Testing New NAVGEM Orographic GWD Parameterization Using DEEPWAVE Data

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Acknowledgements

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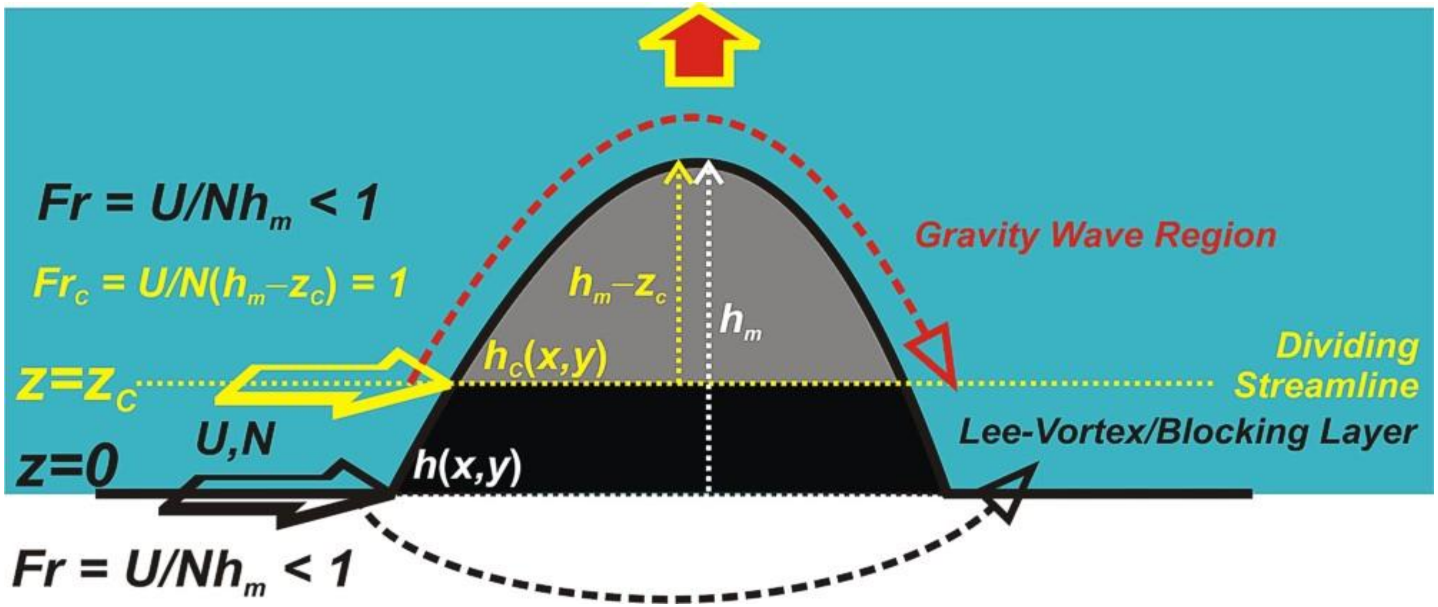
- NASA through the Heliophysics Division SR&T and GI programs.

Source Parameterizations Based on Surface Drag for 3D Elliptical Obstacles



1. Fit subgridscale orographic elevations *h(x,y)* to an effective idealized anisotropic 3D obstacle

$$\sigma_{xx}^2 = \overline{\frac{\partial h}{\partial x}\frac{\partial h}{\partial x}} = \frac{\pi}{6}K_0^{3/2}\left[K_U^{3/2} - K_L^{3/2}\right](3a+c),$$

$$\sigma_{xy}^2 = \overline{\frac{\partial h}{\partial x}\frac{\partial h}{\partial y}} = \frac{\pi}{3}K_0^{3/2}\left[K_U^{3/2} - K_L^{3/2}\right]b,$$

$$\sigma_{yy}^2 = \overline{\frac{\partial h}{\partial y}\frac{\partial h}{\partial y}} = \frac{\pi}{6}K_0^{3/2}\left[K_U^{3/2} - K_L^{3/2}\right](a+3c),$$

2. Infer pressure "drag" D_p from linear relations D_L

$$|\mathbf{D_L}| = D_L = \frac{\pi}{4}a\rho_0 NUh_m^2 = \frac{\pi}{4}a\rho_0\frac{U^3}{N}Fr^{-2},$$

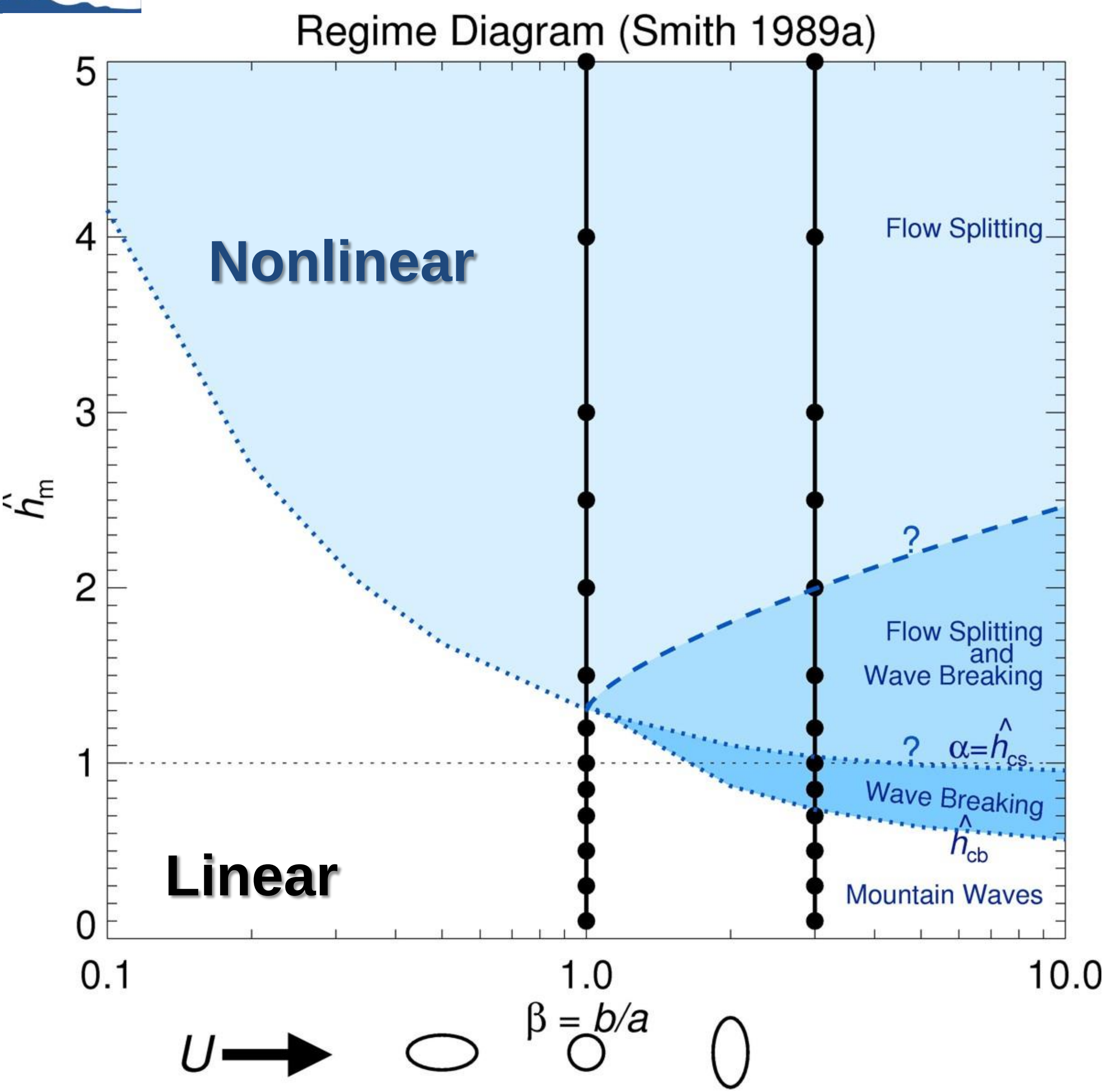
$$\tau_{sx} = \rho u N \widehat{K}^{-1}(\sigma_{xx}\cos\chi + \sigma_{xy}\sin\chi),$$

$$\tau_{sy} = \rho v N \widehat{K}^{-1}(\sigma_{xy}\cos\chi + \sigma_{yy}\sin\chi),$$

