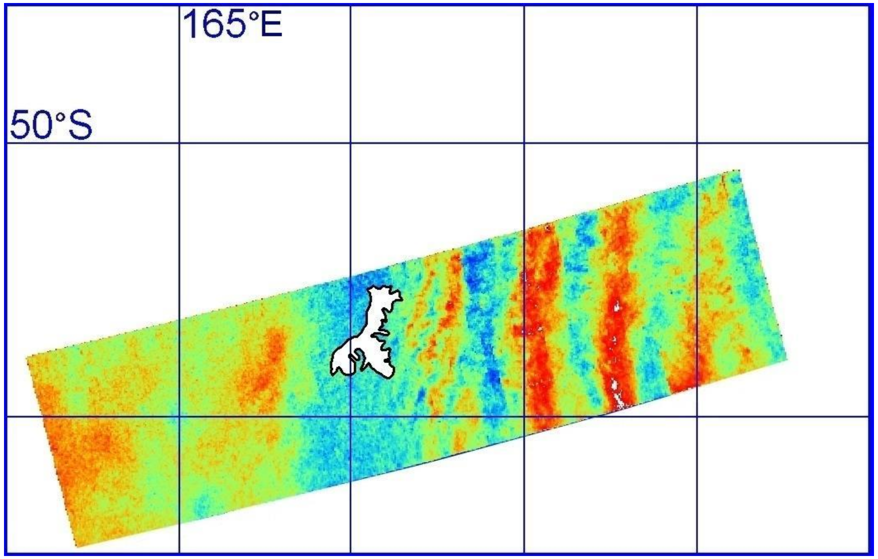
Fourier modeling of mountain waves from Auckland Island.

Dave Broutman
Jun Ma
Stephen Eckermann

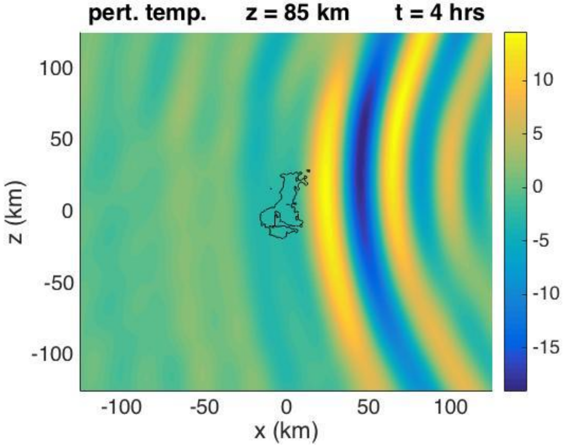
Motivation: Airglow data RF23

Possible mt-waves ~85 km above Auckland Island.



Courtesy of Dominique, Mike Taylor.

z = 85 km



1. How does the Fourier method compare? (Good, but…)
2. Any extra understanding from the Fourier method.

Fourier Method
2D Fourier integral over horizontal waveno's *k,l*

$$\eta(\mathbf{x},\mathbf{y},\mathbf{z}) = \iint_{-\infty}^{\infty} \tilde{\mathbf{h}}(\mathbf{k},\mathbf{l})\ \underbrace{\mathbf{A}(\mathbf{k},\mathbf{l},\mathbf{z})\,\mathbf{e}^{\imath\int \mathbf{m}\mathbf{d}\mathbf{z}}}\ \ \mathbf{e}^{\mathbf{i}(\mathbf{k}\mathbf{x}+\mathbf{l}\mathbf{y})}\,\mathbf{d}\mathbf{k}\,\mathbf{d}\mathbf{l}$$

3. IFT at each z to get vertical displacement, etc.

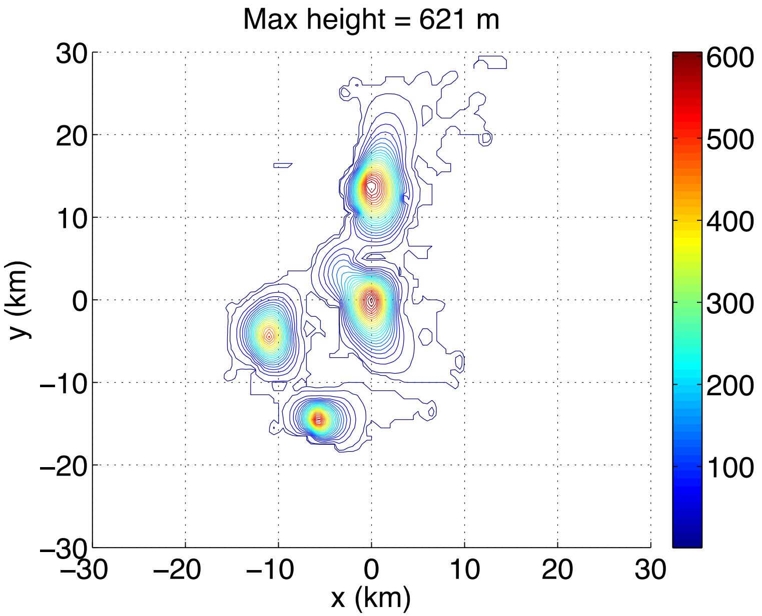
2. Weight with Fourier transform of topography.

1. Solve for the vertical (z) dependence of each Fourier component (k,l), with m = vert.waveno.

Linear method, horizontally uniform background.

Exclude downgoing waves reflected from turning points.

Auckland Island



460 km south of NZ.

