Mountain Wave Attenuation in Positive and Negative Ambient Vertical Wind Shear

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Yale University

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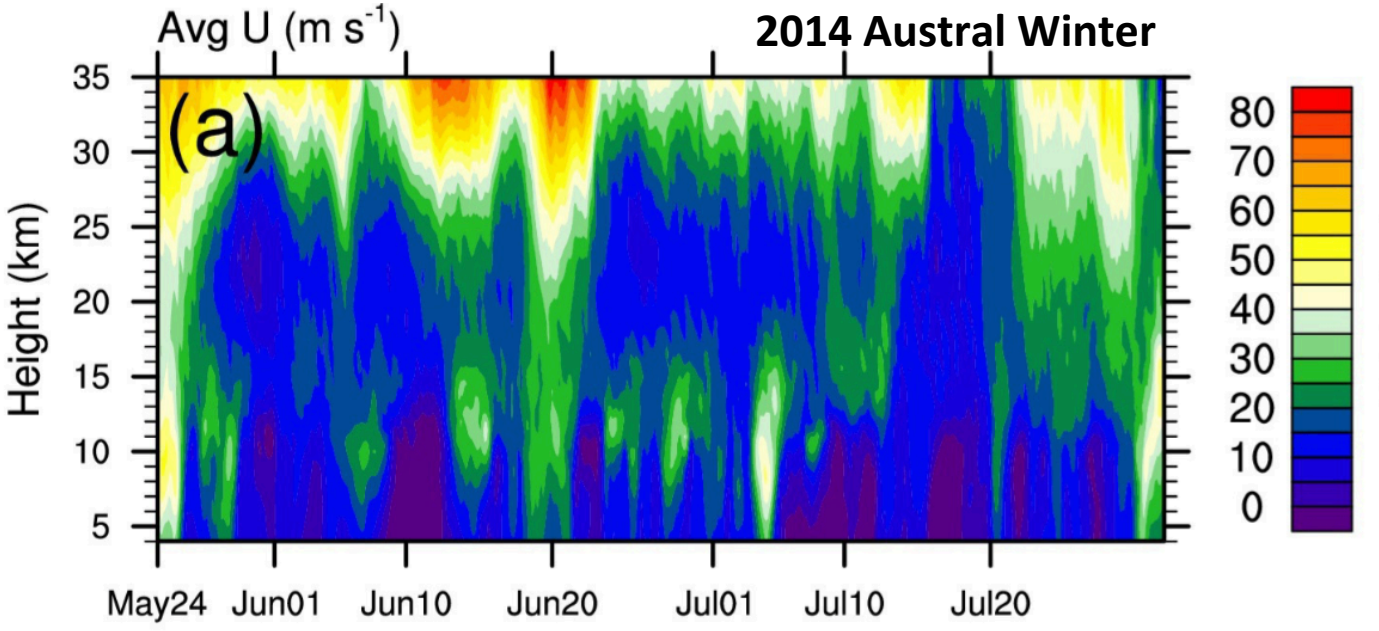
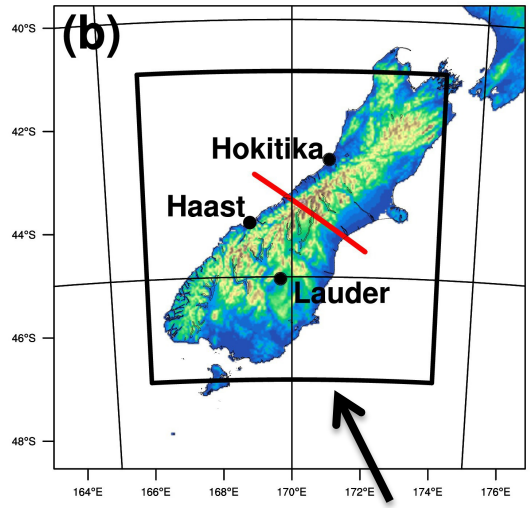
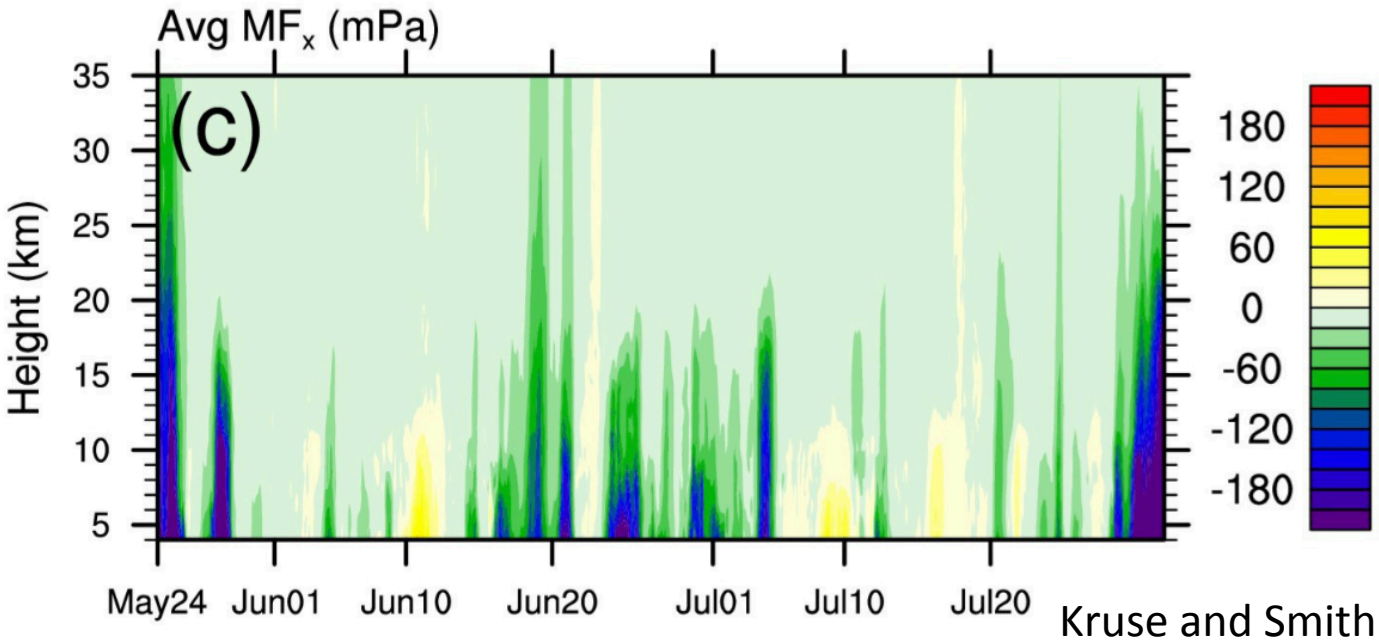
NCAR's Computational Resources (CISL, Yellowstone)