3. Split D_p into wave (D_w) and surface (D_p-D_w) components, based on a dividing streamline z_c.

$$\mathbf{D_p} = \int_{-\infty}^{\infty}\!\!\int p'\left(\frac{\partial h}{\partial x},\frac{\partial h}{\partial y}\right)dx\,dy, \quad \mathbf{D_w}(z) = \rho(z)\int_{-L/2}^{+L/2}\!\!\int (u'w', v'w')\,dx\,dy,$$

3D OMD Regime Diagram



Elliptical Three-Dimensional Hill

$$h(x, y) = \frac{h_m}{[1 + (x/a)^2 + (y/b)^2]^{3/2}}.$$

Constant Upstream Flow Profile

U = 10 m s^{-1} N = 0.01 rad s^{-1}

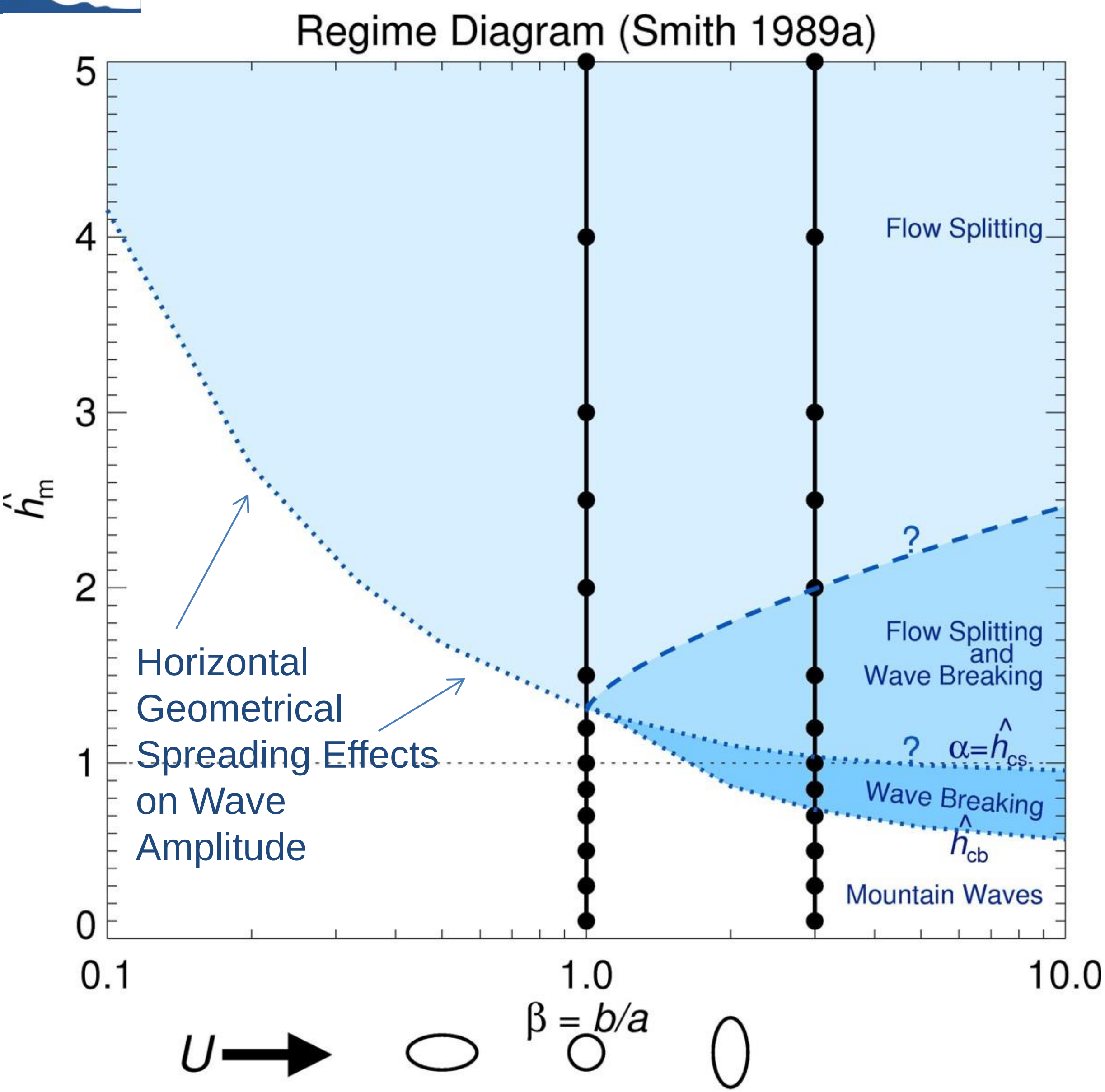
Normalized Hill Height
(inverse Froude number)

$$\hat{h}_m = Nh_m/U = \mathrm{Fr}^{-1}$$

Obstacle Aspect Ratio $\beta = \frac{b}{a}$

Cases Simulated Using Mesoscale Model

3D OMD Regime Diagram



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Cases Simulated Using Mesoscale Model

Horizontal Geometrical Spreading

$$\frac{\partial A}{\partial t} + \nabla . (\mathbf{c_g} A) = 0.$$

A = wave action density = E/ω
E = total wave energy density (KE + PE)
ω = wave intrinsic frequency
$\mathbf{c_g}$ = vector group velocity

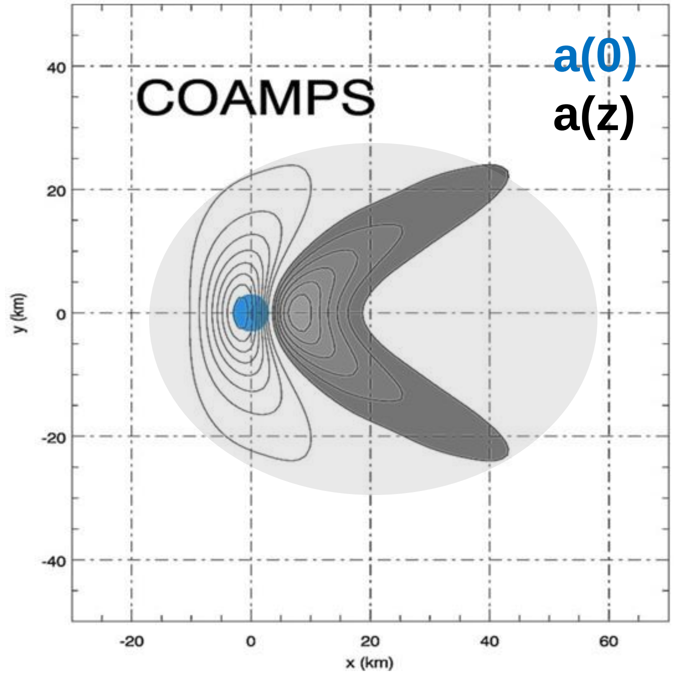
$$c_{gz} A J_h = \text{constant}$$

$$J_h = \partial(x, y)/\partial(x_0, y_0)$$

The Jacobian J_h tracks change in horizontal cross-sectional area a(z) of a "ray tube"

$c_{gz} A(0)a(0) \approx c_{gz}A(z)a(z)$:

$a(z)/a(0) >> 1 \rightarrow A(z)/A(0) << 1$



1. Wave breaking is a local criterion: $\boldsymbol{A(x,y,z) \geq A_{break}}$.
2. Parameterizations all assume no spreading: $\boldsymbol{a(z)/a(0)=1 \quad J_h=1}$

Geometrical Spreading Theory

- Eckermann, S. D., J. Ma and D. Broutman (2015), Effects of horizontal geometrical spreading on the parameterization of orographic gravity-wave drag. Part 1: Numerical transform solutions, *J. Atmos. Sci*., in press.

- Eckermann, S. D., D. Broutman and H. Knight (2015), Effects of horizontal geometrical spreading on the parameterization of orographic gravity-wave drag. Part 2: Analytical solutions, *J. Atmos. Sci*., in press.

- Knight, H., D. Broutman, and S. D. Eckermann (2015), Integral expressions for mountain wave steepness, *Wave Motion*, in press.

