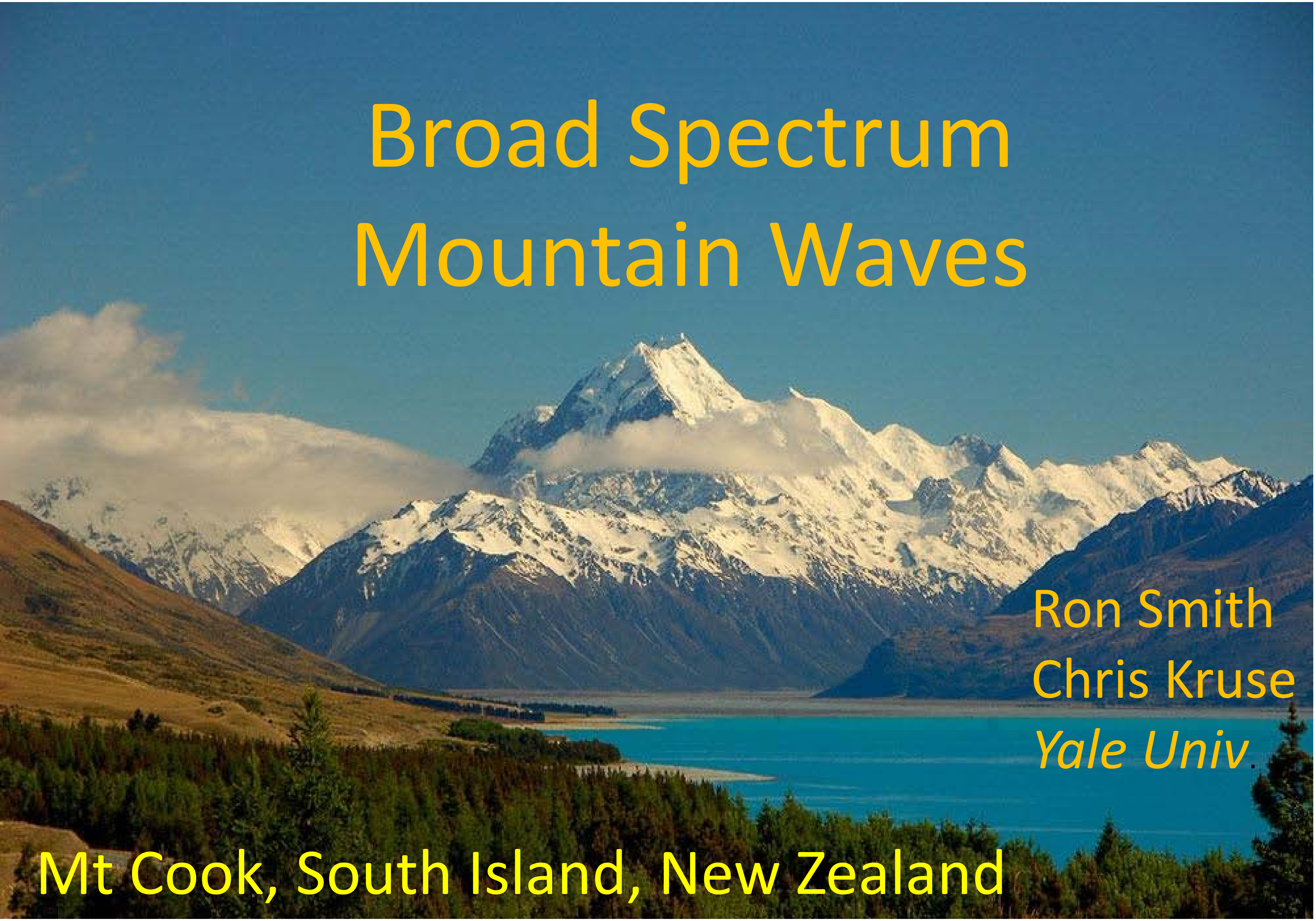
Broad Spectrum Mountain Waves



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Questions

- Why are the DEEPWAVE A/C mountain wave spectra so broad?
- Why are the u-power, MF and w-power spectra so different?
- Is flow over the NZ massif more important than flow into the valleys?
- What wavelength waves carry most of the momentum flux?
- Does WRF also give broad spectra?
- What are the implications of broad spectra?

Spectral Variances

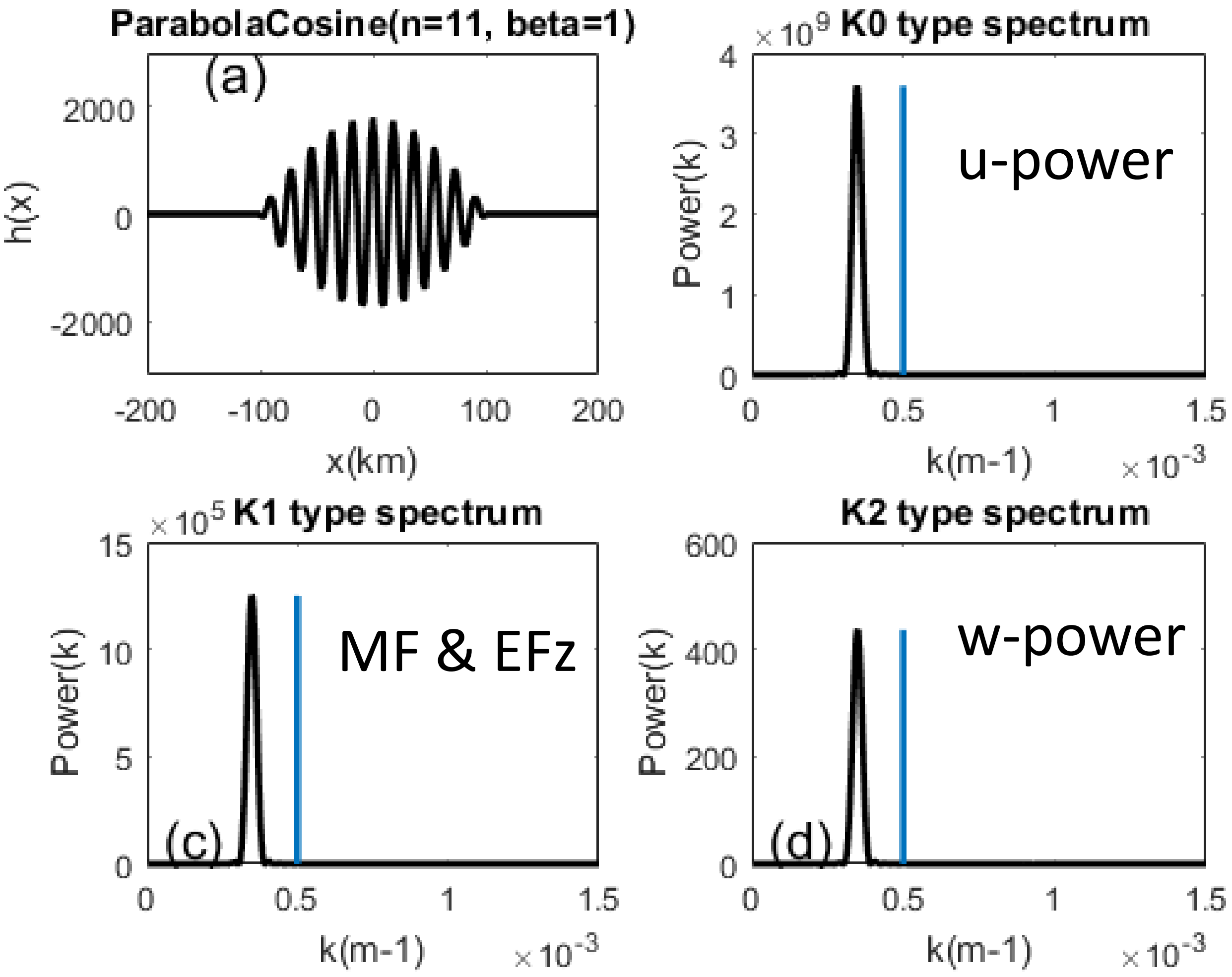
- Mountain shape or streamline displacement $\eta(x)$
- Fourier Transform $\hat{\eta}(k) = \int_{-\infty}^{\infty} \eta(x) \exp(-ikx)\, dx$
- $Var(\eta) = \int_{-\infty}^{\infty} \eta^2(x) dx = \left(\frac{1}{2\pi}\right) \int_{-\infty}^{\infty} \hat{\eta}(k)\hat{\eta}(k)^* dk$
- From hydrostatic mountain wave theory