Background/Motivation





Averaging Area



Kruse and Smith 2016

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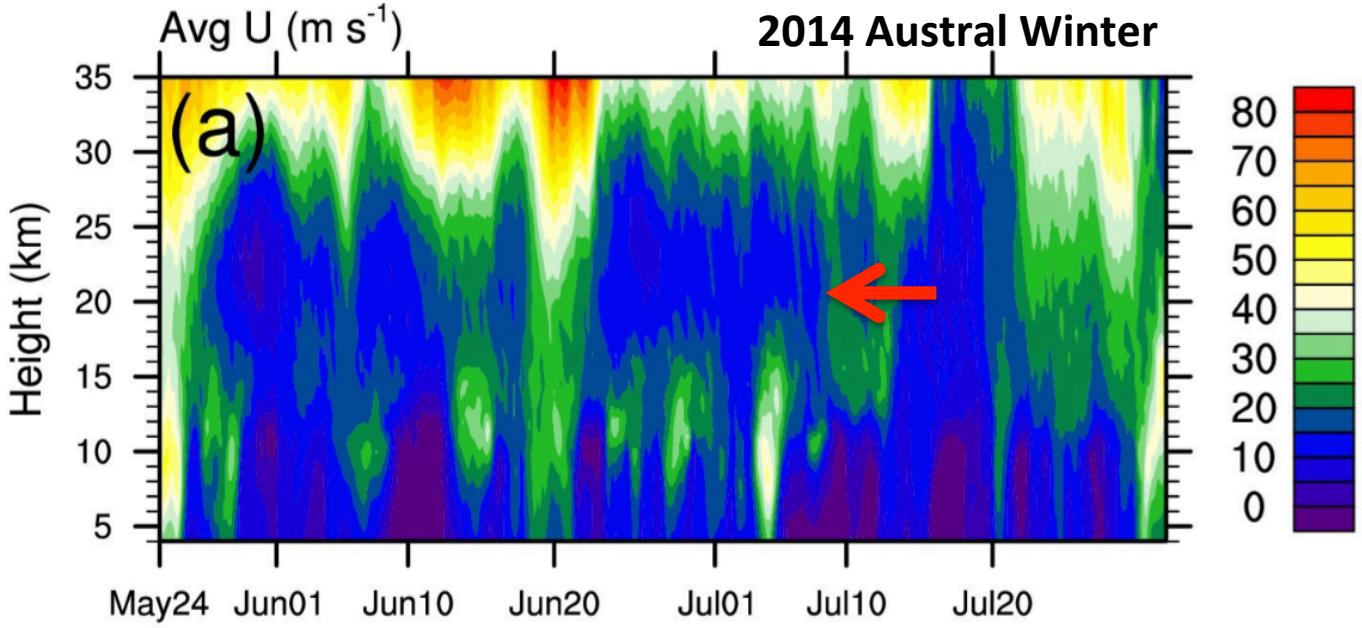

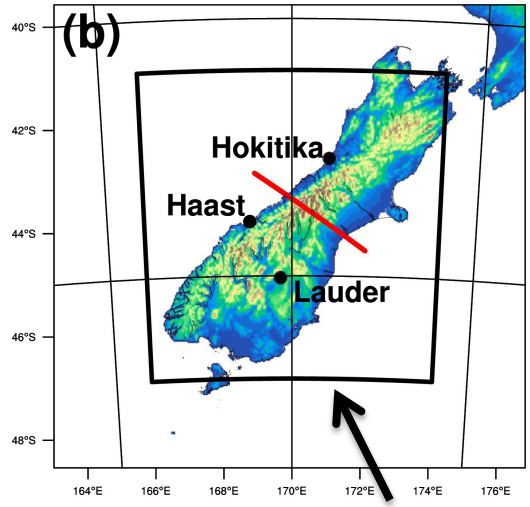

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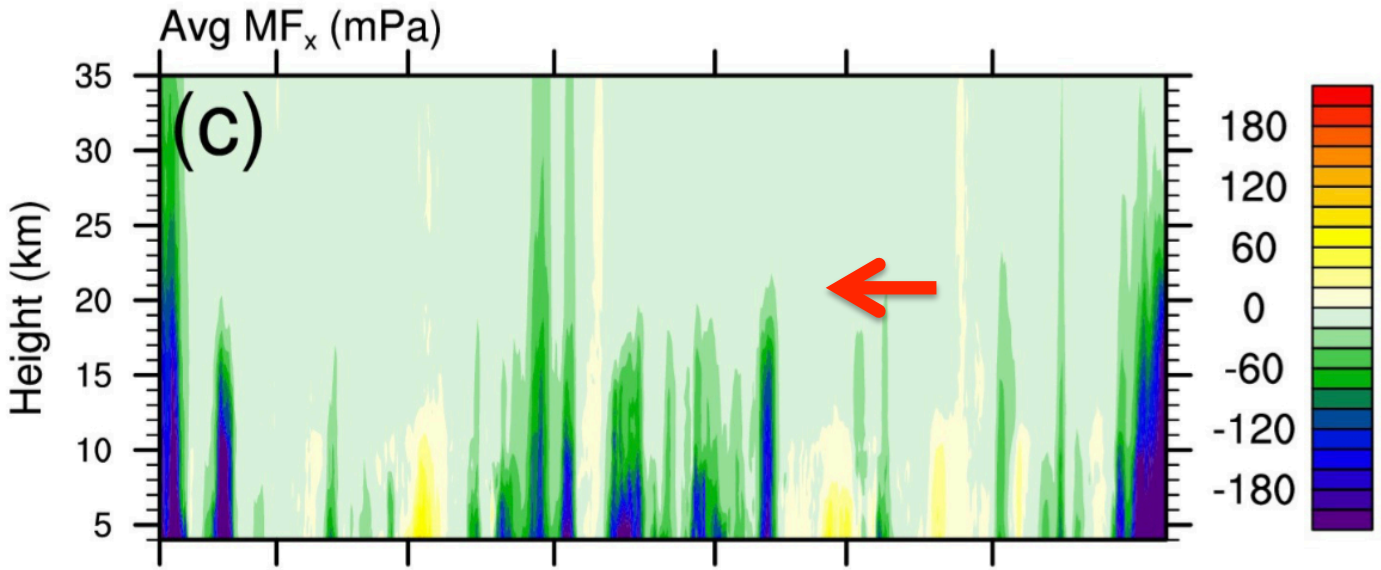