Adding Geometrical Spreading Terms of OGWD Parameterizations

$$\eta_a^{new}(z) = a_\eta(z)\eta_a(z) = \eta_a(0)a_\eta(z)\left[\frac{m(z)\rho(0)N^2(0)}{m(0)\rho(z)N^2(z)}\right]^{1/2}.$$

Small-l Approximation | l/k | << 1

$$\mathring{a}_{\eta_{sl}}(z') = \frac{\mathring{\eta}_{sl}(0,0,z')}{h_m G_\eta(z)} = \exp\left[\imath z'\left(\frac{1}{2\beta^2}-1\right)\right]\,\mathrm{erfc}\left[\left(\frac{z'}{\beta^2}\right)^{1/2}\frac{(1+\imath)}{2}\right].$$

Single-k Approximation $k = 1/\gamma a$

$$\mathring{a}_{\eta_{sk}}(z') = \frac{\mathring{\eta}_{sk}(0,0,z')}{h_m G_\eta(z)} = \frac{e^{-\imath z'}}{(2\imath z'\gamma^2/\beta^2+1)^{1/2}}.$$

$$z' = \int_0^z \left[\frac{N(\tilde{z})}{|U(\tilde{z})|}\right] d\tilde{z},$$

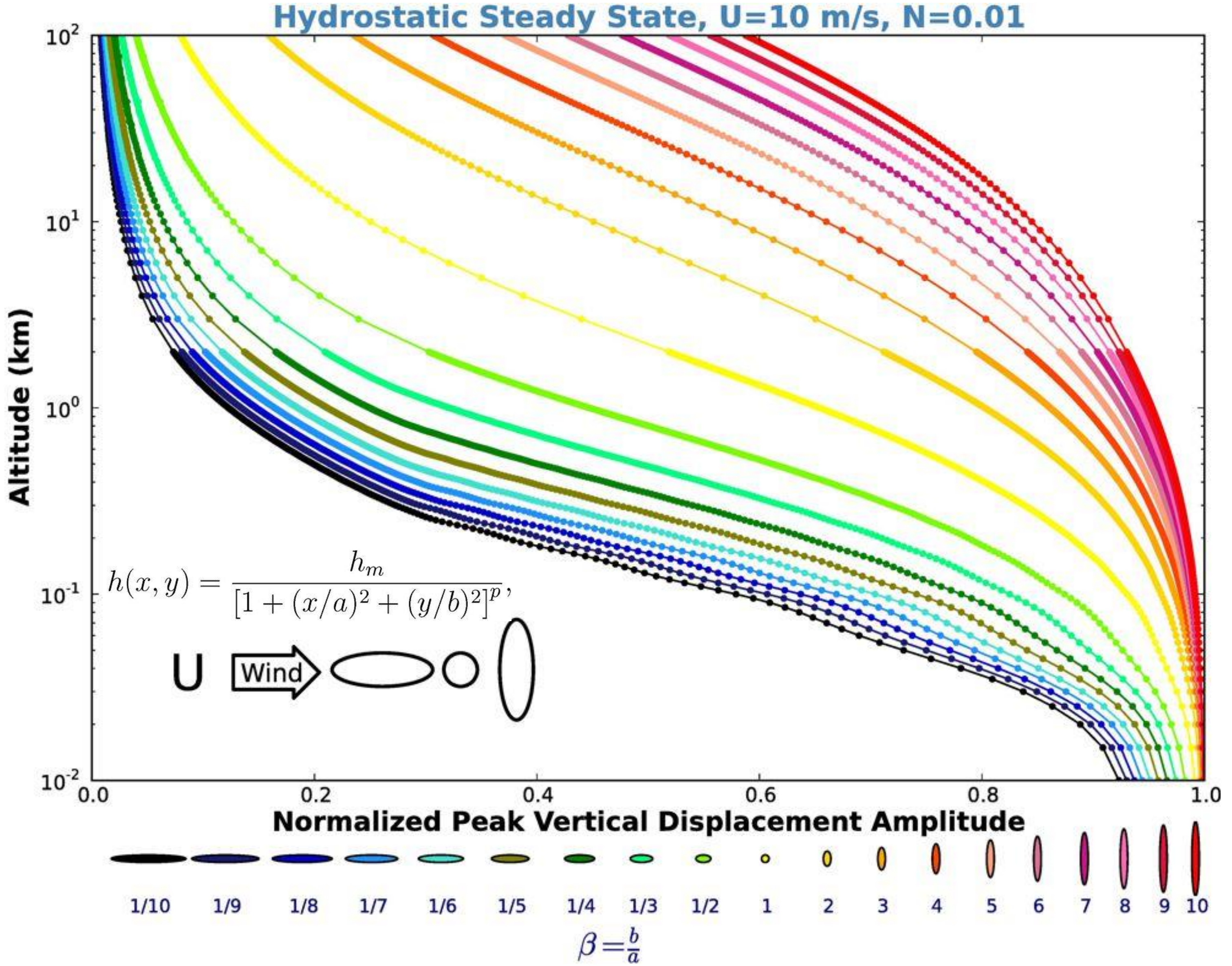
Let $z'/\beta^2 >> 1$

$$|\mathring{a}_{\eta_{z_{sk}}}(z')| \simeq |\mathring{a}_{\eta_{sk}}(z')| \simeq \left(\frac{2}{\pi}\right)^{1/2}\left(\frac{z'}{\beta^2}\right)^{-1/2}$$

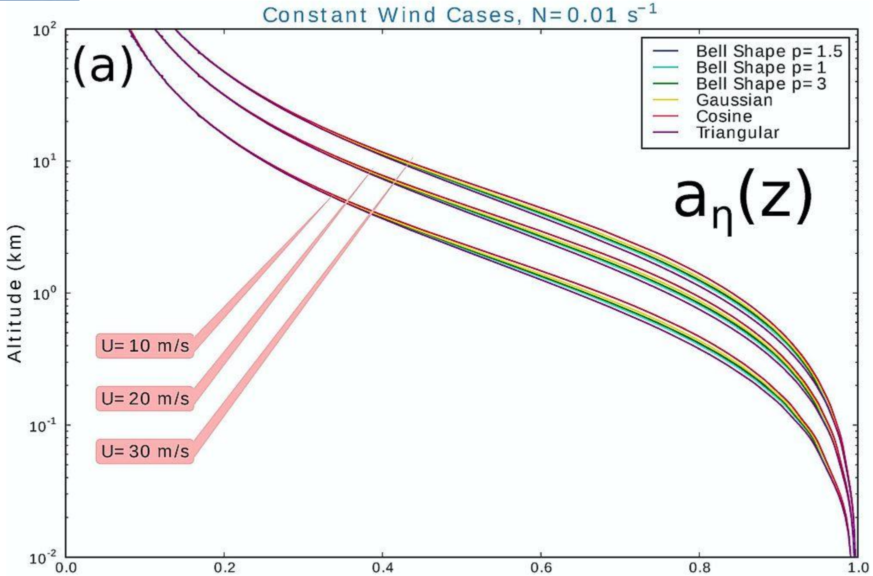
U = 10 ms^{-1}, N=0.01 s^{-1}

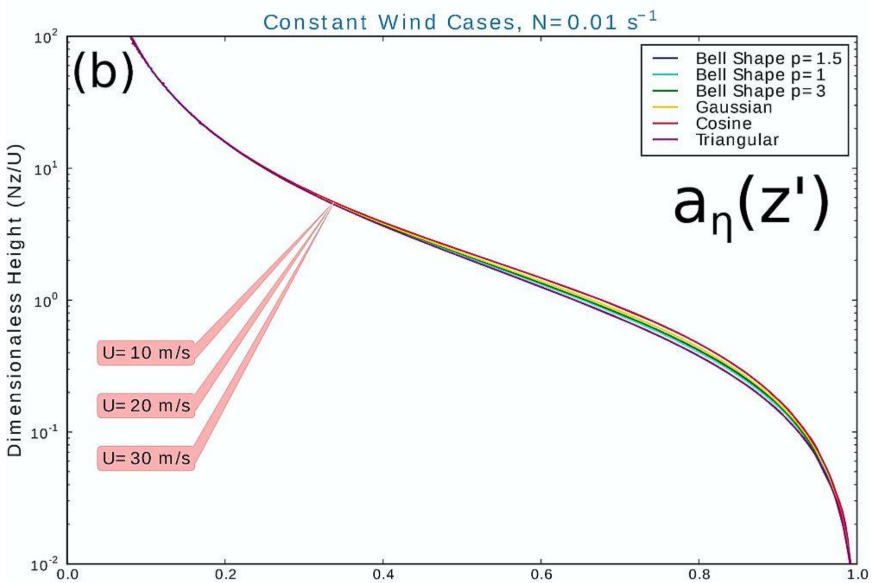
- β =1/10, z'/β^2 = 1 → z=β^2U/N = 10 meters
- β =10, z'/β^2 = 1 → z=β^2U/N = 100 km

