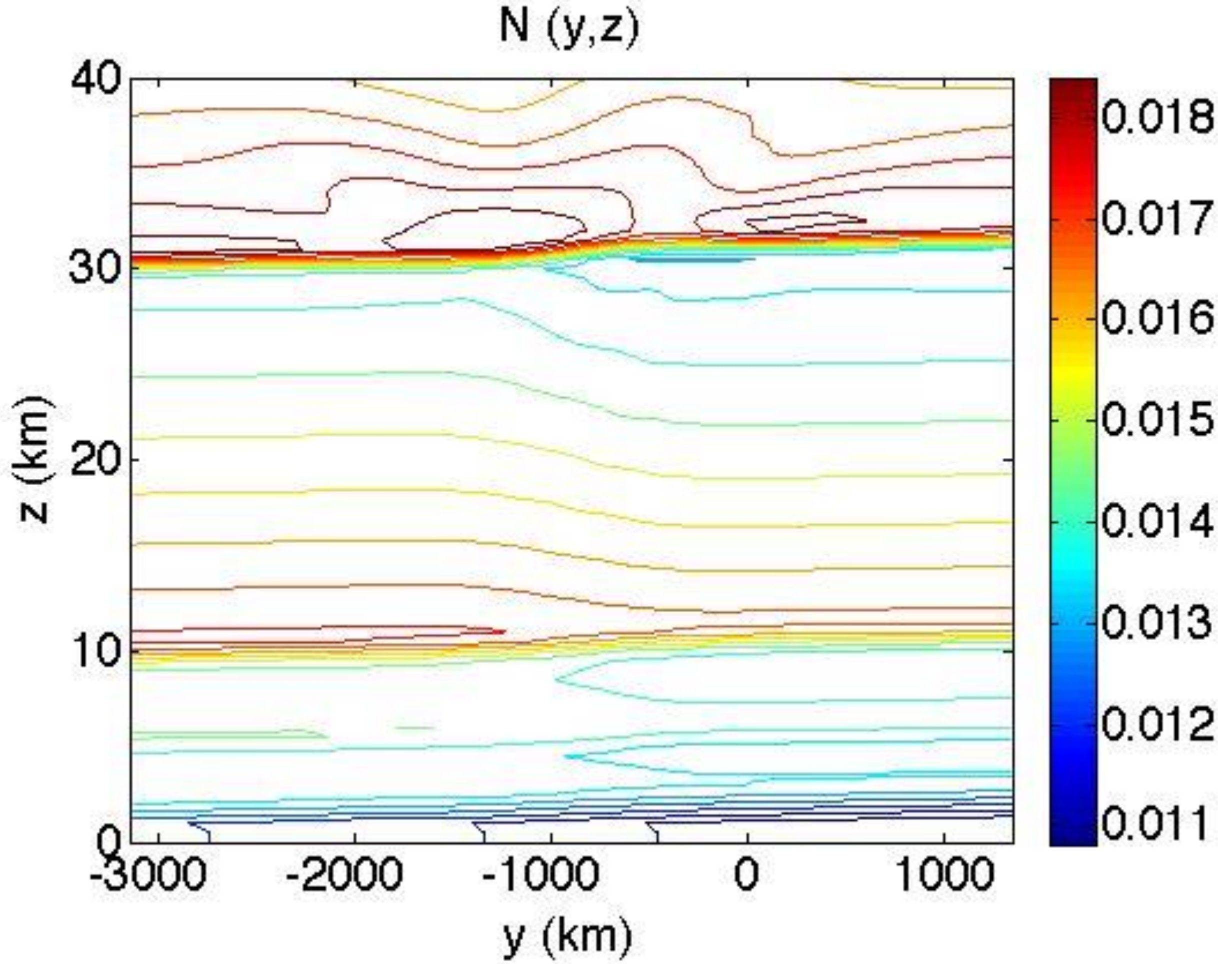
Takes an extra effort because of the Jacobian.
Relative orientation of the rays in the ray tube.