Little mountain wave MF_x gets through a weak wind Mountain Wave "Valve Layer"

The Valve Layer restricts momentum flux analogous to a valve restricting mass flux through a pipe

Kruse and Smith 2016

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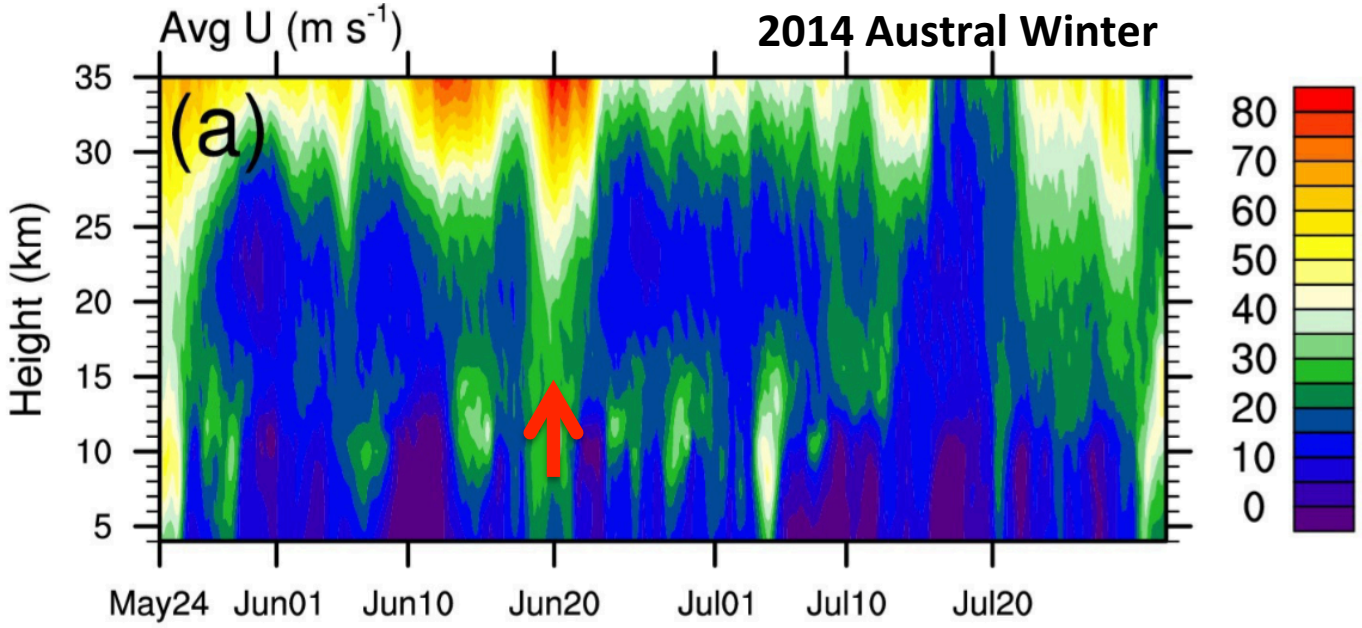