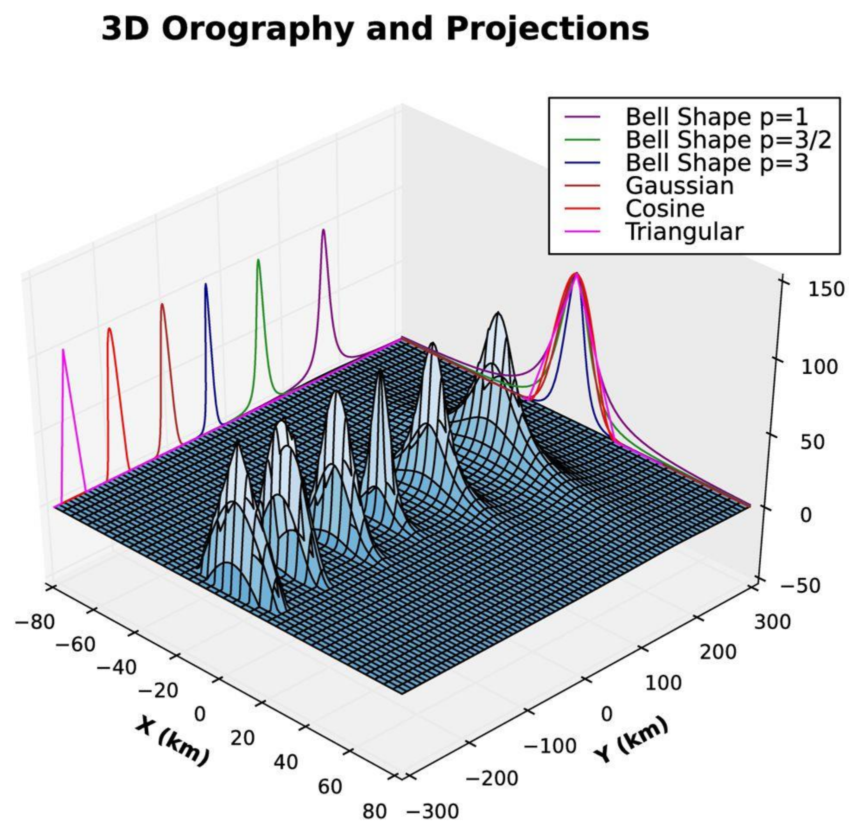
Horizontal Geometrical Spreading Curves



Sensitivity to Mountain Shape & Wind

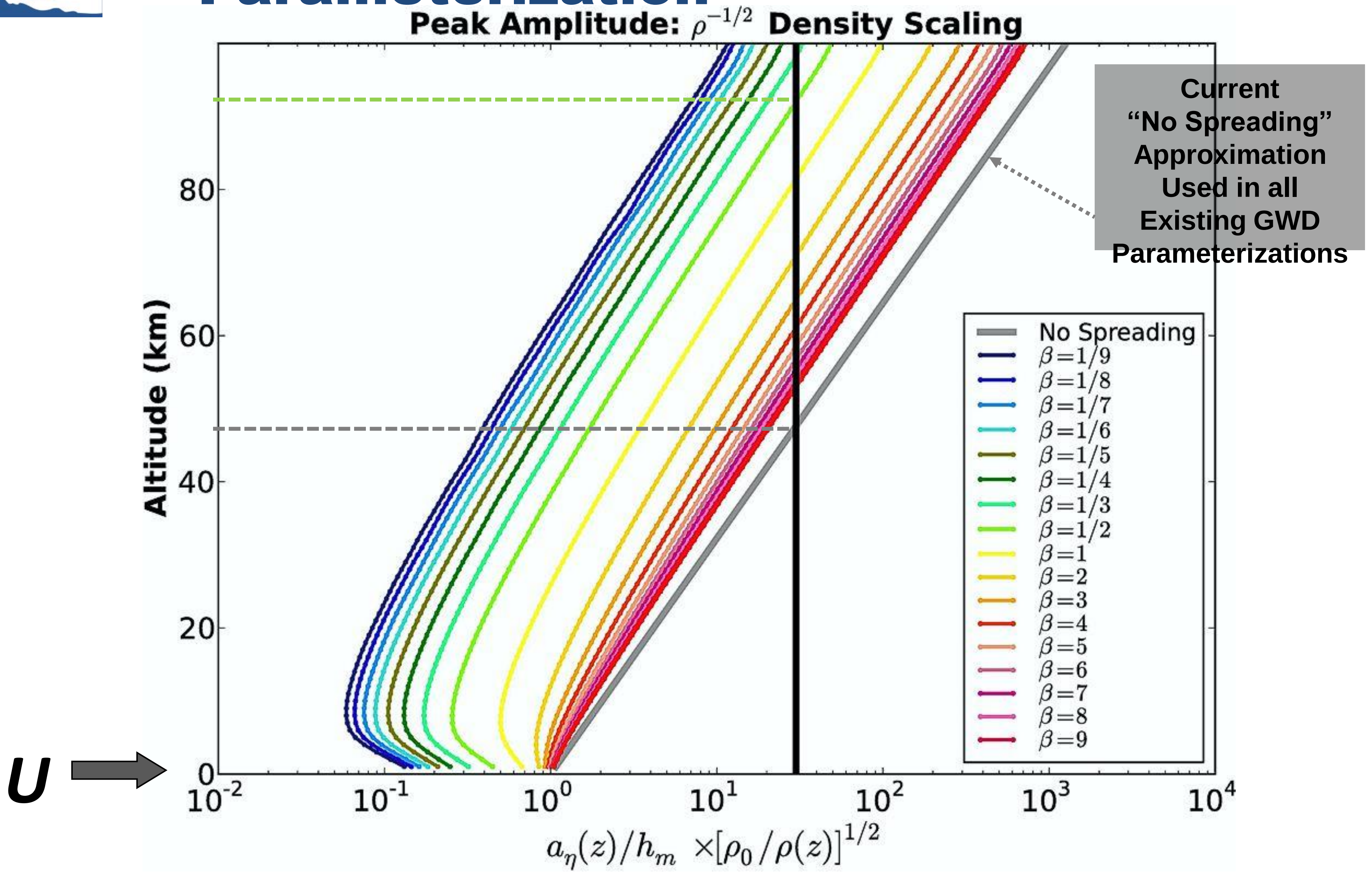






$$z' = \int_0^z \left[\frac{N(\tilde{z})}{|U(\tilde{z})|}\right] d\tilde{z},$$

Relevance to Wave Breaking Parameterization

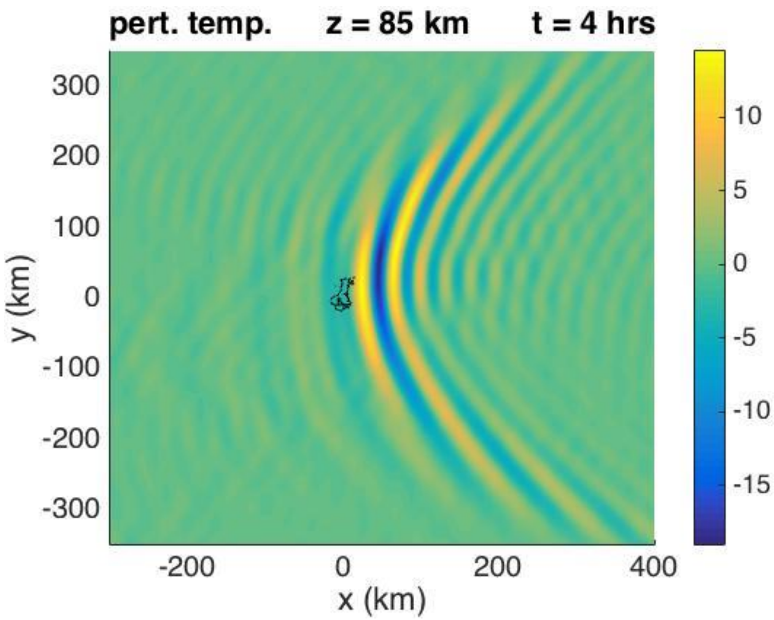


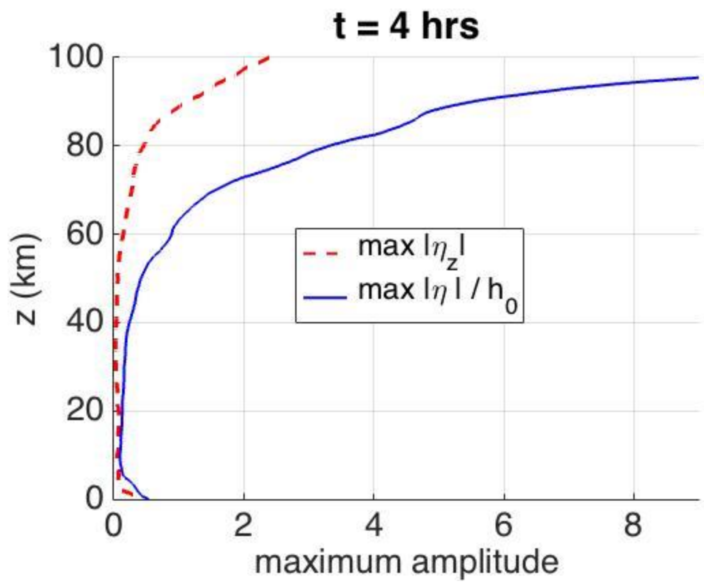
Test these Ideas for DEEPWAVE Auckland Island Case