51ºS, 166ºE

Max ht. ~ 650 m

Fourier-method discretization:

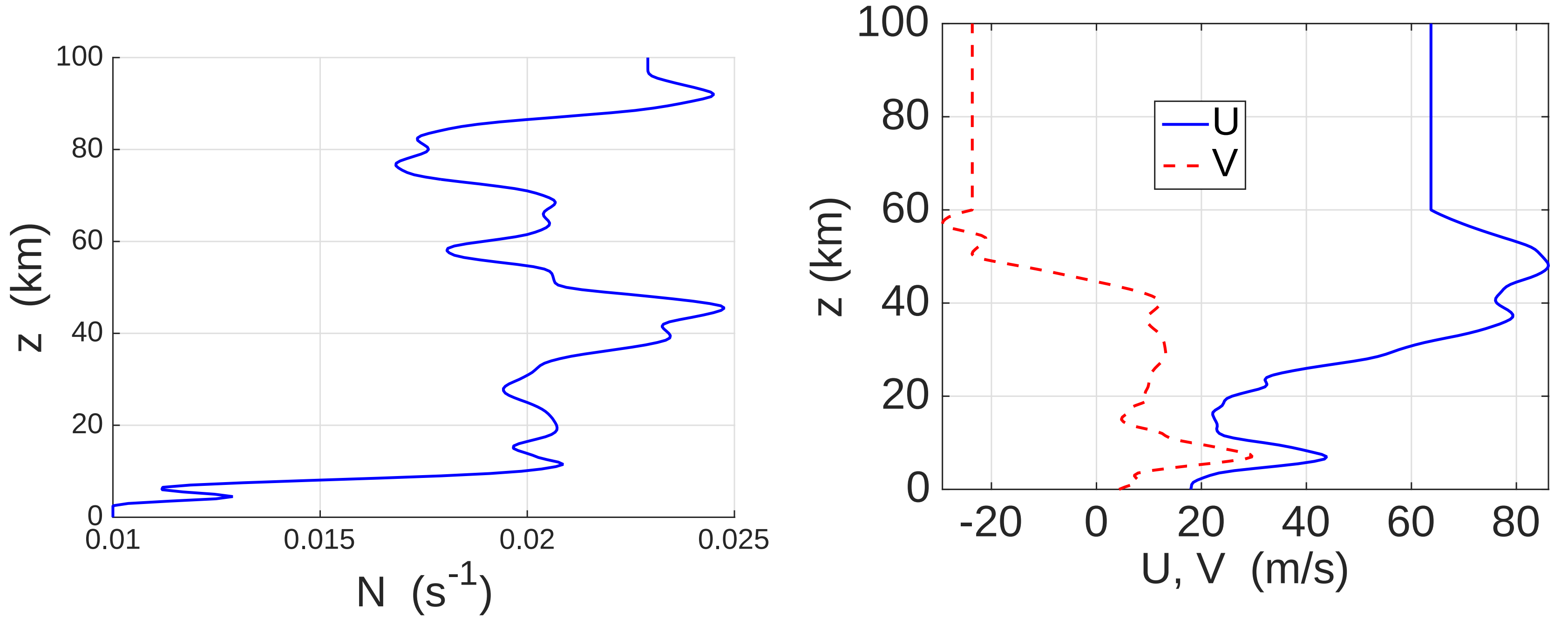
horiz.: ***1024 km x 1024 km, 1 km grid spacing.***

vert: IFT at 1km intervals, 0 -- 100km altitude.

Computation time ~ 10 minutes (Matlab)

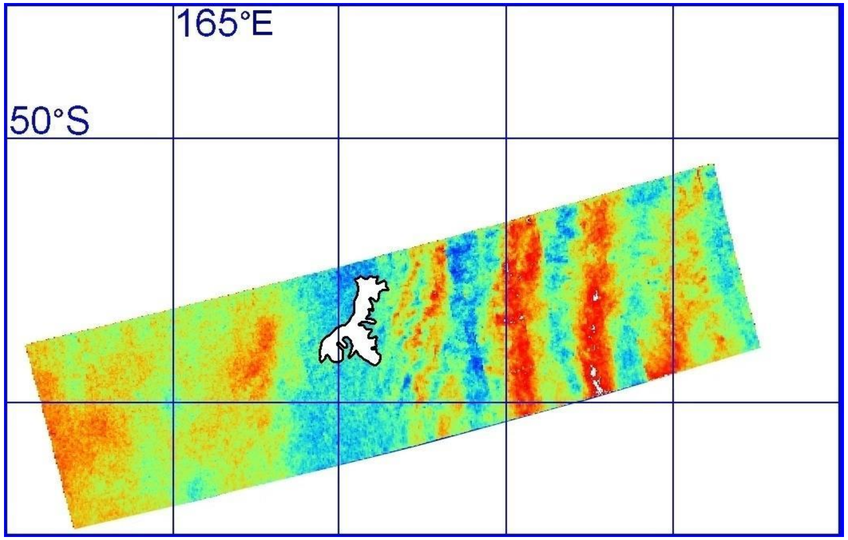
N(z),U(z),V(z). *0600 14 July 2014 (RF23)*

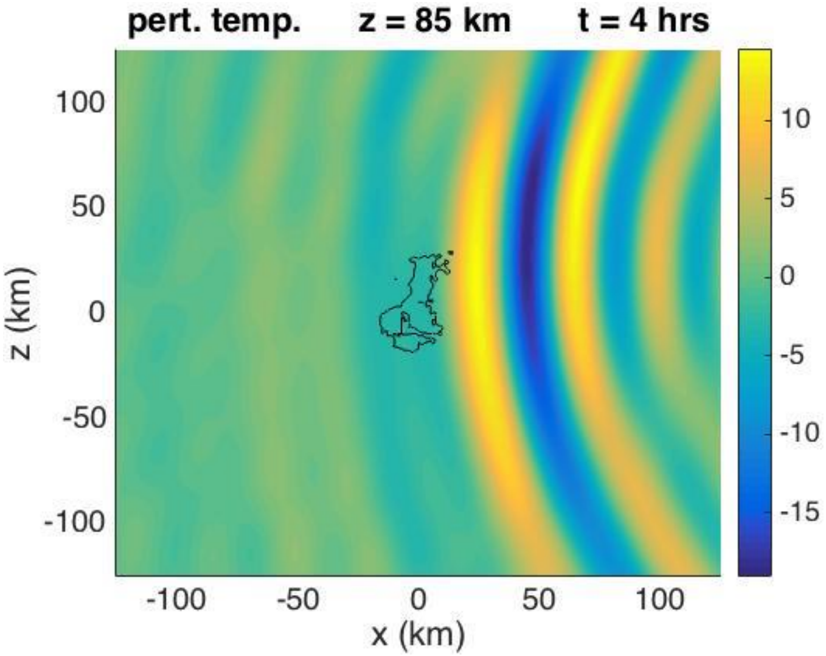
Courtesy of Jun Ma:
dropsonde up to 12km, ECWMF up to 70km, NAVGEM to 100km



First example: U, V constant at z > 60 km.