weights

- $Var(w) = \int_{-\infty}^{\infty} w^2(x) dx = \left(\frac{U^2}{2\pi}\right) \int_{-\infty}^{\infty} k^2\, \hat{\eta}(k)\hat{\eta}(k)^* dk$
- $Cov(u, w) = \int_{-\infty}^{\infty} u(x) w(x) dx = -\left(\frac{NU}{2\pi}\right) \int_{-\infty}^{\infty} |k|\, \hat{\eta}(k)\hat{\eta}(k)^* dk$
- $Var(u) = \int_{-\infty}^{\infty} u^2(x) dx = \left(\frac{N^2}{2\pi}\right) \int_{-\infty}^{\infty} \hat{\eta}(k)\hat{\eta}(k)^* dk$
- Note: P-power and T-power are similar to u-power

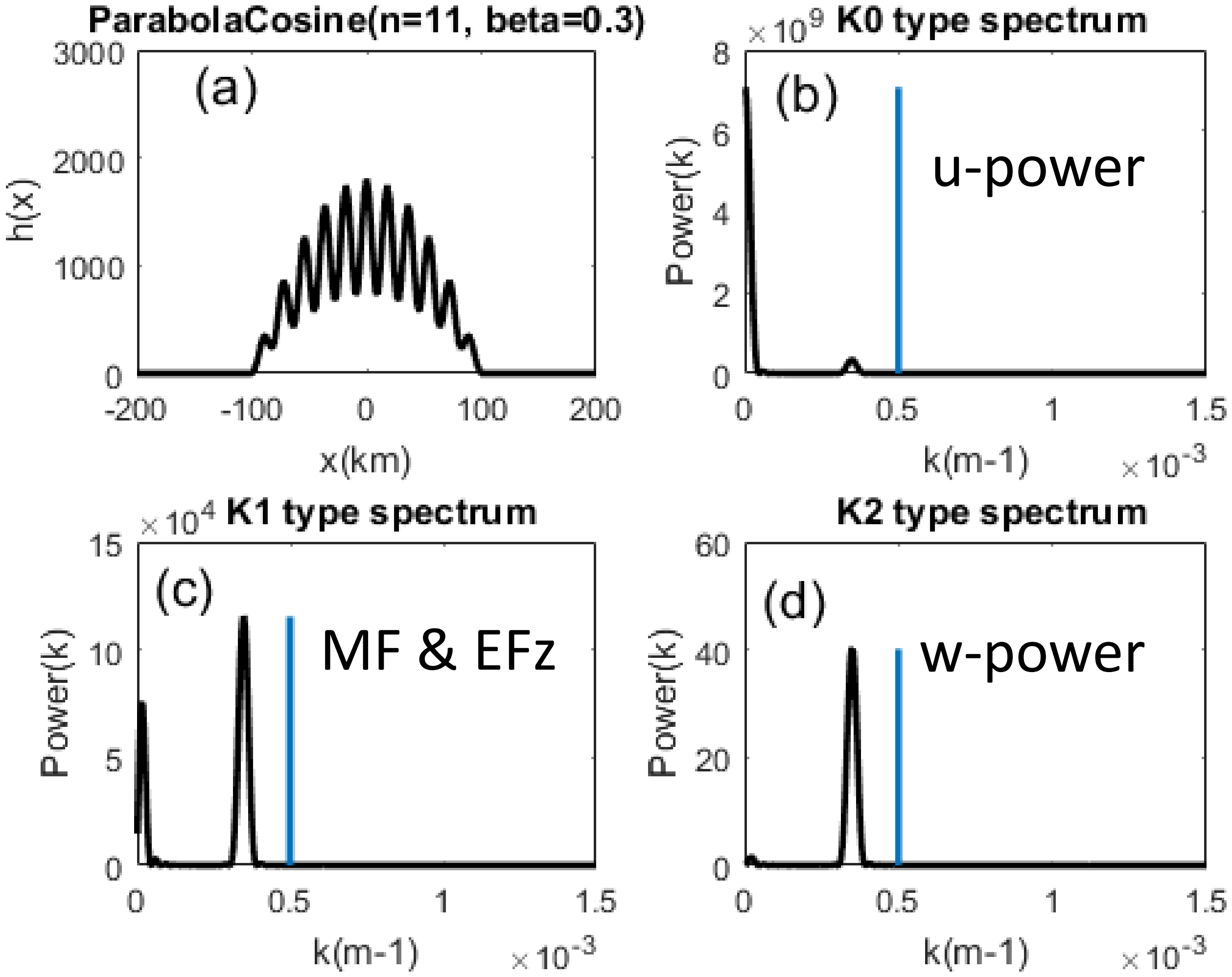
Monochromatic Wave

(blue line: typical buoyancy cut-off)



Ideal Rough hill

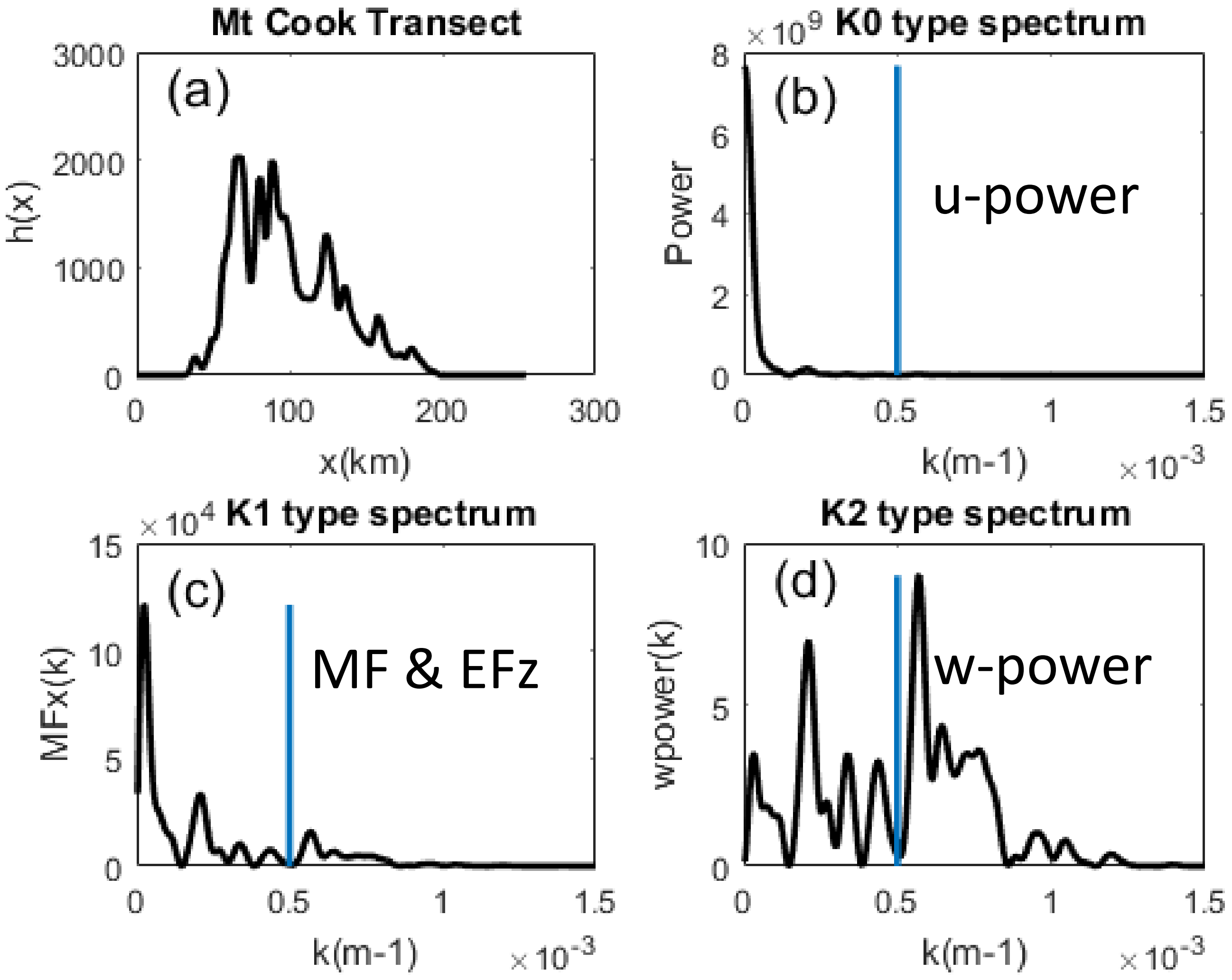
(blue line: typical buoyancy cut-off)