Two ways:

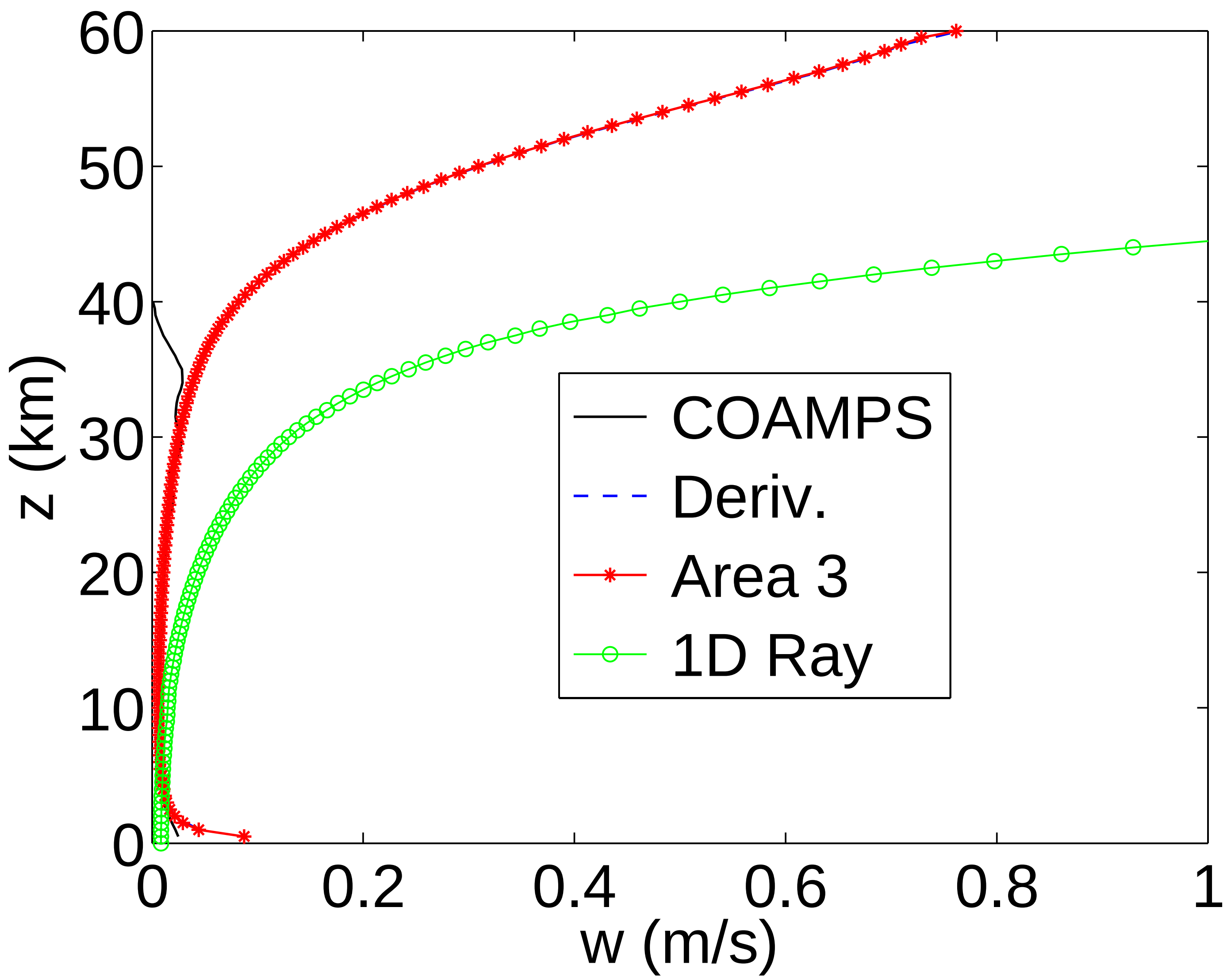
1. Ideally have a Fourier formulation, take stationary phase limit near the ground . Assumes a linearized lower boundary condition.

2. From COAMPS solution at z = constant, possibly above low-level nonlinearity ~ z = 10km).
Not without problems.





N (y,z)
40
30
20
10
0
z (km)
-3000
-2000
-1000
0
1000
y (km)
0.018
0.017
0.016
0.015
0.014
0.013
0.012
0.011



60
50
40
30
20
10
0
z (km)
0
0.2
0.4
0.6
0.8
1
w (m/s)
COAMPS
Deriv.
Area 3
1D Ray