1. Use 0-100 km winds and temperatures from NAVGEM reanalysis to define background
    a. Note: surface Froude numbers $Fr_0=|U_0|/N_0h_0 \geq 3$
2. Derive exact 3D linear transform solutions using Auckland Island topography and *U*(z), *V*(z) and *T*(z) profiles from (1)
3. Compare to RF23 AMTM images
4. Compare to orographic gravity wave drag parameterizations

Tests: Leverage Linear RF23 Solutions

T' at z = 85km



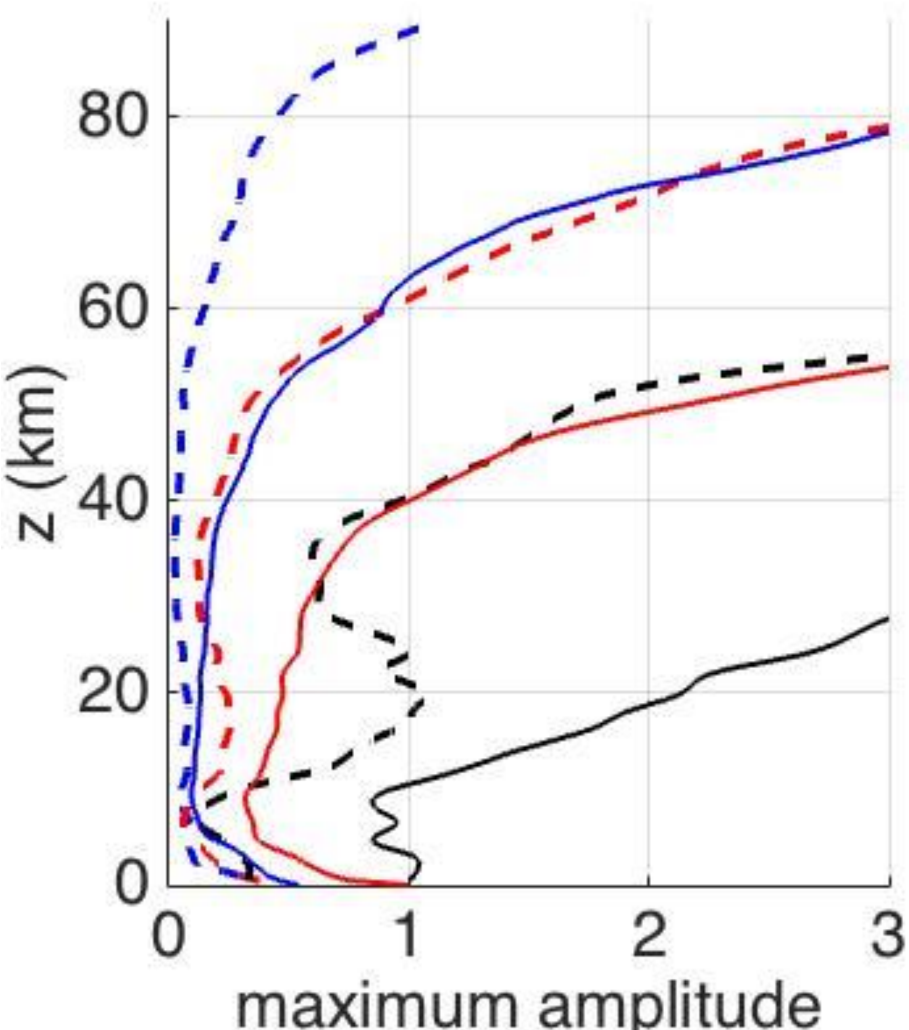


No wavebreaking until ~ 90 km altitude.

Wave activity gets to 90 km rapidly! (2-4 hours)

At lower altitudes, filtering by turning points and critical layers helps keep wave amplitudes small.

Comparison of RF23 OGWs to Parameterization Approximations



Vertical Displacement η

Old Parameterization

New Parameterization

Exact 3D Solution

Steepness η_z

Old Parameterization

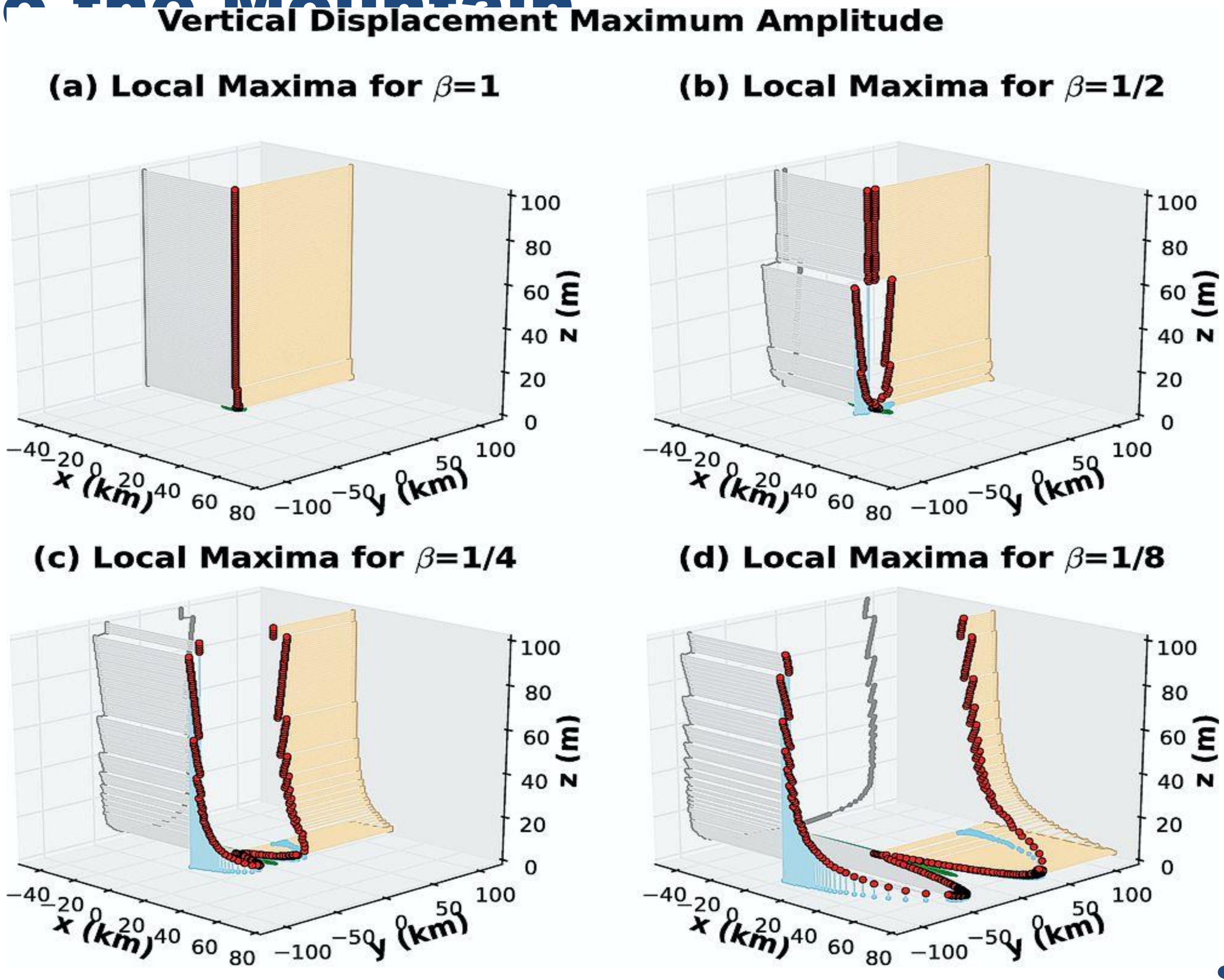
New Parameterization

Exact 3D Solution

Summary So Far……

- Validating/improving OGWD parameterizations is a major impetus for Navy involvement in DEEPWAVE
- Specific DEEPWAVE OGW cases are already providing a very useful environment for objectively testing new features developed for the NAVGEM parameterization of subgridscale OGWD
- Early work, need more guidance from 3D modelers and measurement teams