Airglow comparison with Fourier solution



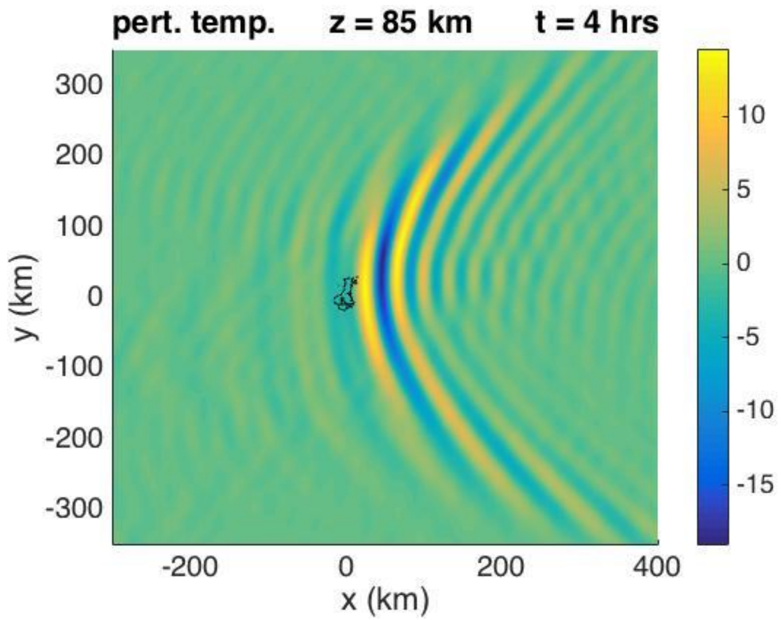


Airglow -- red is T’ > 0.
30 K T range

Fourier. T’ at z = 85km

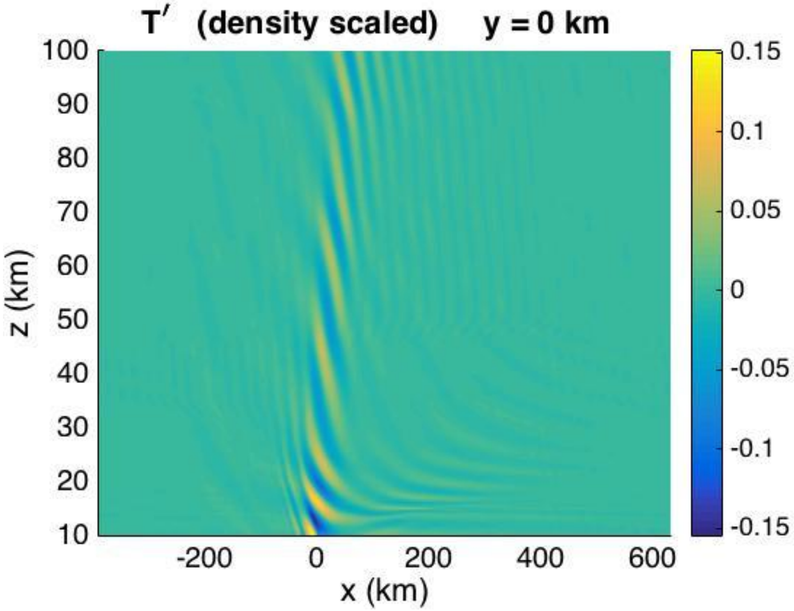
Similar wavelength (~40 km), similar phase orientation, similar T’ range of 30 K, and in phase.

Fourier solution for pert. temp. T’



T’ at z = 85km.

T Range -20 K -- 15 K.



T’ at y = 0

(Density scaled.)

- ***Standard lee wave pattern.***
- ***Lots of geometrical spreading.   Phases tilt upwind.***
- ***Waves do not reach wave-breaking amp. until z > 90 km altitude.***

How do the waves get to 90 km altitude without wavebreaking?

Processes:

- *Density effect*: ~exp($z/2H_\rho$) ~ 1000 at z = 90 km.
- *Geometrical spreading:* amps. decrease as $1 / z^{1/2}$
- (R. Smith 1980, idealized case.)
- *Filtering (trapping at lower altitudes) by turning points or critical layers.*

  Large part of the spectrum is left behind….

  Helps limit amplitudes as waves propagating upward.

DE only: wavebreaking at z ~ 25km. (for these N,U,V)

GS only: no wavebreaking.

DE + GS only: wavebreaking at z ~ 45 km.