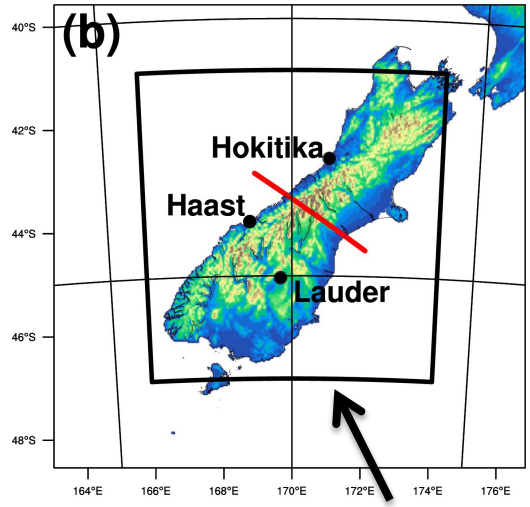

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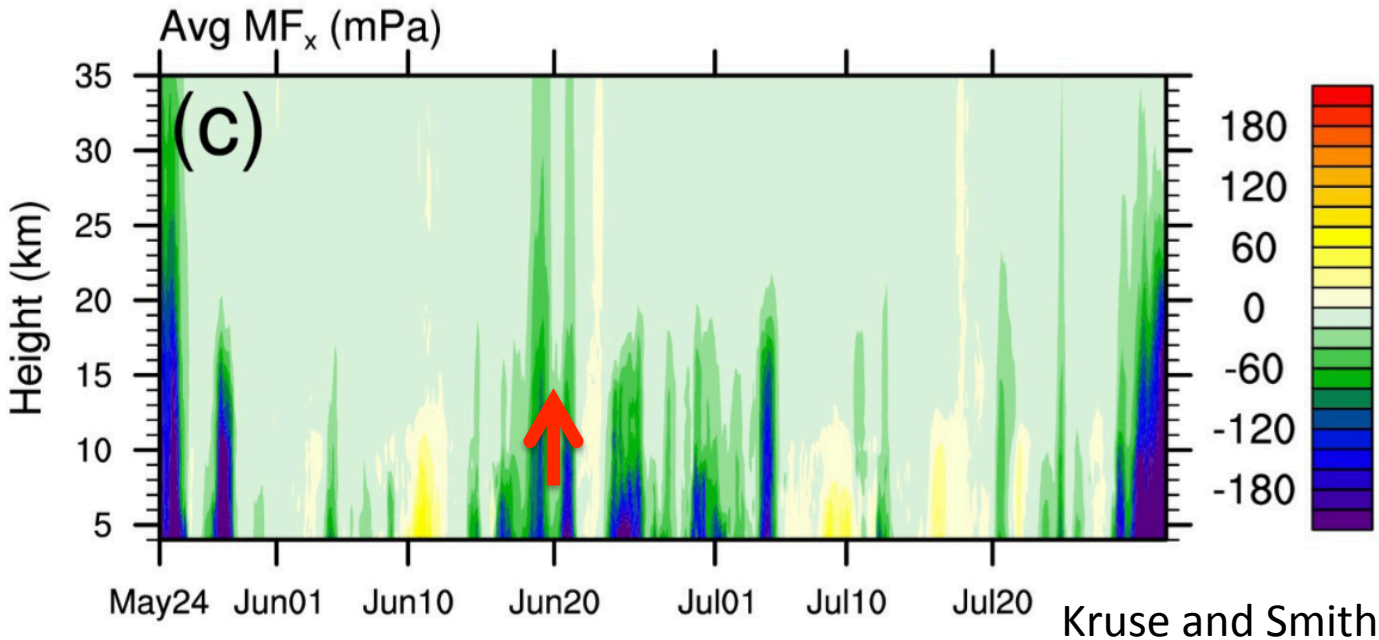
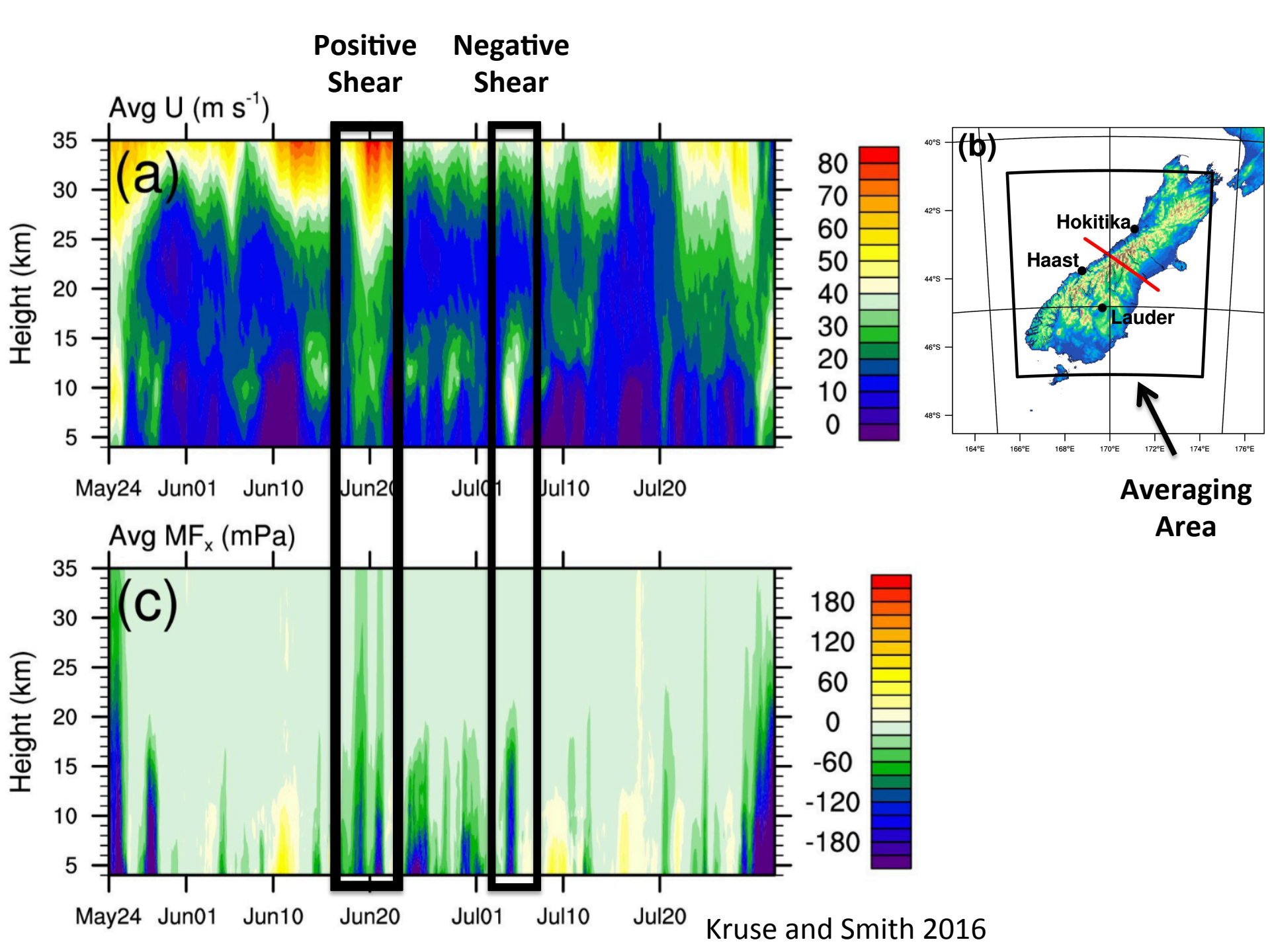


Largest mid-stratospheric MF$_x$ occurs when vertical wind shear is **positive.**

Kruse and Smith 2016



Kruse and Smith 2016

Questions

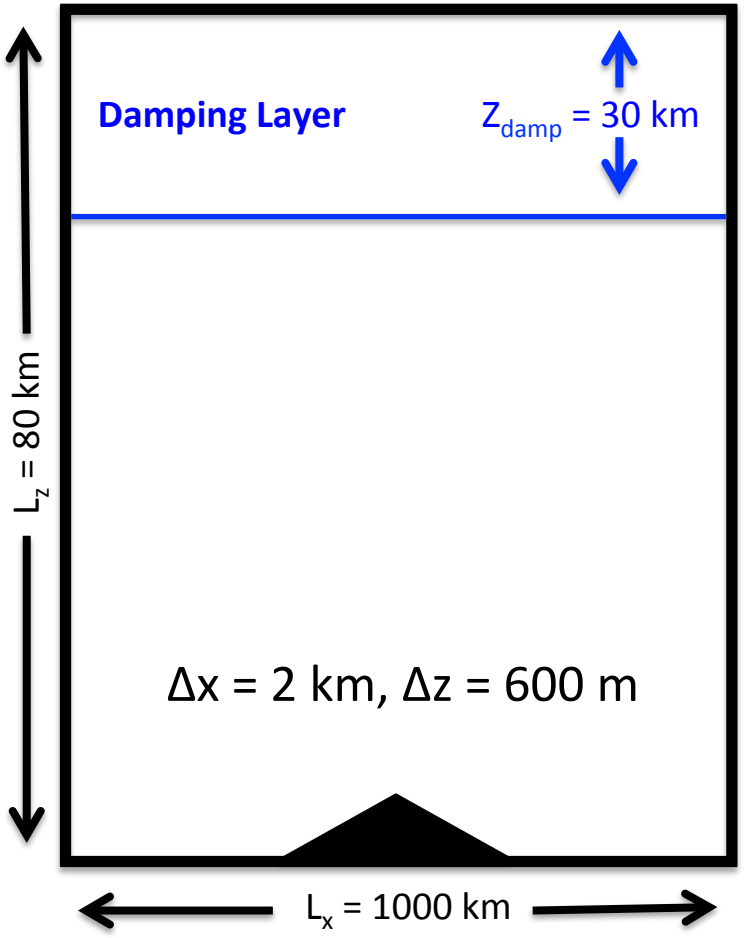
1. How does attenuation and resulting momentum deposition vary with altitude?