Locations of Wave Amplitude Maxima Remain Surprisingly Close to the Mountain



Mathematical Solution Summary

Eckermann, Broutman & Knight JAS, 2015

Gaussian Elliptical Orography

Hydrostatic Dispersion Relation

e Fourier transform of this Gaussian hill function is

$$\hat{h}(k,l) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)\, e^{-\imath(kx+ly)}\, dx\, dy\,,$$
$$= \frac{h_m ab}{4\pi}\exp\left(-\frac{k^2a^2}{4}-\frac{l^2b^2}{4}\right).$$

$$m = s\left(\frac{N}{|U|}\right)\left(1+l^2/k^2\right)^{1/2}$$

FR vertical displacement solution (5) and the Gaussian form (4) yields

$$\mathring{\eta}(0,0,z') = \frac{h_m ab}{2\pi}G_\eta(z)\int_{-\infty}^{\infty}\int_{-\infty}^{0} e^{\chi(k,l,z')}\, dk\, dl\,,$$

where

$$z' = \int_0^z \left[\frac{N(\tilde{z})}{|U(\tilde{z})|}\right] d\tilde{z},$$

$$\chi(k,l,z') = -\imath z'\left(1+\frac{l^2}{k^2}\right)^{1/2} - \frac{k^2a^2}{4} - \frac{l^2b^2}{4}.$$

3D Mountain Wave Transform Solution Above Mountain at *x*=*y*=0

How to approximate the *exp*(χ) integral?

Small-*l* Approximation | *l*/*k* | << 1

$$\left(1+\frac{l^2}{k^2}\right)^{1/2} \simeq 1+\frac{l^2}{2k^2}.$$

The small-l approximation of (16) is then

$$\chi_{sl}(k,l,z') \simeq -\imath z'\left(1+\frac{l^2}{2k^2}\right) - \frac{k^2a^2}{4} - \frac{l^2b^2}{4}.$$

$$\mathring{a}_{\eta_{sl}}(z') = \frac{\mathring{\eta}_{sl}(0,0,z')}{h_m G_\eta(z)} = \exp\left[\imath z'\left(\frac{1}{2\beta^2}-1\right)\right]\ \mathrm{erfc}\left[\left(\frac{z'}{\beta^2}\right)^{1/2}\frac{(1+\imath)}{2}\right].$$

Single-*k* Approximation *k* = 1/γ*a*

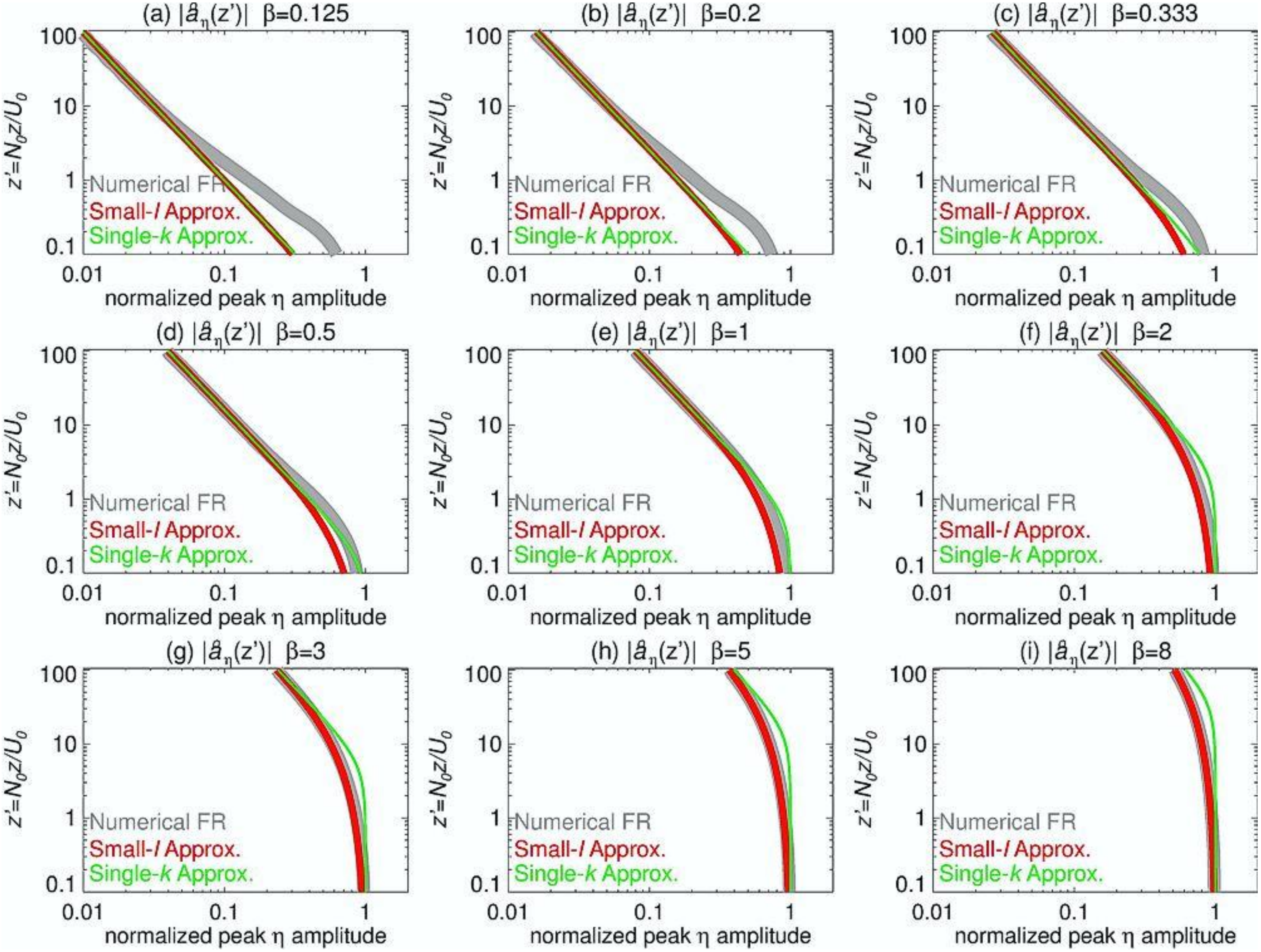
$$(1+l^2/k^2)^{1/2} \simeq 1+\tfrac{1}{2}l^2\gamma^2a^2\,,$$

leading to a single-k approximation to the exponential argument (16) of

$$\chi_{sk}(k,l,z') \simeq -\imath z'\left(1+\frac{l^2\gamma^2a^2}{2}\right) - \frac{k^2a^2}{4} - \frac{l^2b^2}{4},$$
$$= -\imath z' - \frac{k^2a^2}{4} - \frac{l^2b^2}{4}\left(2\imath z'\gamma^2/\beta^2+1\right).$$

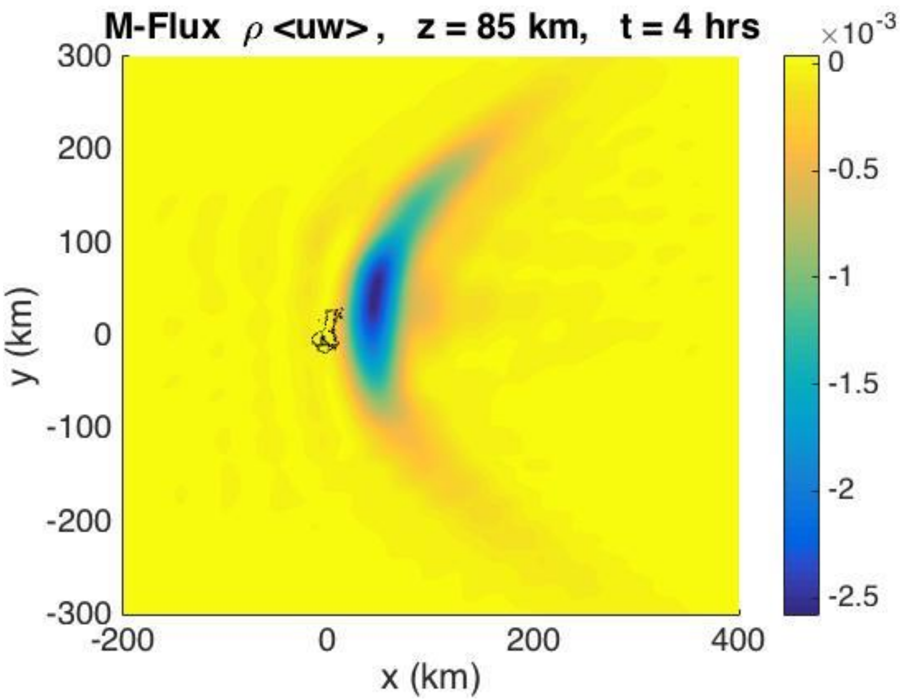
$$\mathring{a}_{\eta_{sk}}(z') = \frac{\mathring{\eta}_{sk}(0,0,z')}{h_m G_\eta(z)} = \frac{e^{-\imath z'}}{(2\imath z'\gamma^2/\beta^2+1)^{1/2}}.$$