DE + GS + FILTERING -- wavebreaking ~90 km

Fourier Components: Bookkeeping

524,288 ($=1024^2 / 2$) independent Fourier components *k, l.*

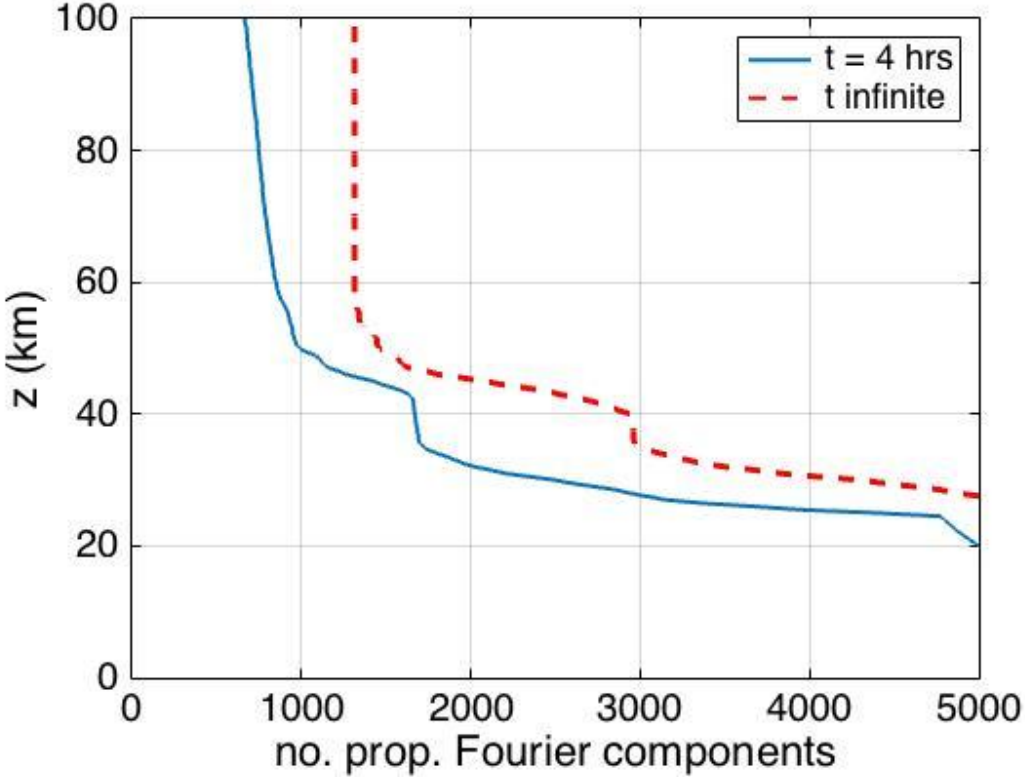
90,000 are initially propagating ($m^2 > 0$ at $z = 0$).

40,000 are *barely* propagating, with turning points
or critical layers at $z < 1$ km.

35,000 have turning points or critical layers
between 1 km $< z <$ 5km.

(These numbers are for the above U,V,N profiles.)

No. Propagating Fourier components vs. altitude.



From 90,000 at z = 0 to 666 at z = 100 km (with t = 4hr filter).
Typical in AI results: ~100 Fourier components at z = 100km.

Histogram of TP's , CL's with altitude.

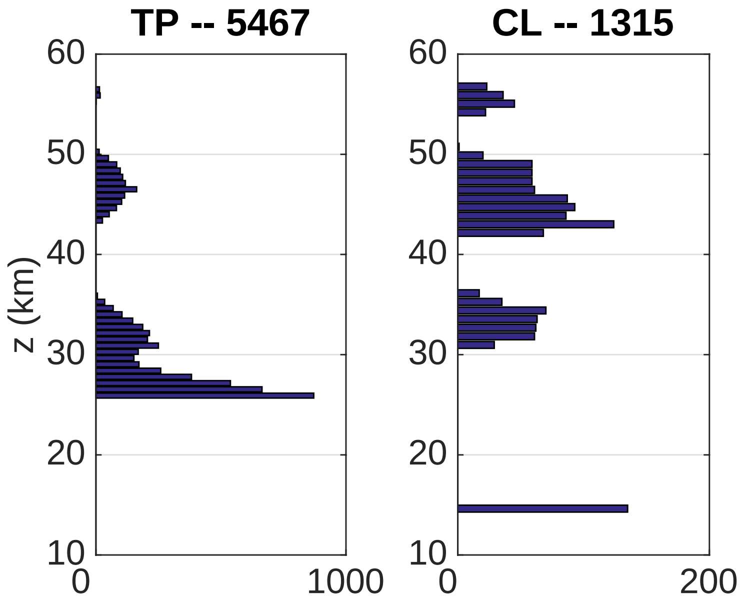
TP = turning point

=> $m = 0$

CL = Critical layer

=> $m \rightarrow \infty$

z = 10 km – 60 km.