define Volume and Roughness Modes

New Zealand transect, Mt. Cook

(blue line: typical buoyancy cut-off)

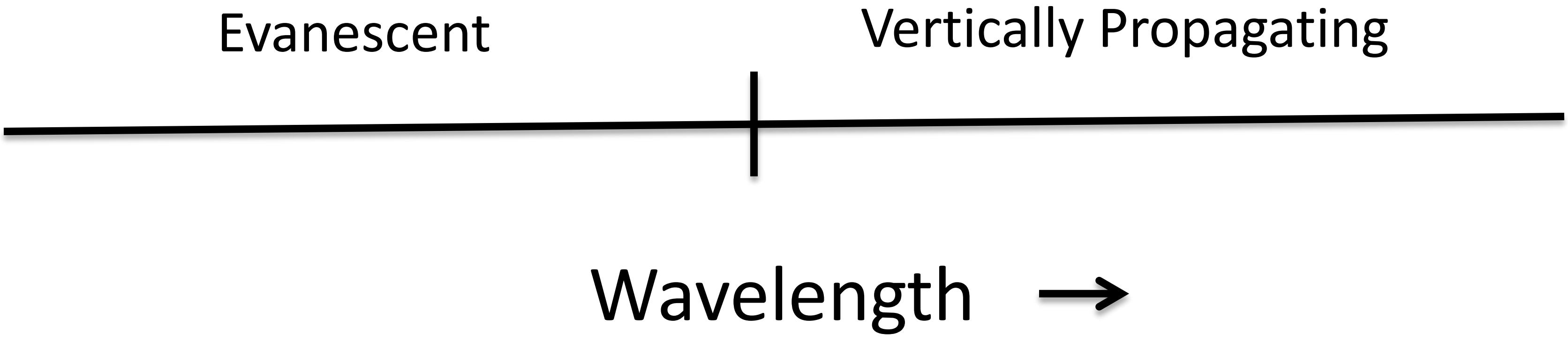


Terrain with volume and roughness

(hydrostatic results)

- Variance spectra are broad
- Volume mode dominates the u-power
- Roughness mode dominates the w-power
- Both modes contribute to MF and EFz
  - Volume mode: large u’ and small w’
  - Roughness mode: small u’ and large w’

Non-hydrostatic waves near the buoyancy cut-off



Evanescent

Vertically Propagating

Wavelength →

$$k = N/U$$

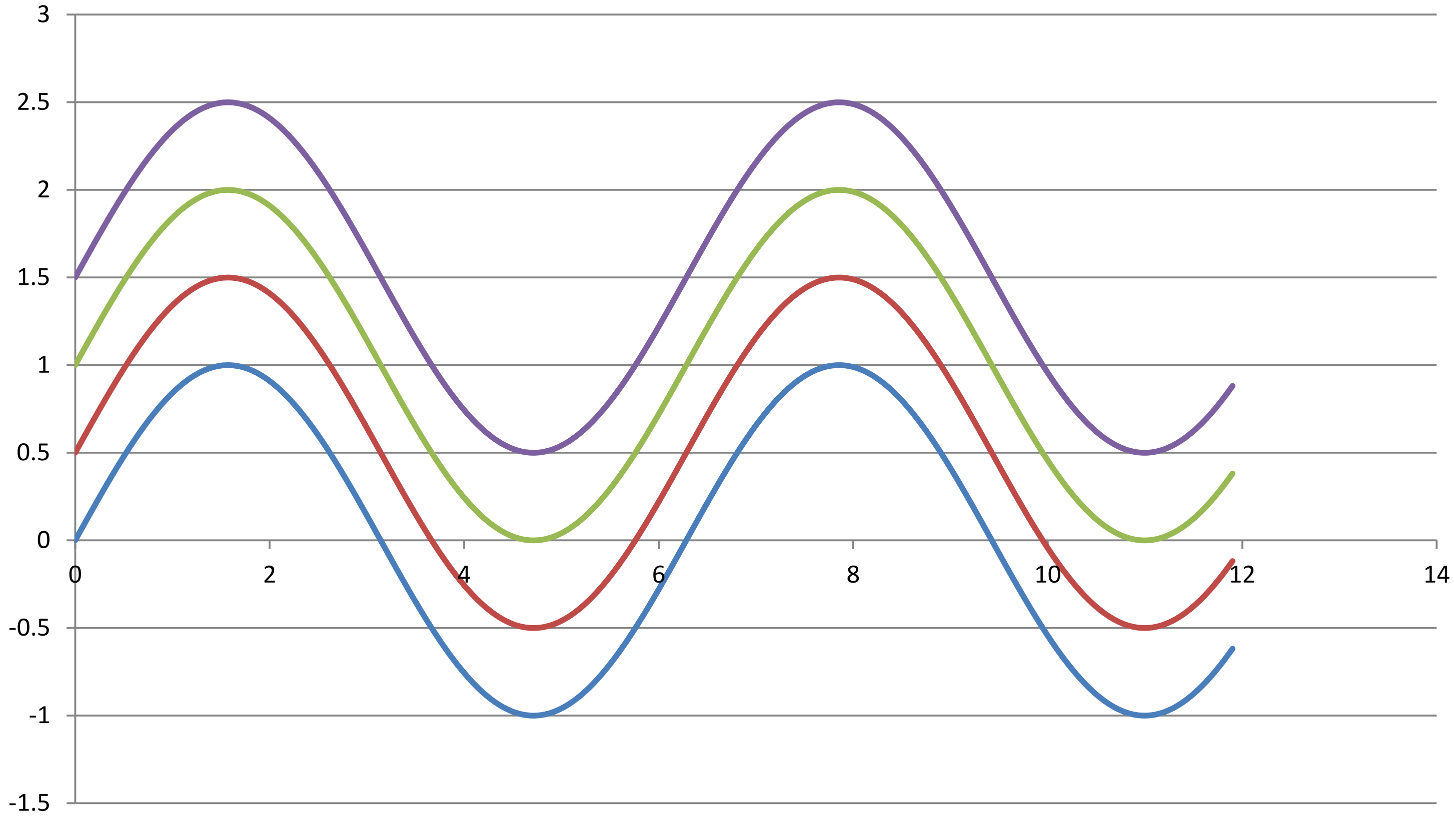
$$\lambda = 2\pi U/N \sim 8\mathrm{km}$$