Analytical Approximations

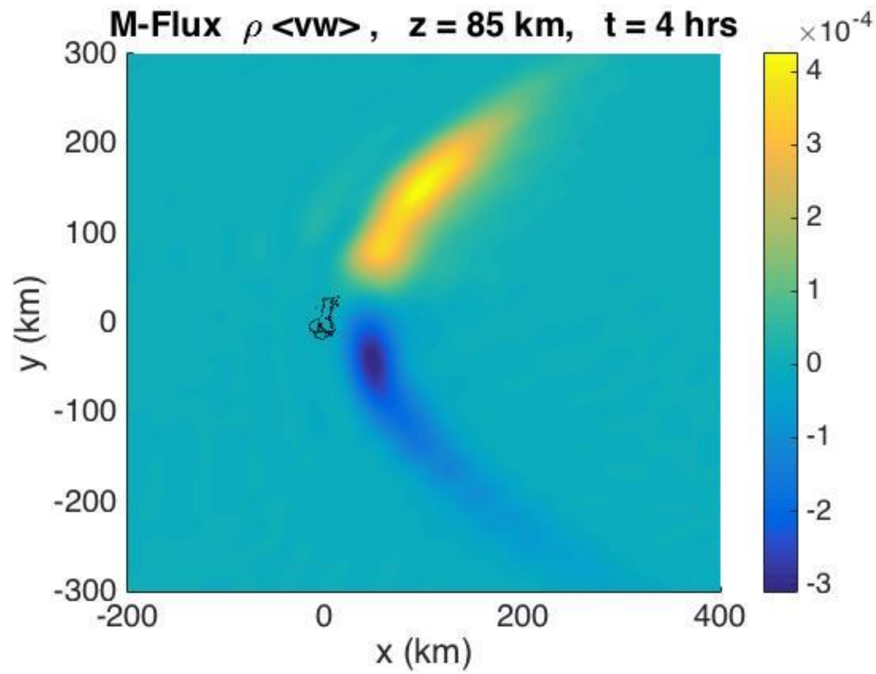


Momentum Fluxes at 85 km

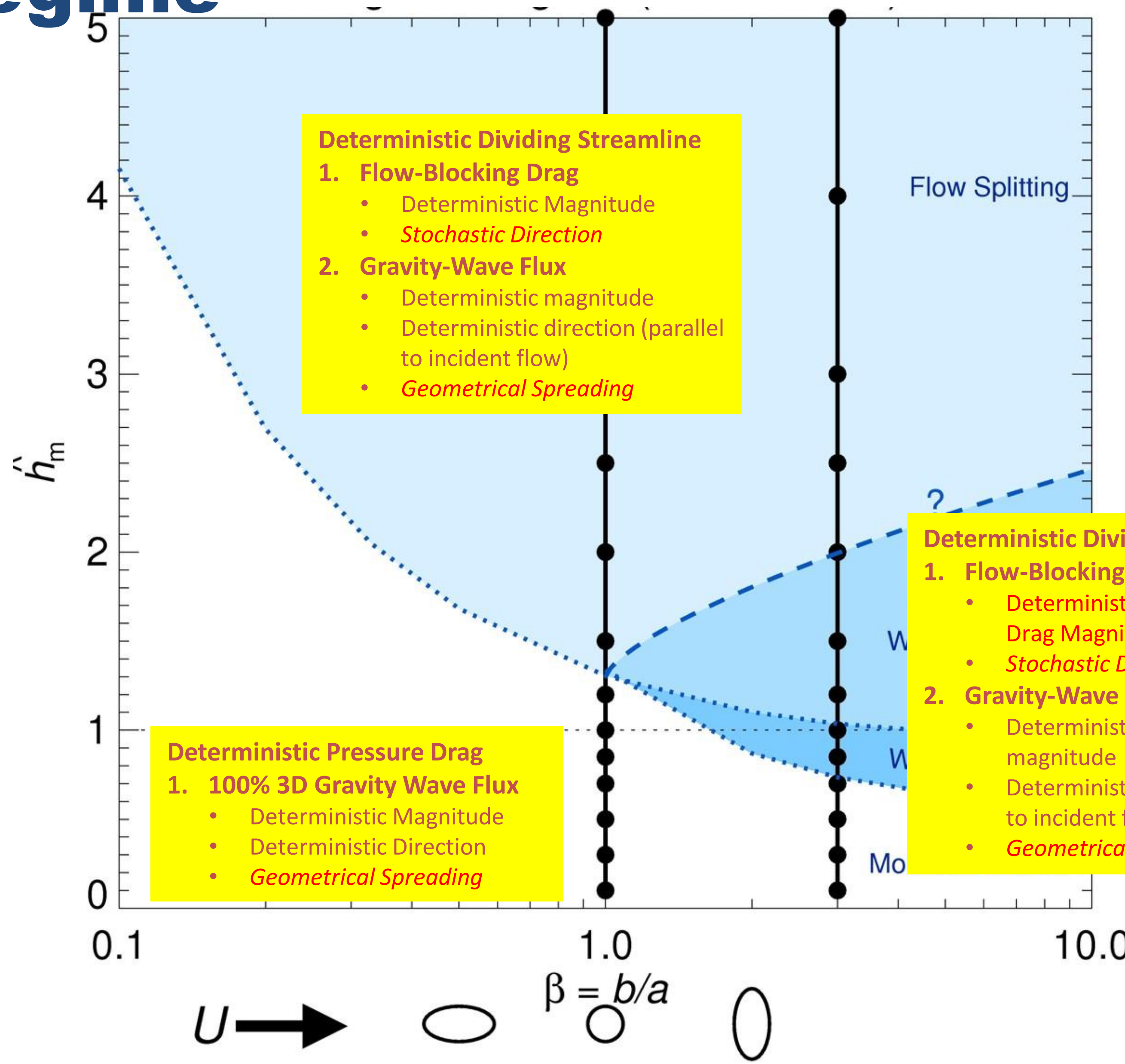
ρ < uw >



ρ < vw >



Parameterization of 3D OMD Regime



Deterministic Dividing Streamline

1. **Flow-Blocking Drag**
   - Deterministic Magnitude
   - *Stochastic Direction*
2. **Gravity-Wave Flux**
   - Deterministic magnitude
   - Deterministic direction (parallel to incident flow)
   - *Geometrical Spreading*

Deterministic Pressure Drag

1. **100% 3D Gravity Wave Flux**
   - Deterministic Magnitude
   - Deterministic Direction
   - *Geometrical Spreading*

Deterministic Dividing Streamline

1. **Flow-Blocking Drag**
   - Deterministic+Stochastic High-Drag Magnitude
   - *Stochastic Direction*
2. **Gravity-Wave Flux**
   - Deterministic+*Stochastic* magnitude
   - Deterministic direction (parallel to incident flow)
   - *Geometrical Spreading*