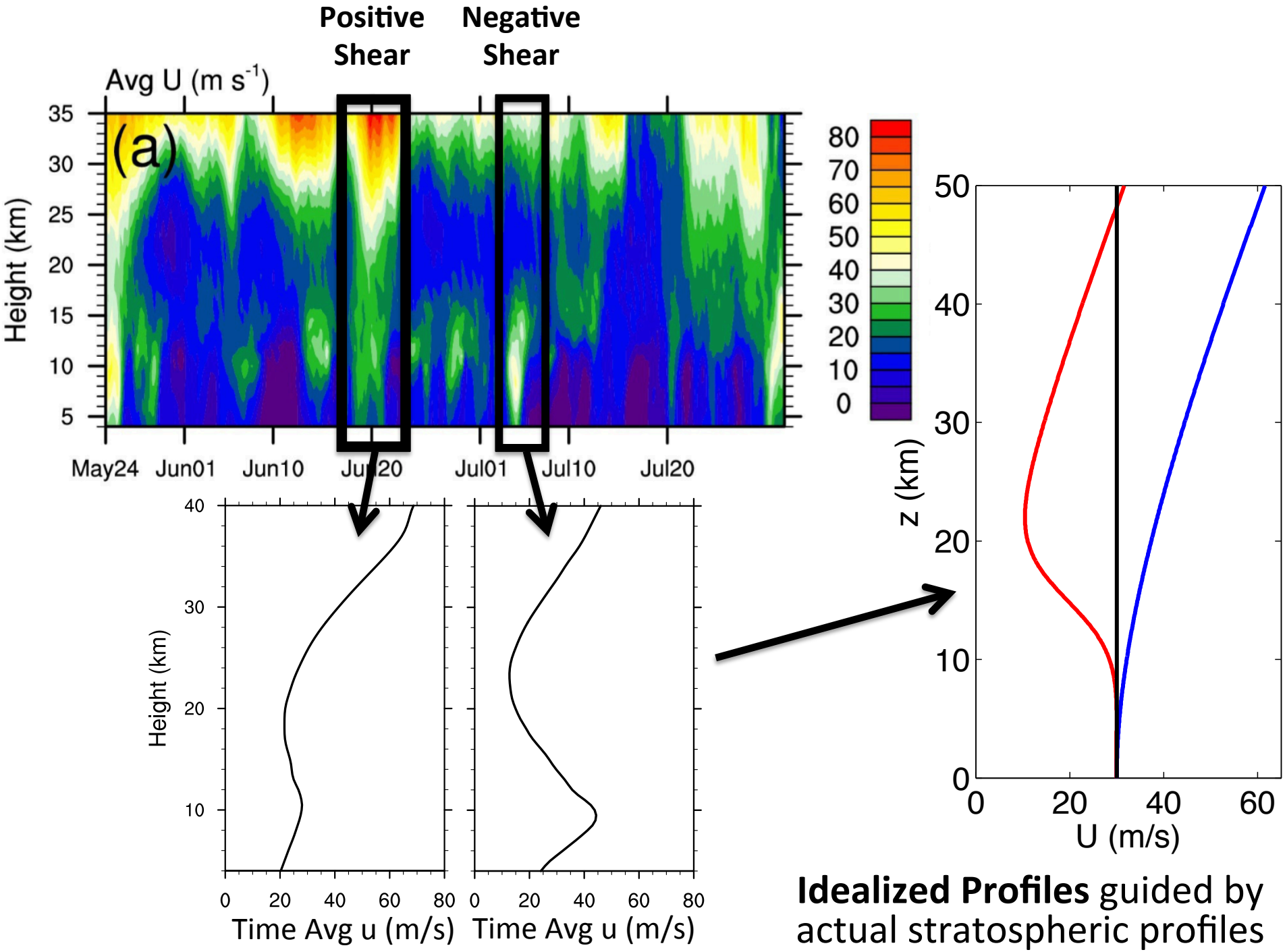
2. Does wind shear affect wave attenuation?

3. How do different scales interact during wave breaking?

Idealized Model Setup

- **Weather Research and Forecasting Model**
- **Setup (for now)**
  - 2-D
  - Horizontally Periodic
  - Constant N = 0.02 s^{-1}
  - No Coriolis Force
  - Inviscid
- **3 Wind Profiles**
- **2 Compact Terrains**