Number of Fourier components

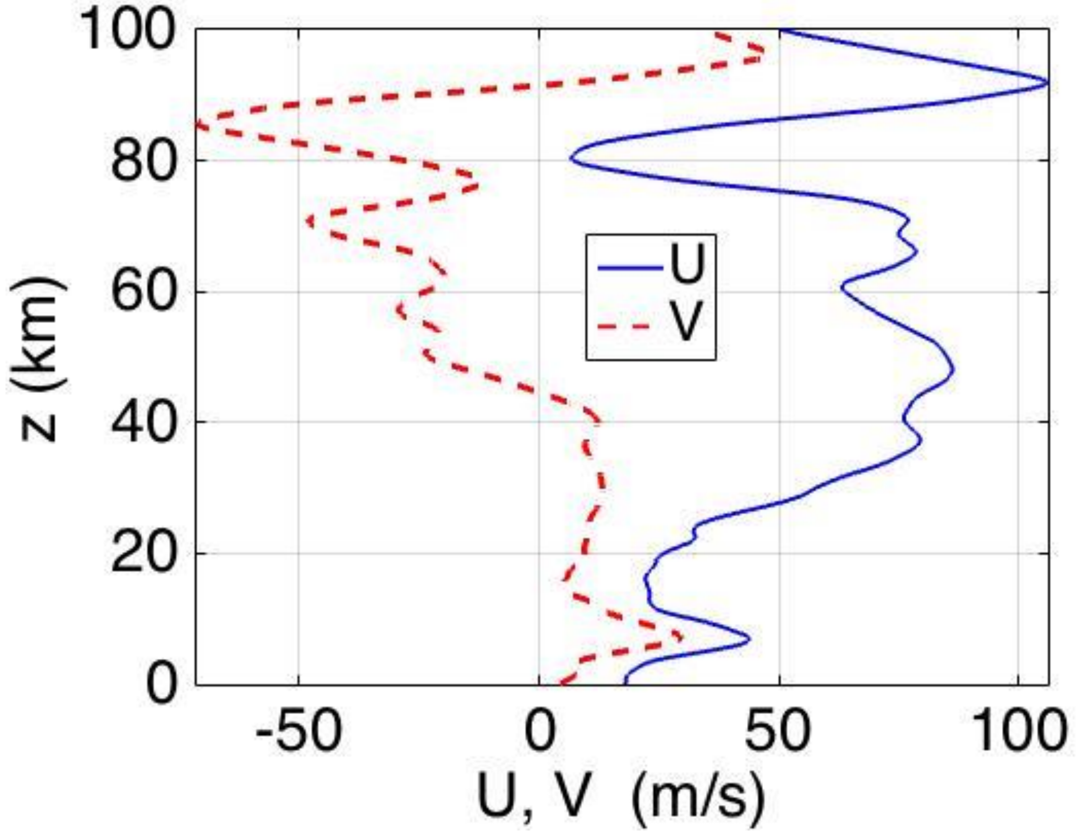
- *Most TP's, CL's below z = 10 km. None above 60 km.*
- *No wavebreaking at any of these critical layers, according to the Fourier method. (More later.)*

2nd EXAMPLE
Variable winds for z > 60km.

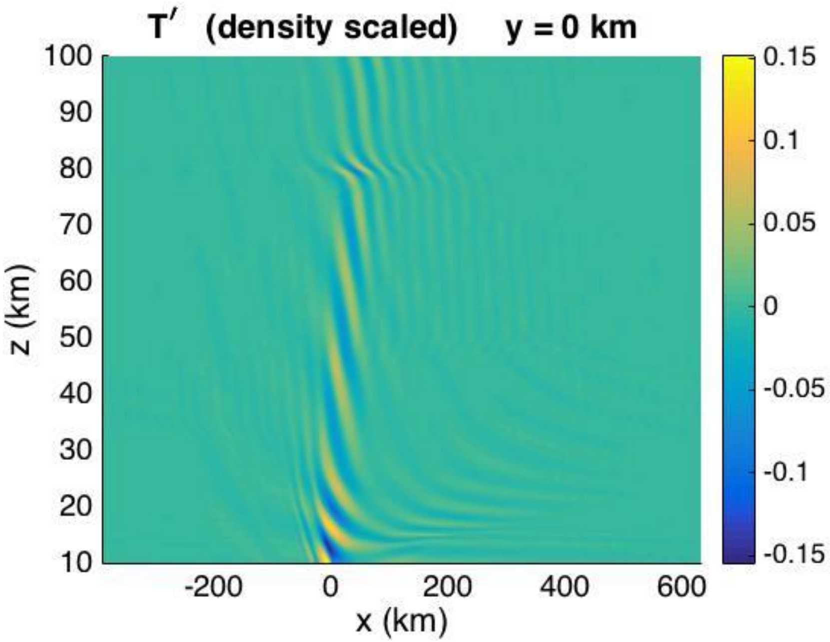
z > 60 km.
BEFORE: winds constant

Now winds from NAVGEM for RF 23, z > 60km .

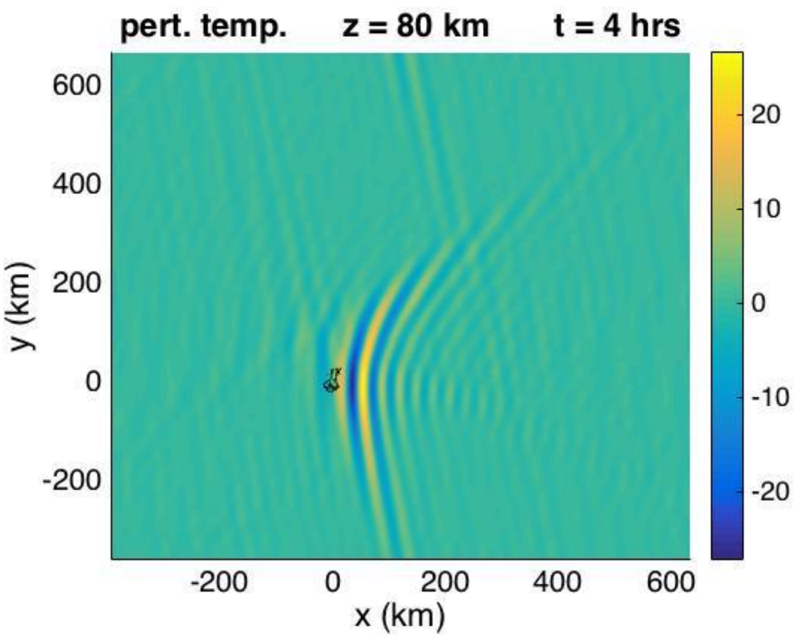
Strong tidal component.



Fourier solution for T’.



Density scaled.