Quantifying Horizontal Geometrical Spreading Effects on Wave Amplitude

Eckermann et al., JAS, in press, 2015a 2015b

- Use a Hilbert transform technique to drive local wave amplitudes from exact numerical transform solutions for linear three-dimensional mountain waves

$$\mathring{X}(x,y,z) = X_A(x,y,z) e^{\imath \psi_X(x,y,z)}$$

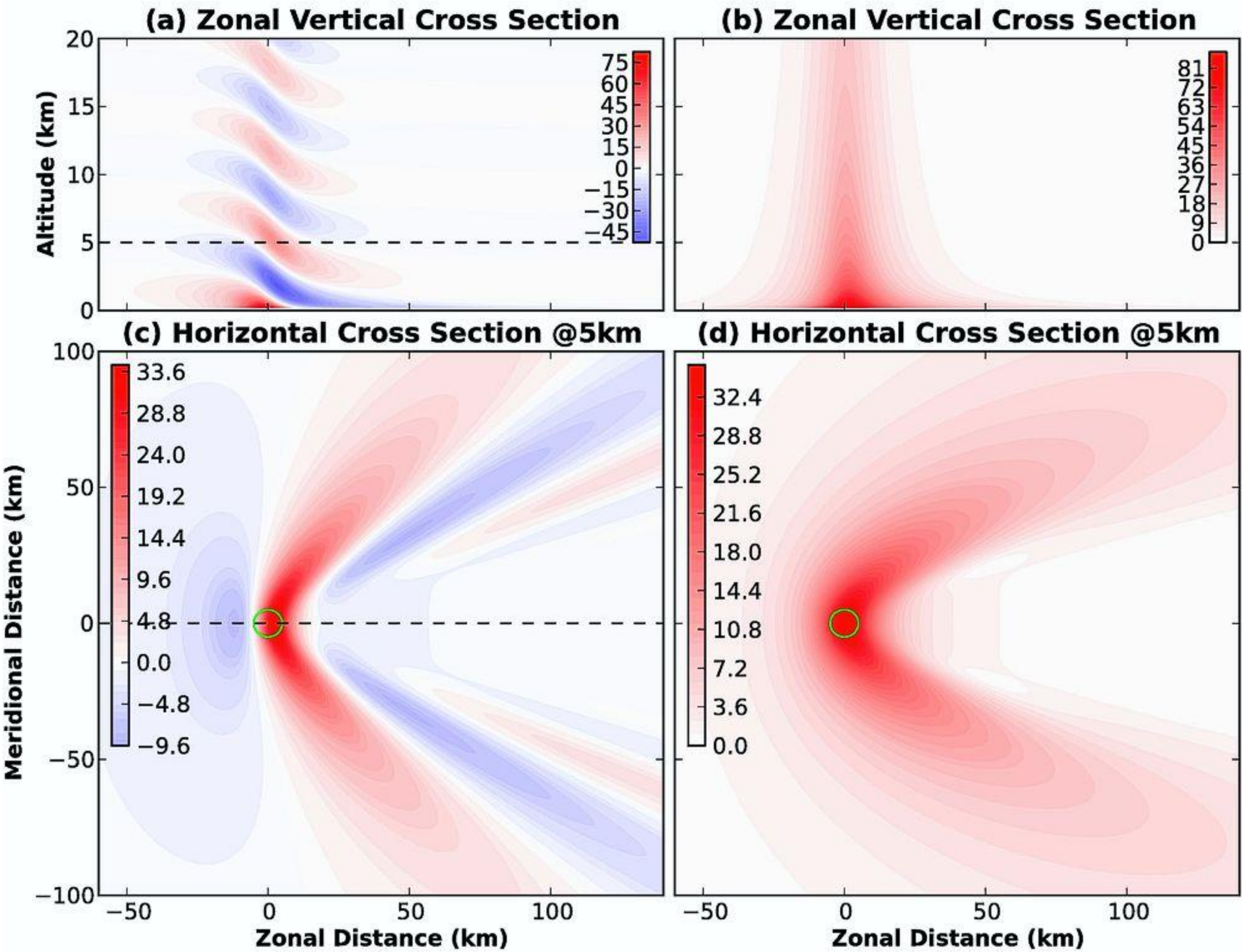
- Locate and quantify largest wave amplitude at each altitude (most likely location for wave breaking)

$$X_A^{max}(z) = \max\left[X_A(x,y,z)\right].$$

- For hydrostatic solutions, vertical refraction terms that affect wave amplitudes can be well approximated by simple height profiles ***G(z)*** that depend only on background atmospheric parameters: e.g.,
  - Hydrostatic WKB solutions have an $[m(z)/(m(0)]^{1/2}$ amplitude dependence with height for vertical displacements $\eta(x,y,z)$
  - yet vertical wavenumbers $m(z) \approx N(z)/U(z)$ where $N(z)$ is buoyancy frequency and $U(z)$ is horizontal wind profile, thus $\mathbf{G(z)=[N(z)U(0)/N(0)U(z)]^{1/2}}$.
- Normalize the peak wave amplitudes to isolate the horizontal geometrical spreading effect on wave amplitude evolution with height

$$a_\eta(z) = \frac{\eta_A^{max}(z)}{h_m G_\eta(z)}$$

Hilbert Transform Removal of Phase to Yield Peak Amplitude Solutions



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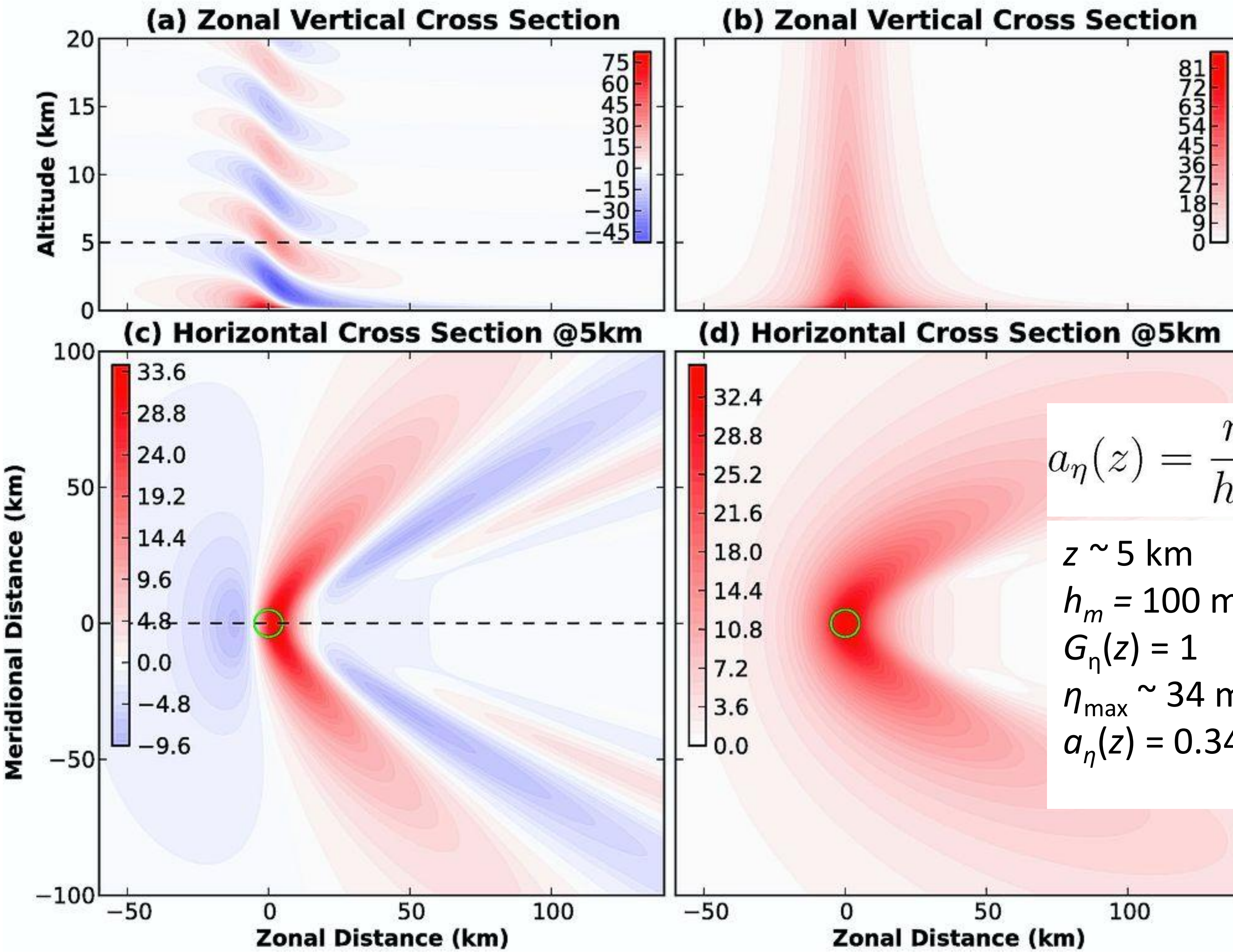

$$a_\eta(z) = \frac{\eta_A^{max}(z)}{h_m G_\eta(z)}$$

z ~ 5 km
h_m = 100 m
$G_\eta(z)$ = 1
η_{max} ~ 34 m
$a_\eta(z)$ = 0.34