Non-hydrostatic waves near the buoyancy cut-off

- *Vertical wavenumber (m) approaches zero for k=N/U*
- $m = \left(\frac{N}{U}\right)[1 - (\frac{kU}{N})^2]^{1/2}$
- *Deep vertical penetration*
- Wind perturbation approaches zero
- $\hat{u} = \left(\frac{m}{k}\right)\widehat{w}$
- Momentum flux approaches zero

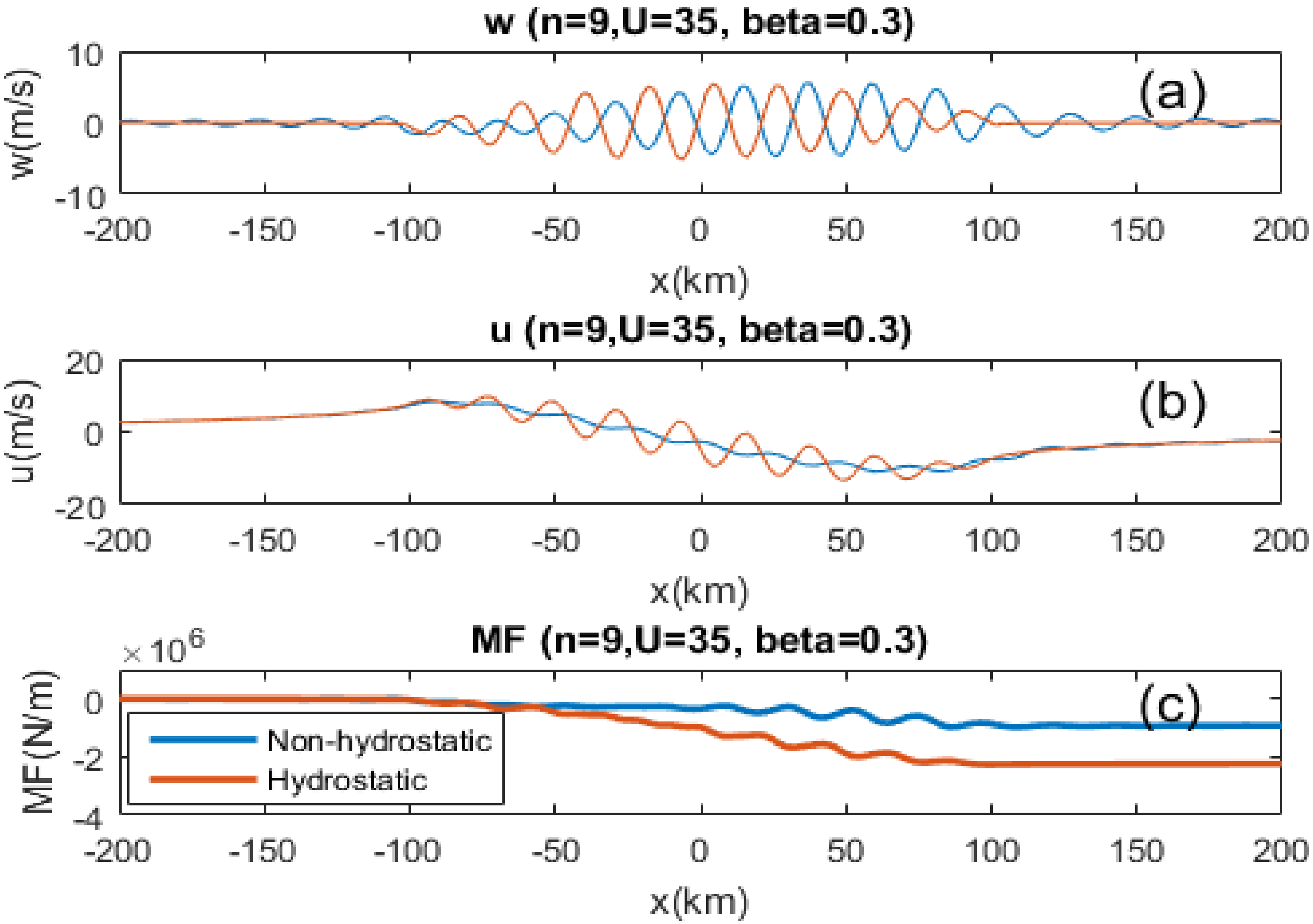
Non-hydrostatic waves near the buoyancy cut-off

(m=0; constant streamline spacing: u’=0)

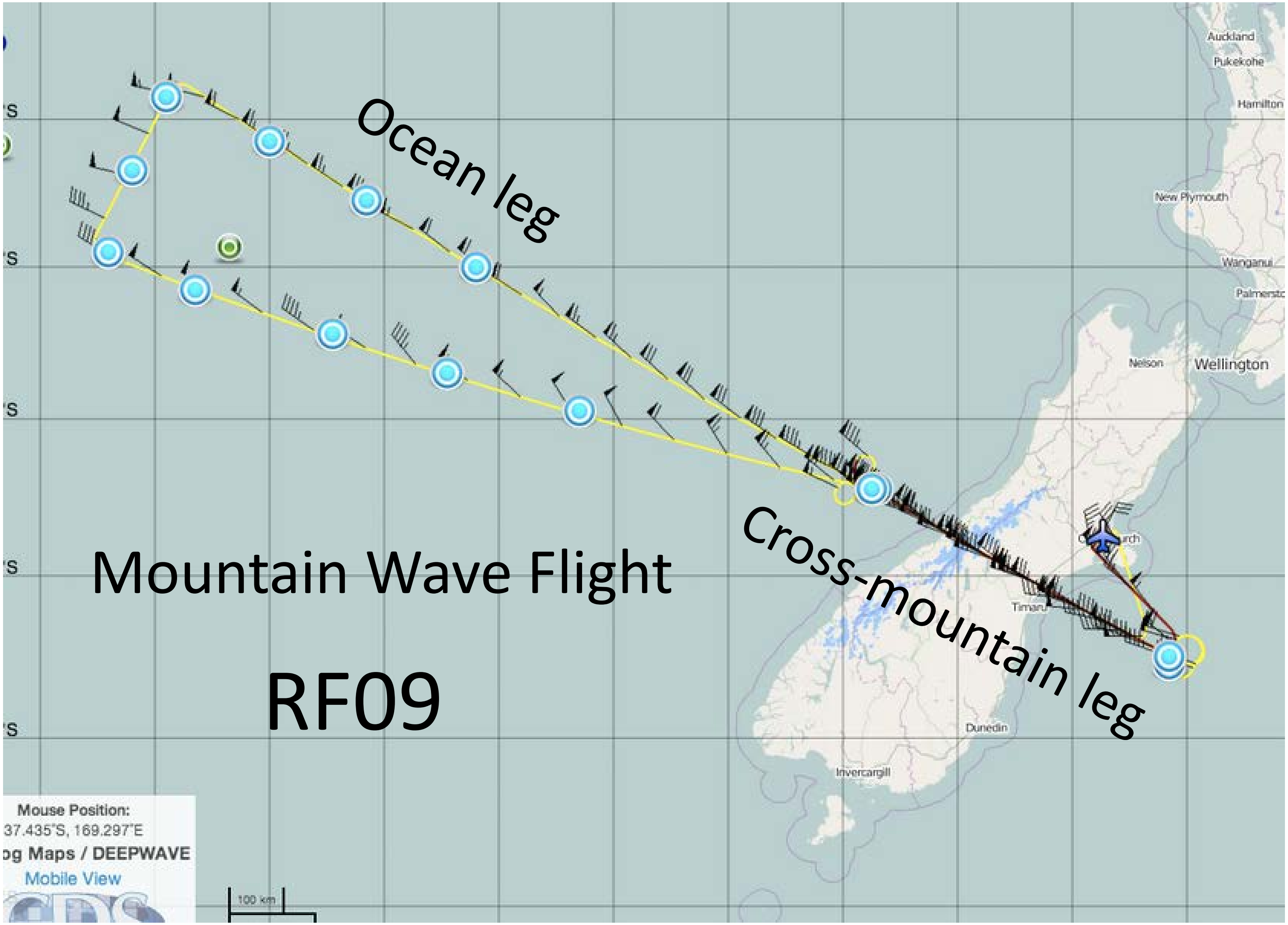


Waves near the buoyancy cut-off

(good w penetration, little u’ or MF)