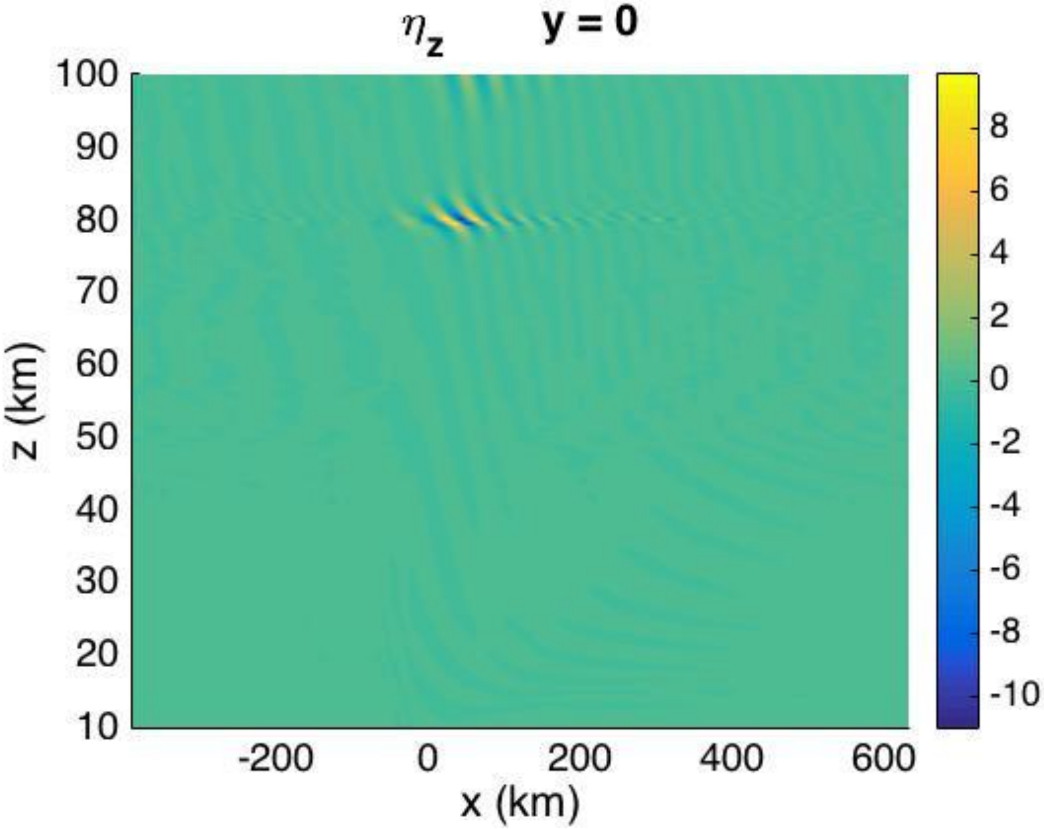
z = 80 km

Vertical cross-section – strong refraction near z = 80 km.

Horizontal cross-section. Larger T’ than before.

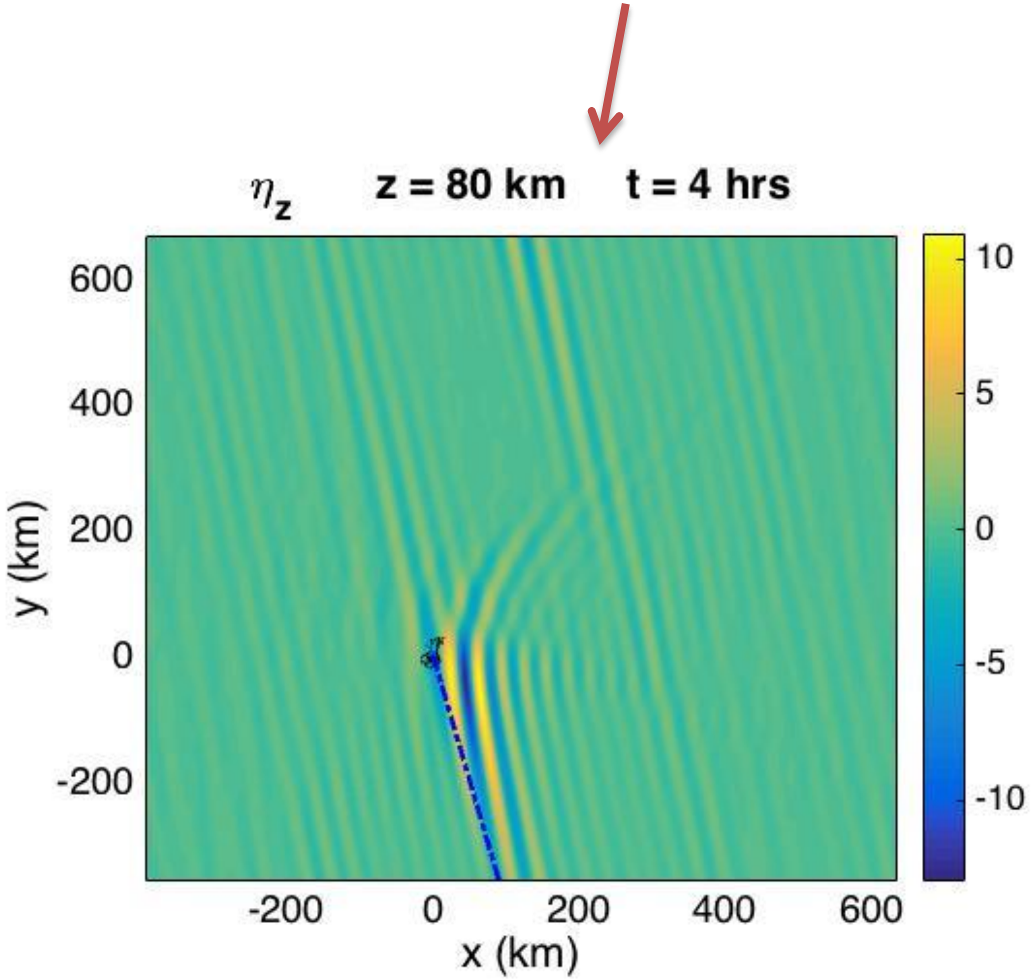
Wavebreaking? Critical layers?

Wave steepness η_z .
η_z > 1 convective overturning (linear theory).