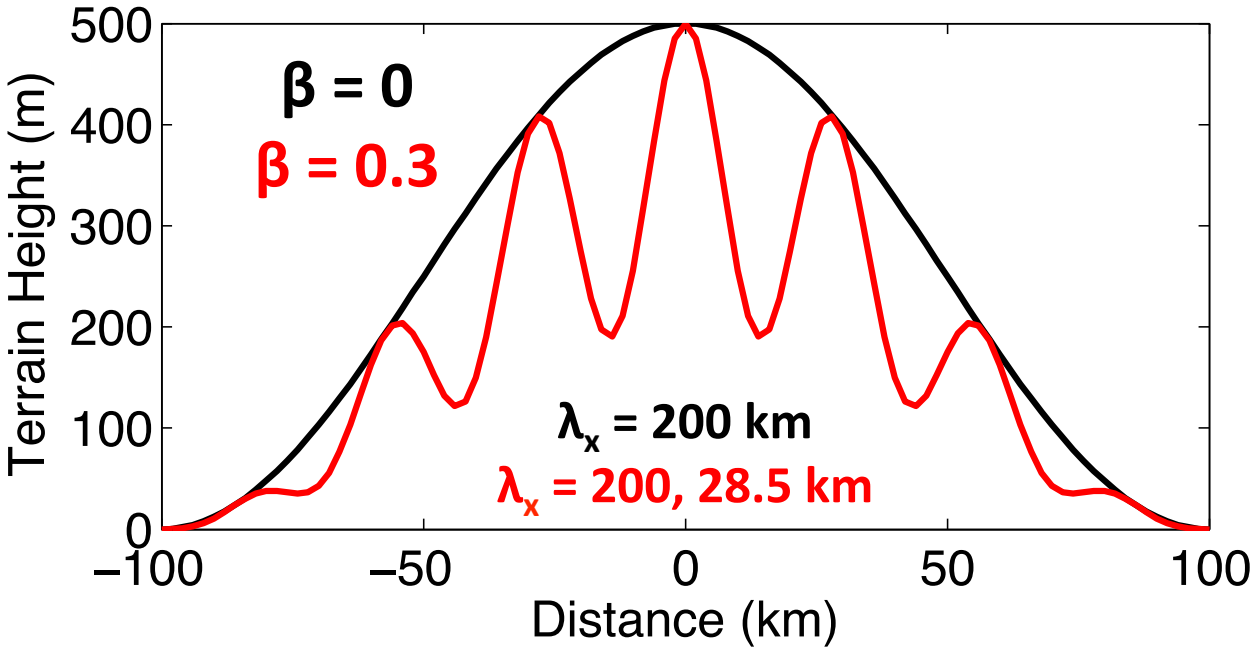


Idealized Profiles guided by actual stratospheric profiles

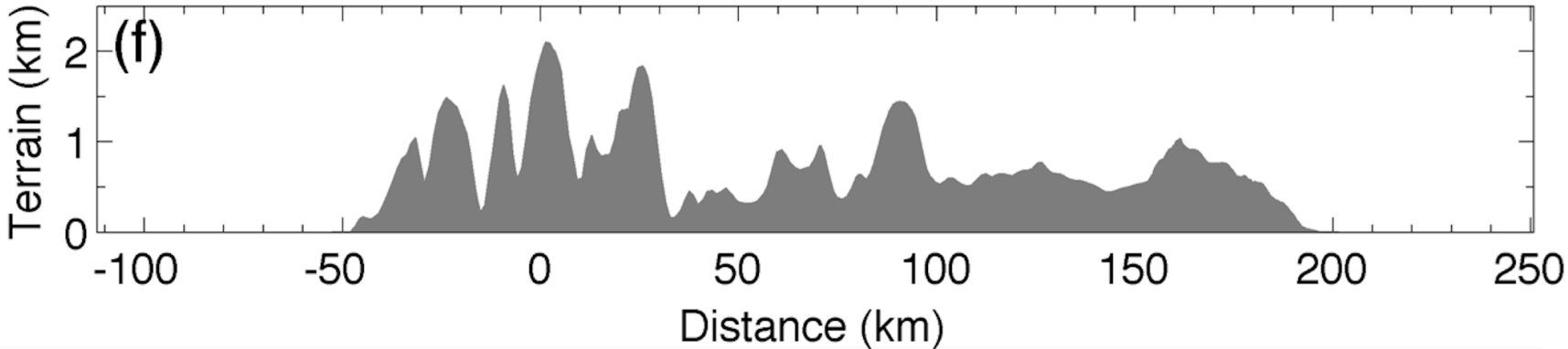
Idealized Terrains

$$h(x) = \begin{cases} \frac{h_m}{2}\left(1+\cos(kx)\right)\left(1-\beta+\beta\cos\left(nkx\right)\right) & , \quad |x| \le d \\ 0 & , \quad |x| > d \end{cases}$$

- h_m = 500 m: max terrain height
- k = π/d where d is the range half-width
- β controls valley depth
- n = 7: number of smaller scale peaks



Actual New Zealand Transect



Outline

1. Linear Theory Predictions

2. Levels of Wave Overturning/Attenuation
   - Many? Periodic?

3. Attenuation Diagnostic

4. Scale Interaction

1. WKB Predicted Overturning Altitude (z_b)

- Wave reaches overturning amplitude when u-amplitude equal to ambient wind
  - i.e. wave causes stagnation
- WKB Non-Linearity Ratio