Example at z=12km: roughness wavelength close to the buoyancy cut-off

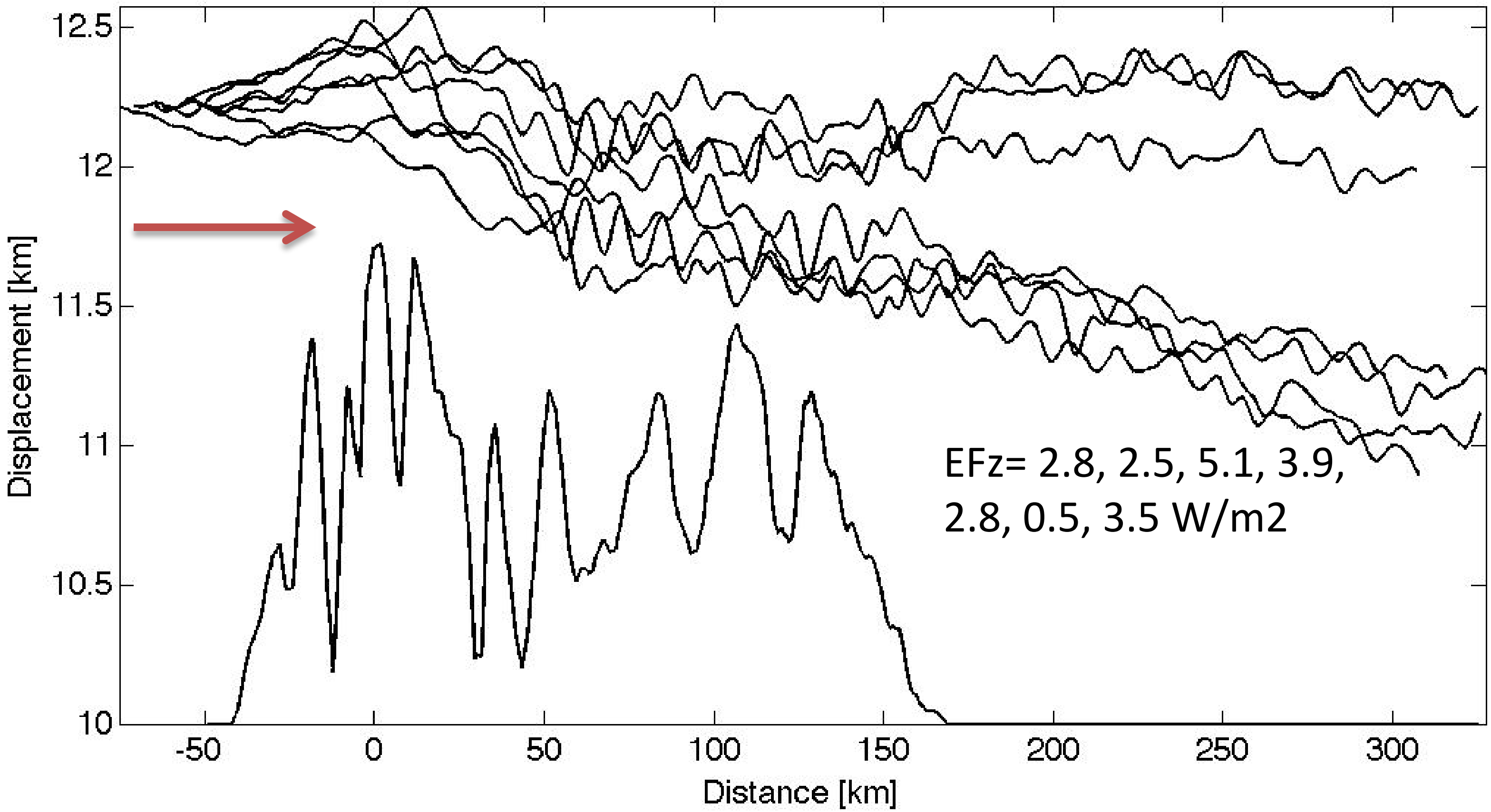


Mountain Wave Flight

RF09

Mt Aspiring

RF04: 7 legs
Vertical displacement

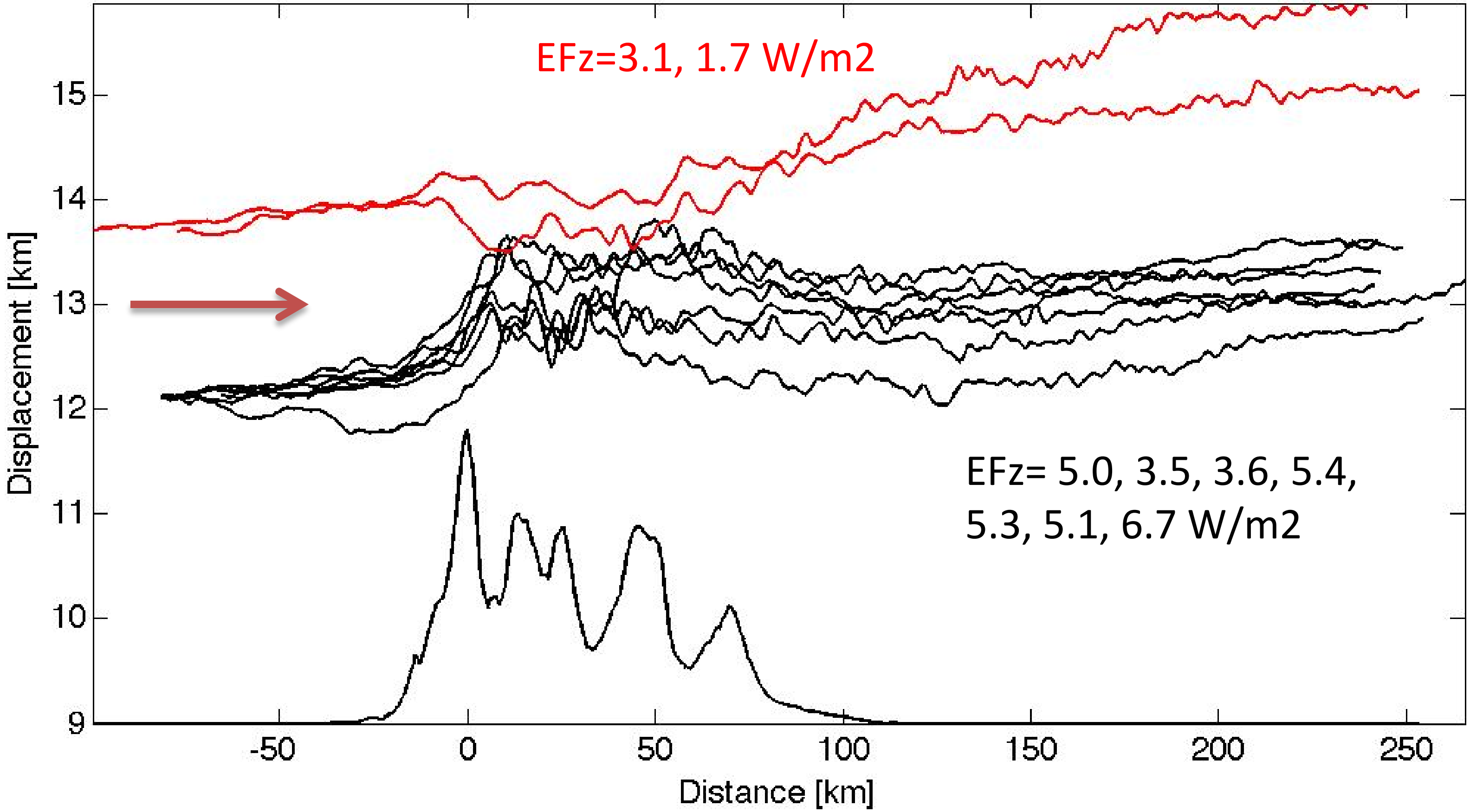


EFz= 2.8, 2.5, 5.1, 3.9, 2.8, 0.5, 3.5 W/m2

Mountain to scale but offset vertically

Mt
Cook

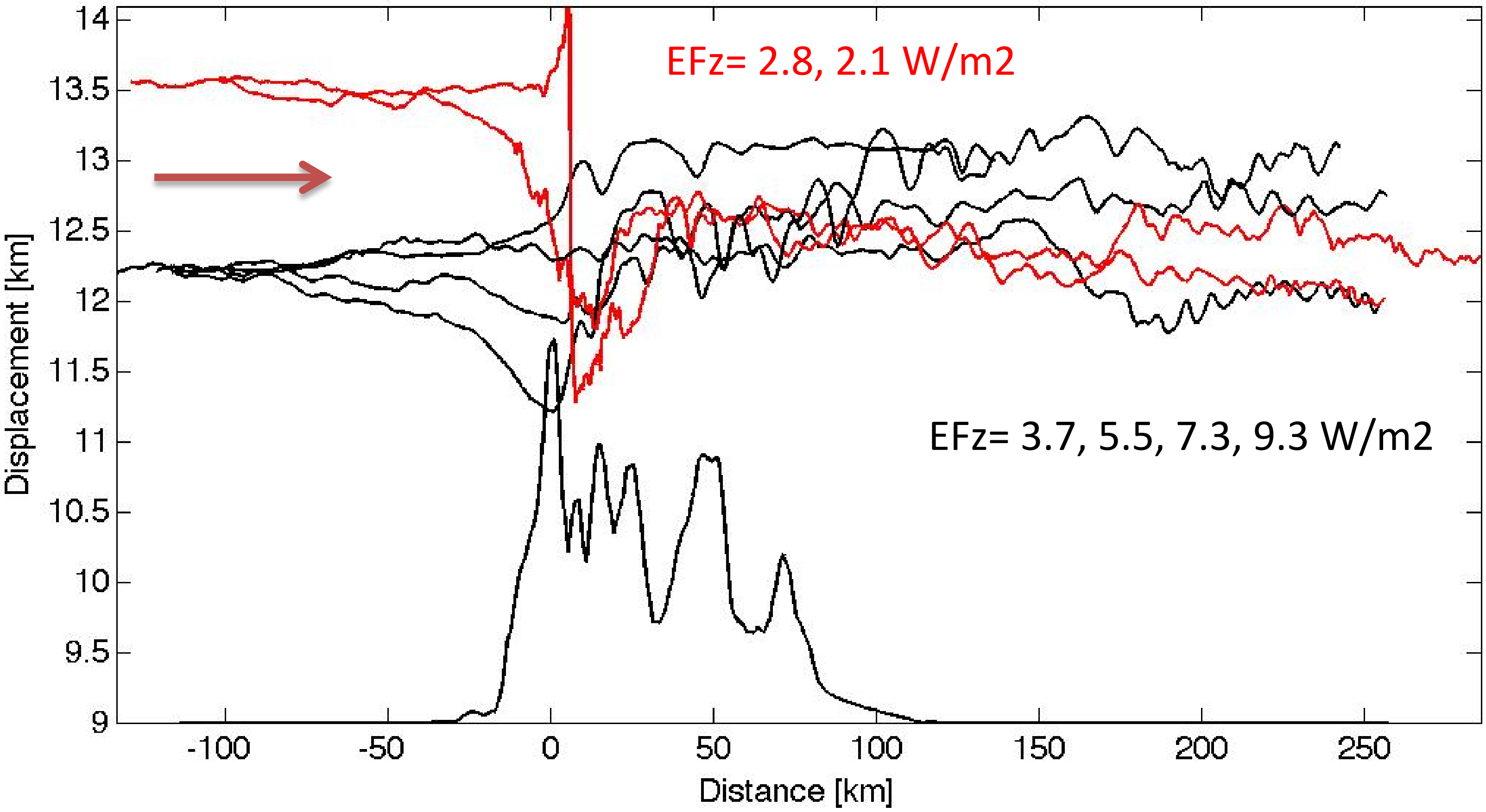
RF05: 9 Legs
Vertical displacement



Mountain to scale but offset vertically

Mt
Cook

RF09: 6 legs
Vertical displacement

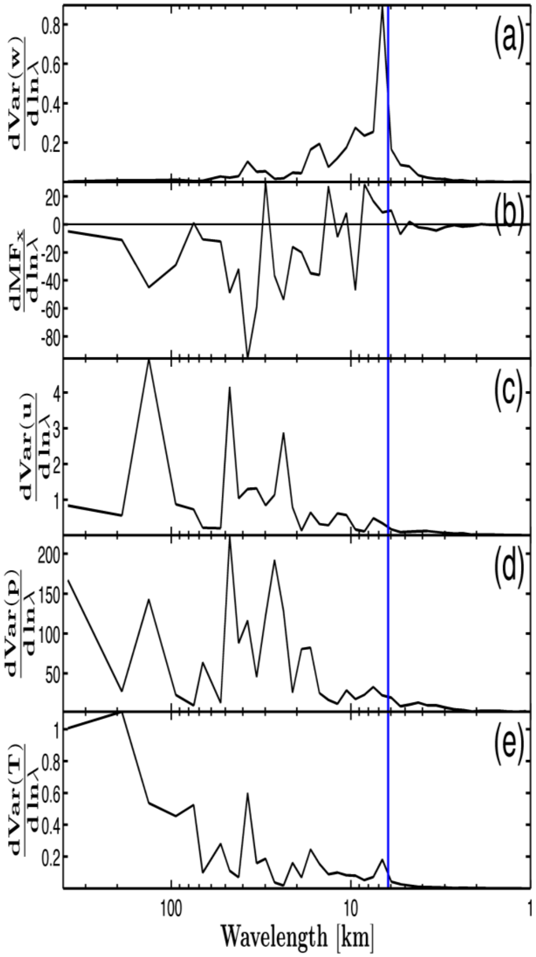


EFz= 2.8, 2.1 W/m2

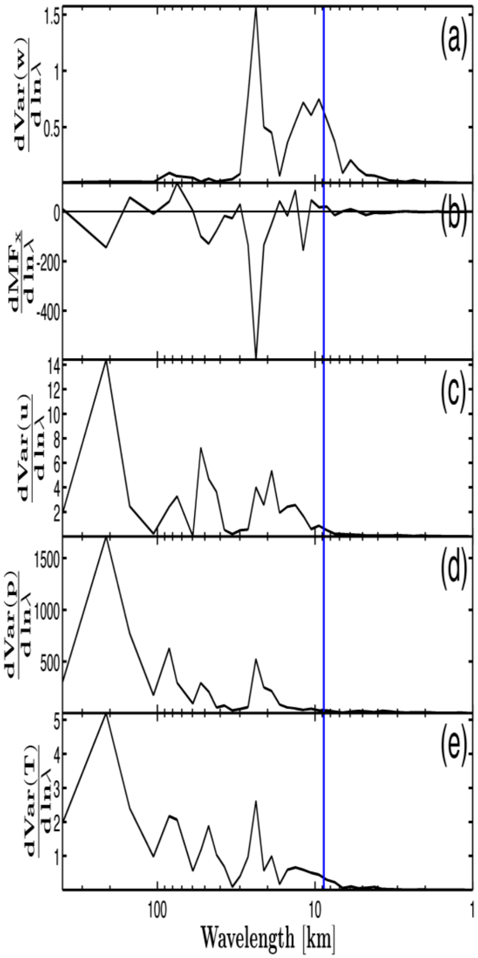
EFz= 3.7, 5.5, 7.3, 9.3 W/m2

Mountain to scale but offset vertically

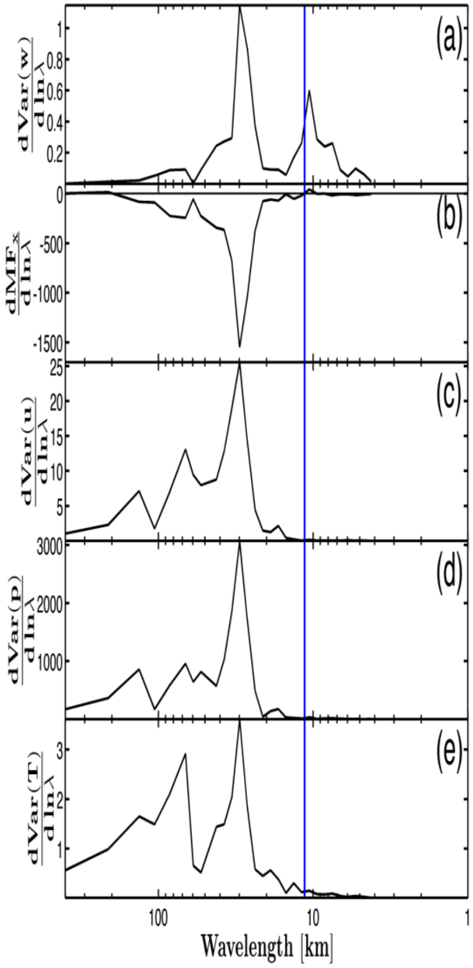
Aircraft Transect Spectra