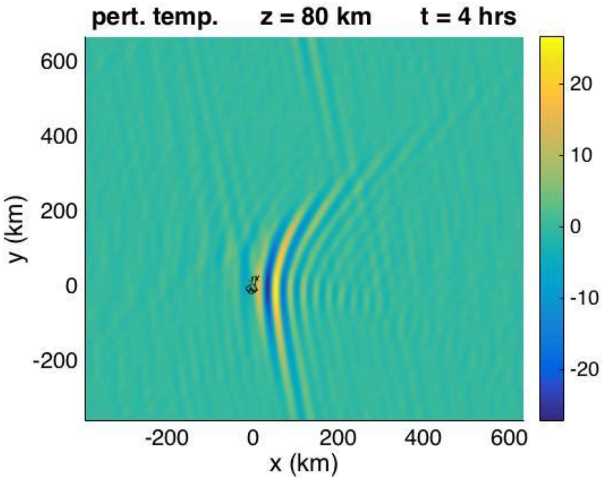
Wave steepness > 1 only at altitudes 78 -84 km.
Critical layers associated with tidal winds.

Wave steepness η_z at z = 80 km.

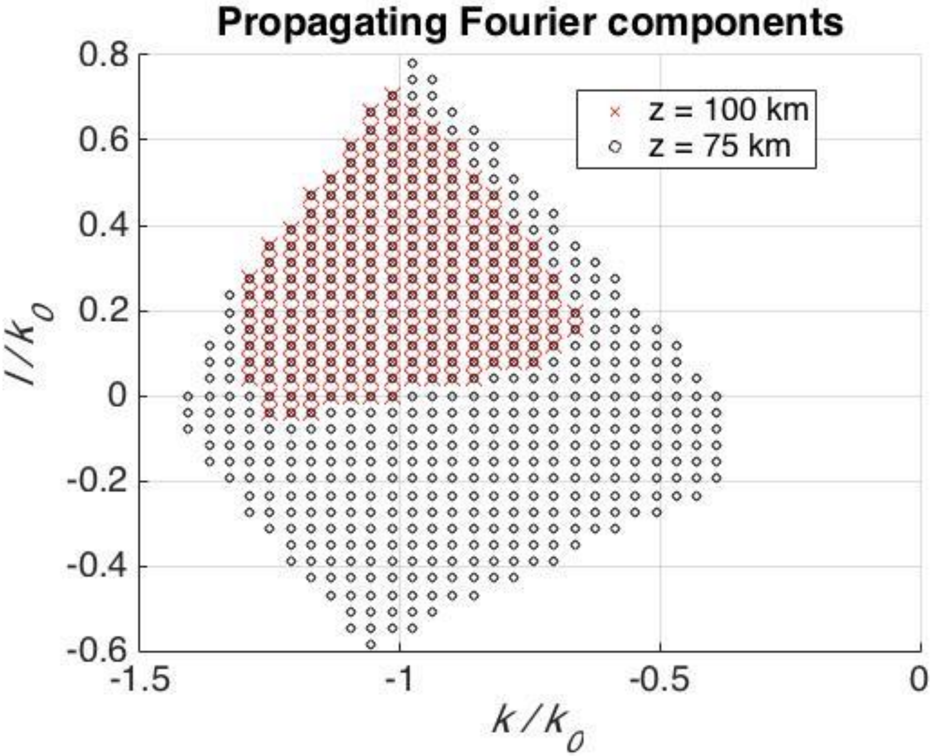


Near 3D critical layers, wave steepness tends to spread out in the direction of the local wind. Here: South of Auckland Island

Perturbation temp.



Solid line – wind direction at z = 80 km.
N-S asymmetry due to direction wind shear