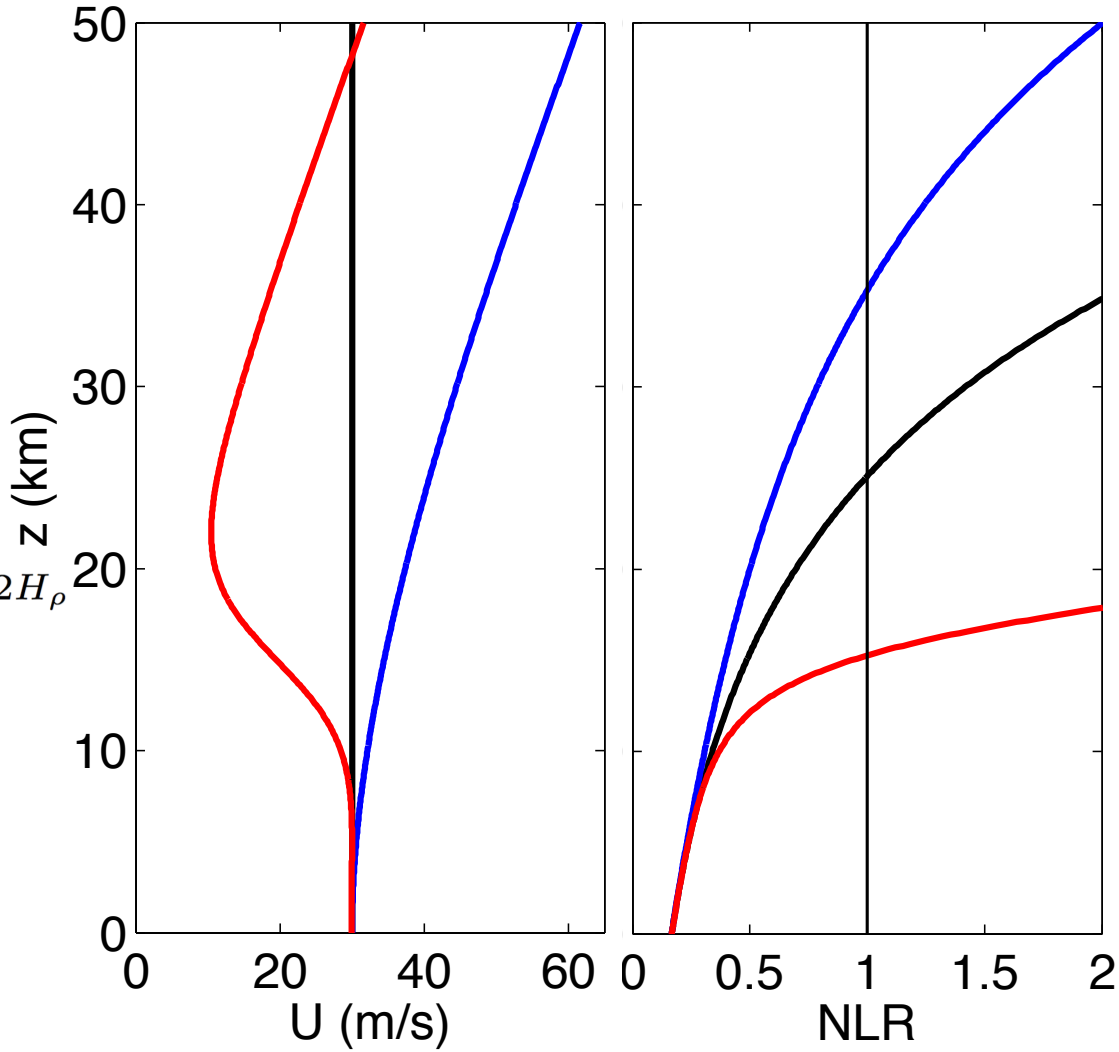
$$NLR(z) = \frac{\hat{u}(z)}{U(z)}$$

$$NLR(z) = NLR_0 \left(\frac{U_0}{U(z)}\right)^{3/2} e^{z/2H_\rho}$$

$$NLR_0 = \frac{Nh_m}{2U_0} = \frac{1}{6}$$

h_m=500 m, U_0=30 m s^{-1}, N=0.02 s^{-1}

- **NLR(z_b) = 1**



z_b	No Shear	Positive Shear	Negative Shear
	25.1 km	35.3 km	15.3 km

1. WKB Predicted Time to Overturning (τ_b)

- Steady, Hydrostatic vertical group velocity:

$$c_{g_z} = \frac{U^2 k}{N}$$

- Time to propagate to overturning altitude:

$$\tau_b = \int_0^{z_b} \frac{1}{c_{g_z}} dz$$

z_b

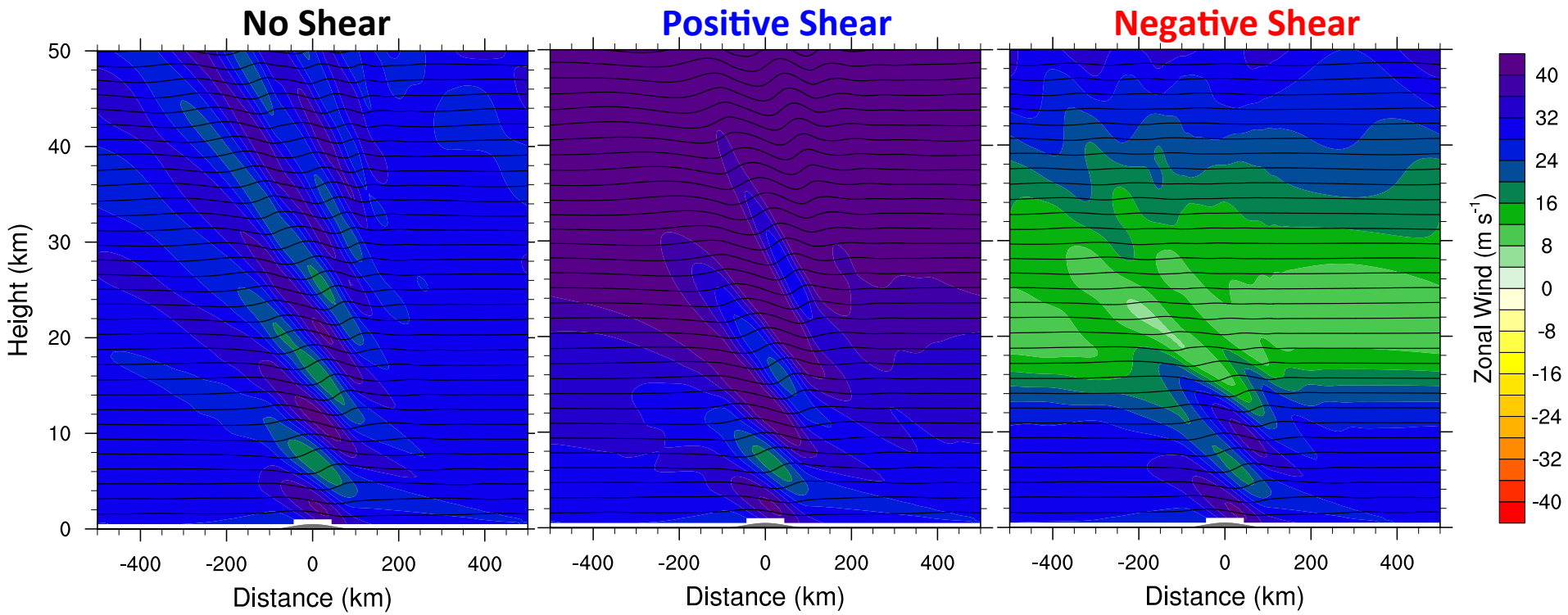
No Shear	Positive Shear	Negative Shear
25.1 km	35.3 km	15.3 km

τ_b

Mode	λ_x	No Shear	Positive Shear	Negative Shear
Volume	200 km	296 min	292 min	218 min
Roughness	28.5 km	42 min	42 min	31 min