RF05



RF09



RF16

w-power

MFx

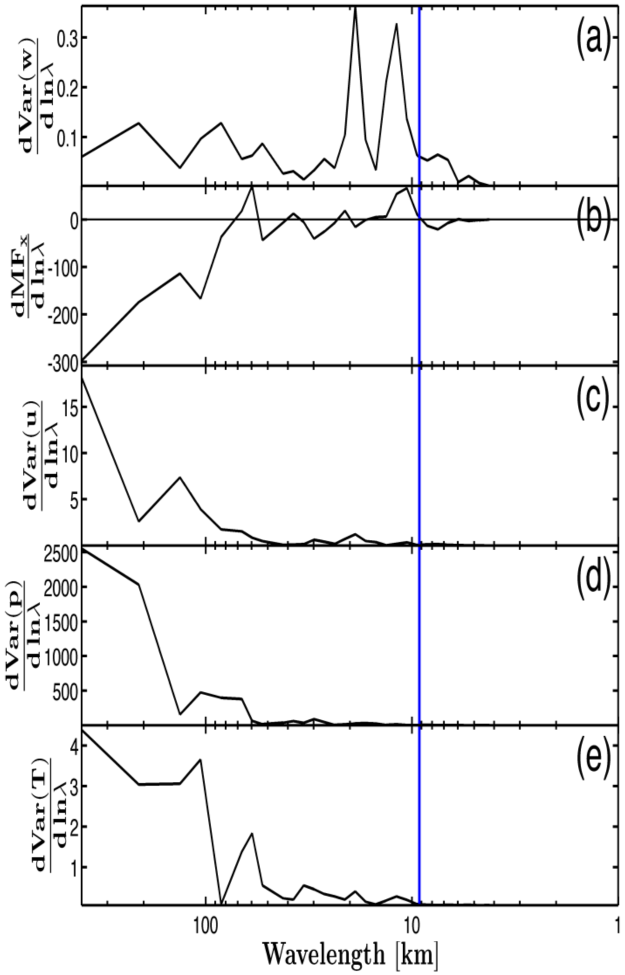
u-power

P-power

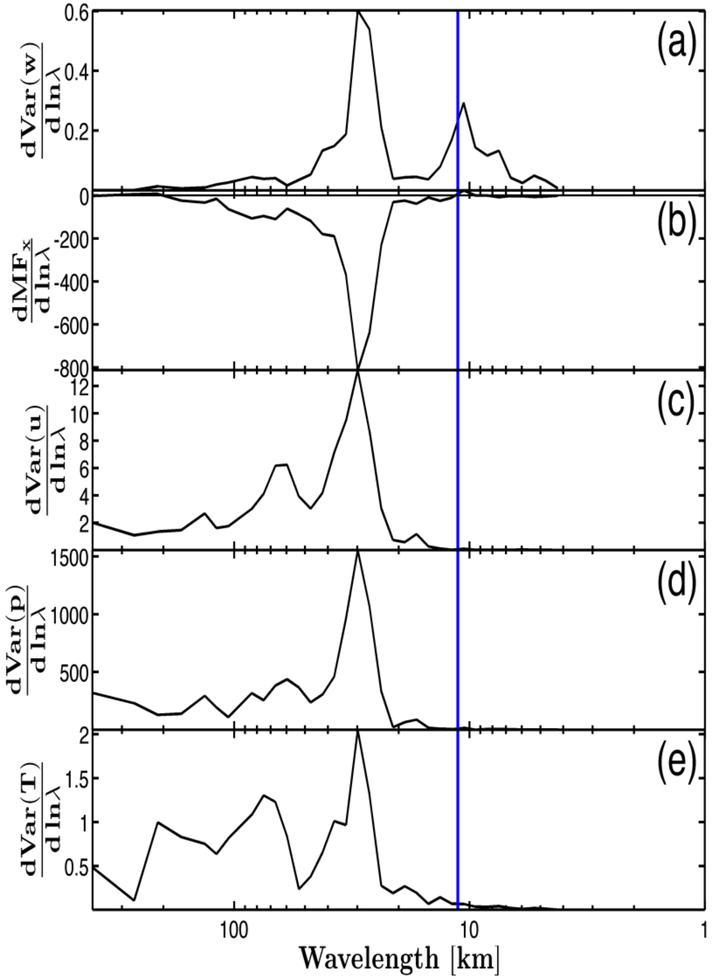
T-power

Blue line is the buoyancy cut-off

2km WRF Simulated Transect Spectra



RF09



RF16

w-power

MFx

u-power

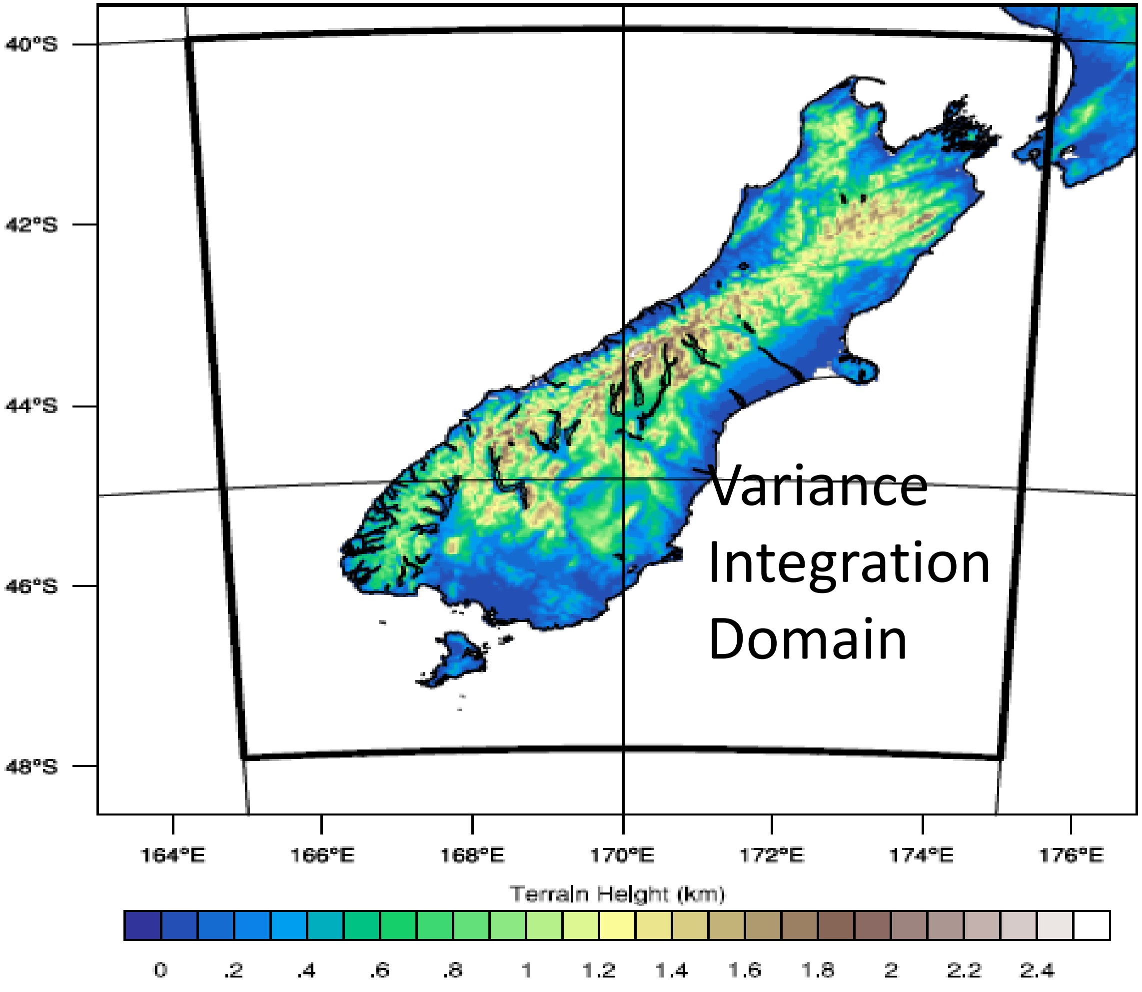
P-power

T-power

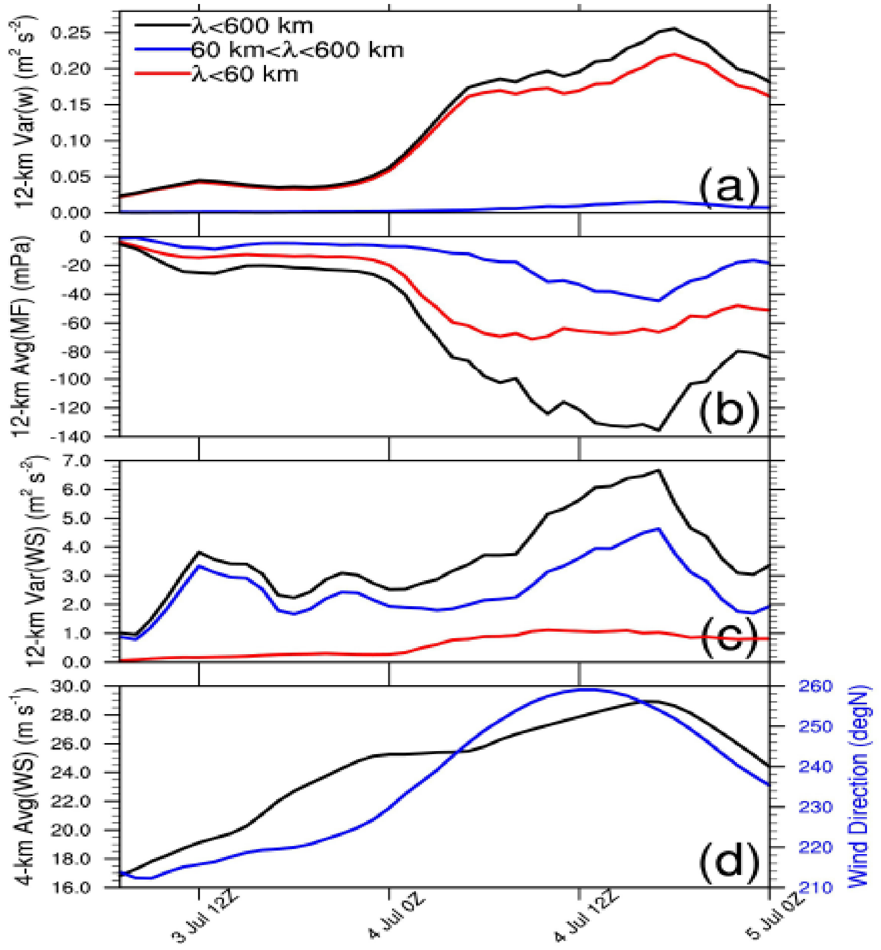
Blue line is the buoyancy cut-off

Compare Variances from Volume and Roughness Modes

- WRF with 2km grid
- 3-day wave event from DEEPWAVE, July 3-5, 2014
- Use a high-pass and low-pass spectral filter


Variance Integration Domain

2km WRF Simulation: 3-day wave event; z=12km



w-power

MFx

u-power

Wind Speed
& Direction

Red = Roughness Mode $\lambda < 60km$

Blue = Volume Mode $\lambda > 60km$