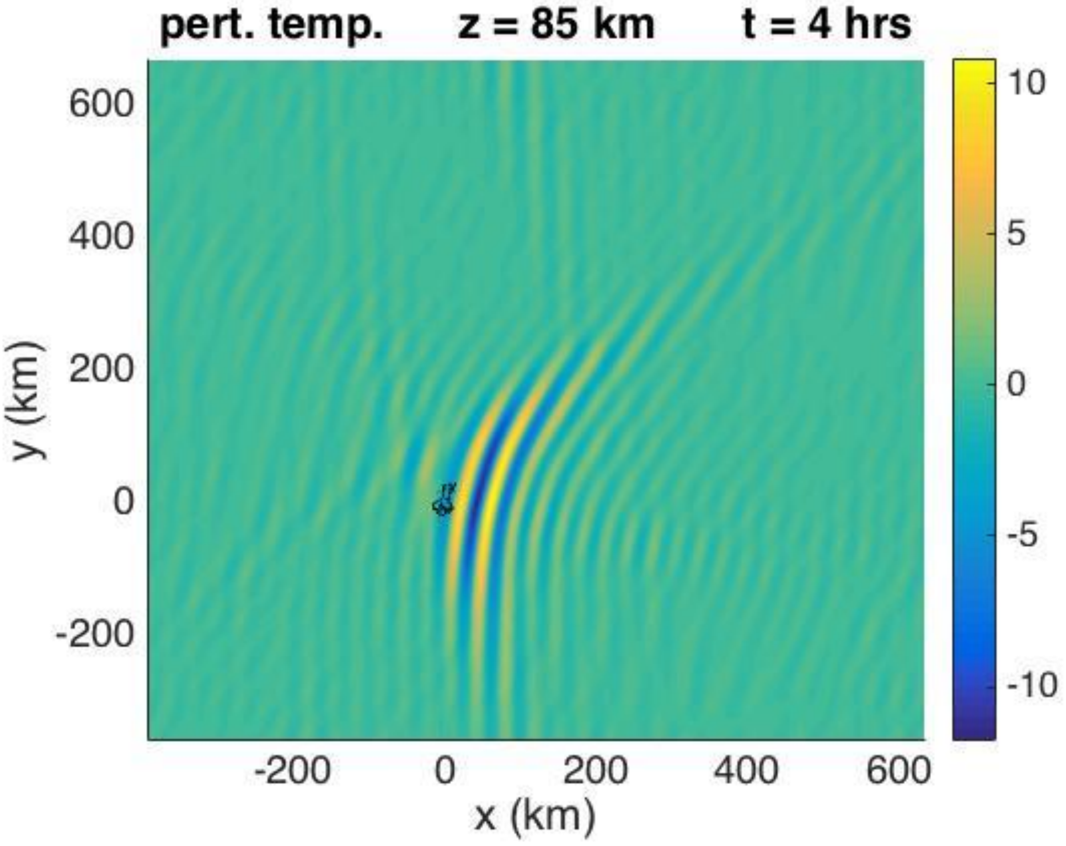


$k_0 = 2\pi/40$ km

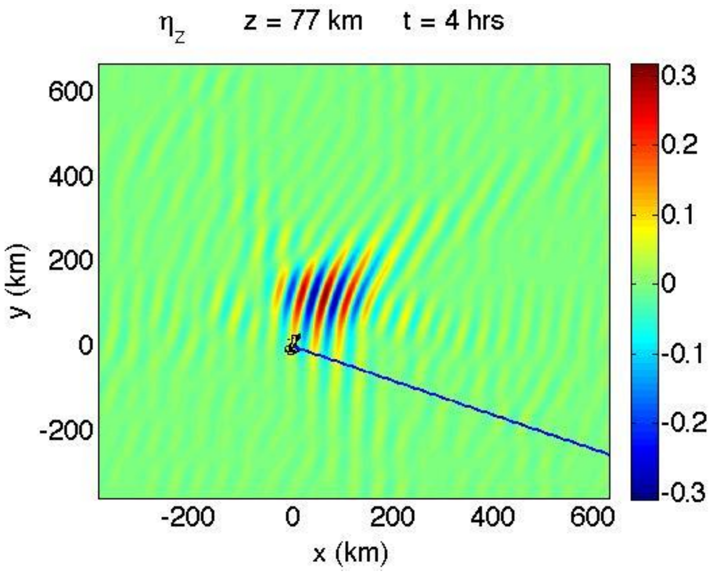
Filter out all Fourier components with critical layers
And only leave those Waves that reach 100 km altitude.

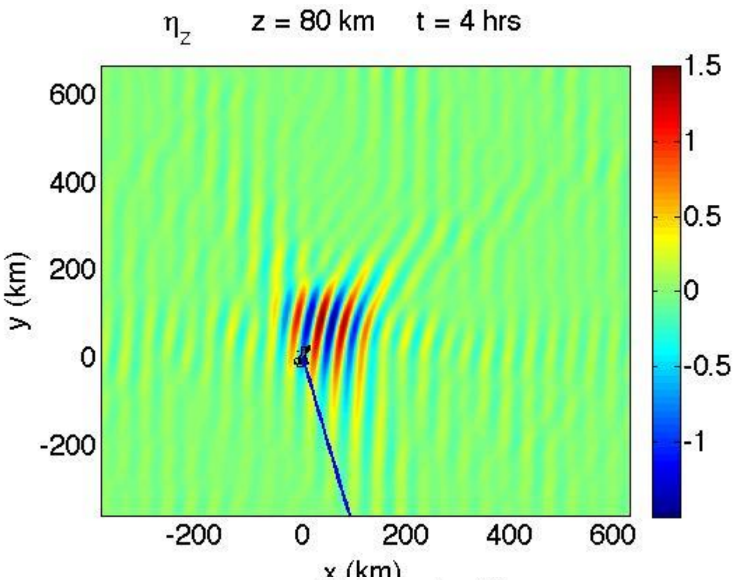
Leaves red crosses..

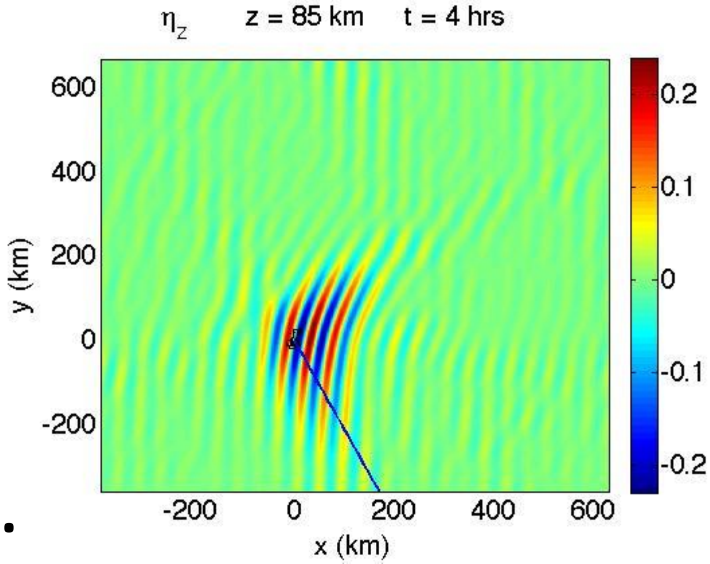
Fourier T’ z = 85 km after critical layer filtering.



Fourier solution for wave steepness for only Fourier components that reach z= 100km.







Conclusion: these tidal winds lead to wavebraking In the Fourier model near z = 80 km.

Different tidal phase – turning points.

Issues

1. ***Winds***: **especially above 60km altitude.**

Critical layers – or turning points -- from tides.

Note – time dependence of tide.

2. Critical layers:

Why are some important for wave breaking and others not? (40,000 of them here.)

Present work: mt-wave ***ray focusing in 3D, CL, geom. spreading, realistic winds.***

3. ***Wavebreaking***: **how to get thru wavebreaking regions to higher altitudes?**