Black = Total

Spectral contributions

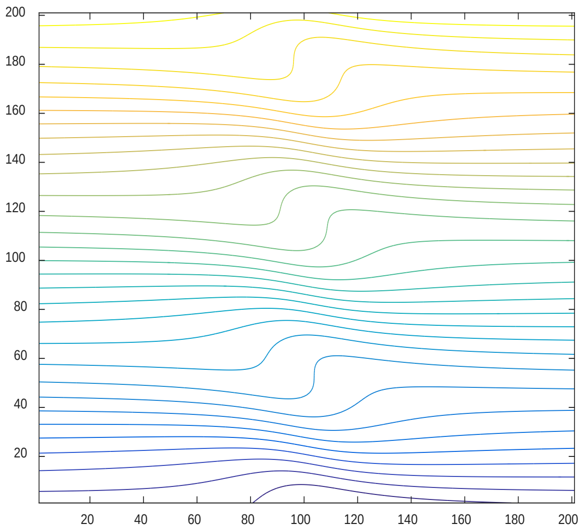
- Roughness Mode I: $\lambda = 8\ to\ 15km$
  - Near the buoyancy cut-off
  - Non-hydrostatic
  - Dominates w-power. Very little u-power.
  - Little MF or EFz
- Roughness Mode II: $\lambda = 15\ to\ 40km$
  - Large w-power
  - Small but significant u-power
  - Dominates MF and Efz
  - Nearly hydrostatic
- Volume Mode: $\lambda = 60\ to\ 300km$
  - Large u-power
  - Small but significant w-power
  - Carries 1/3 or less of the MF and EFz
  - Dominates the P-power and T-power too.
  - Hydrostatic

Implications of Broad Spectra

- Aircraft sensors must have accurate broad-band response. (OK, I think)
- Numerical models must capture the Roughness Mode II: $\lambda = 15\ to\ 40km$ to get the MF and EFz right.
- Satellite IR sensors and Rayleigh Lidar will mostly see the T-power in the Volume Mode: $\lambda = 60\ to\ 300km$; missing the MF in Roughness II.
- Balloon data mostly see the u-power in the Volume Mode
- Usual monochromatic relations between u, w, T, p and MF do not apply.

The wave spectrum will influence wave breaking and GWD

- Volume Mode $(\lambda = 60\ to\ 300km)$
  - Dominates u-power , stagnation and wave breaking
  - Has steepening levels (not included in saturation theory)
- Roughness Mode II $(\lambda = 15\ to\ 40km)$
  - Dominates the MF and Efz
  - Weak u-power
  - MF deposition controlled by the Volume Mode



Queney solution

See talk by Chris Kruse.