1. Wave Field at time=τ_b

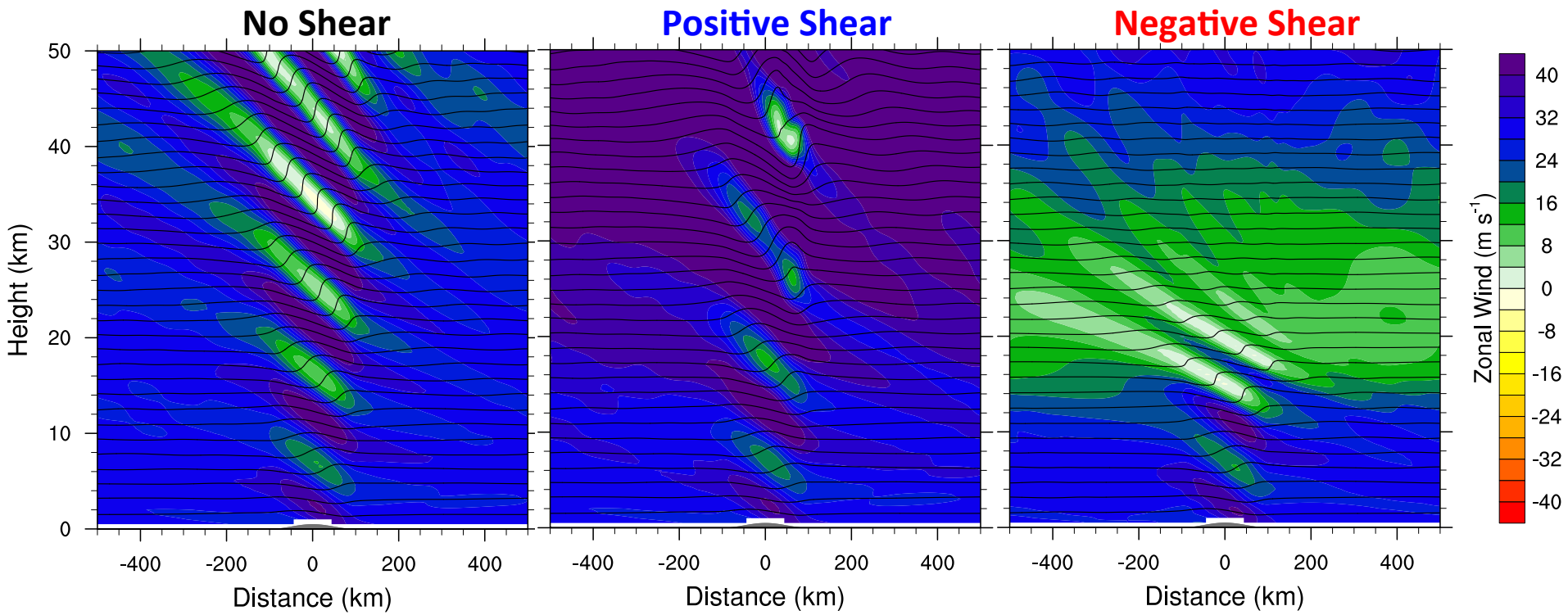
- Wave gets to z_b, but no wave overturning



Color Shading: Zonal Wind
Contours: Isentropes

1. Wave Field at time=$2\tau_b$

- Wave overturning occurs near t = $2\tau_b$



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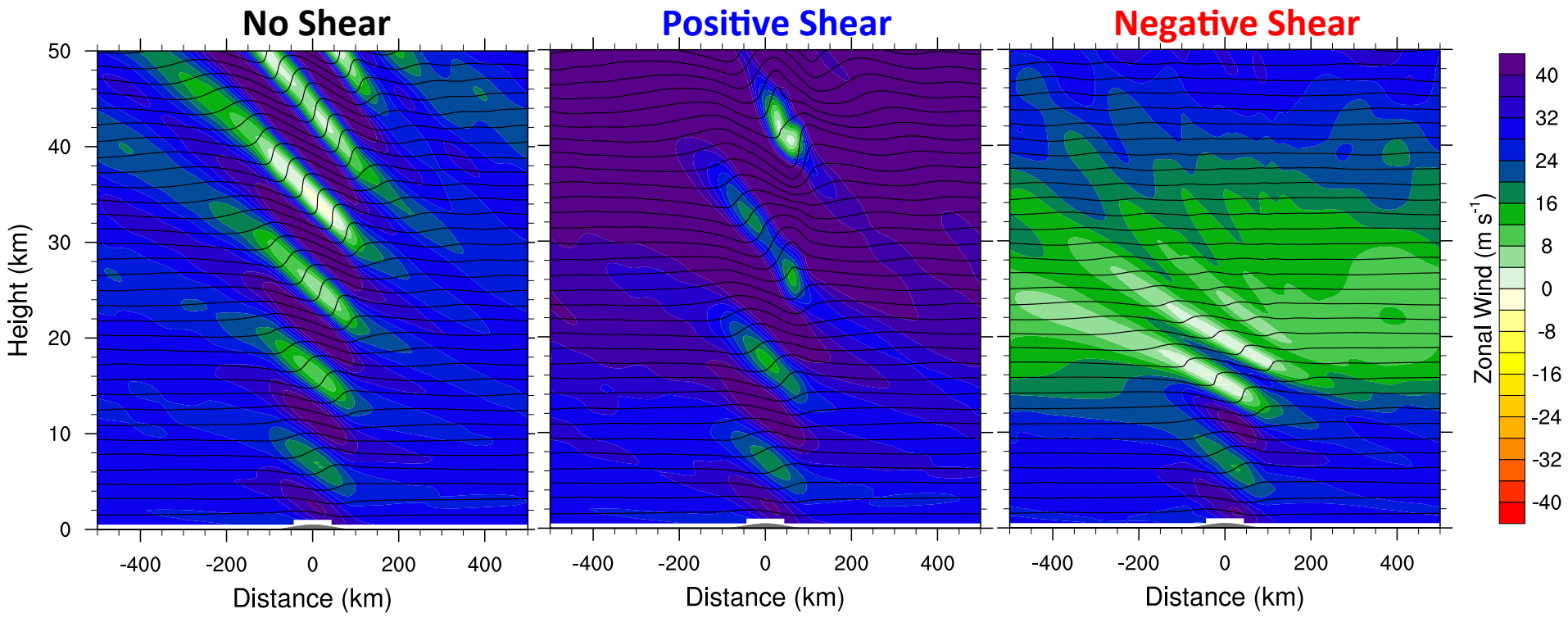


No Shear: **Many overturning levels**

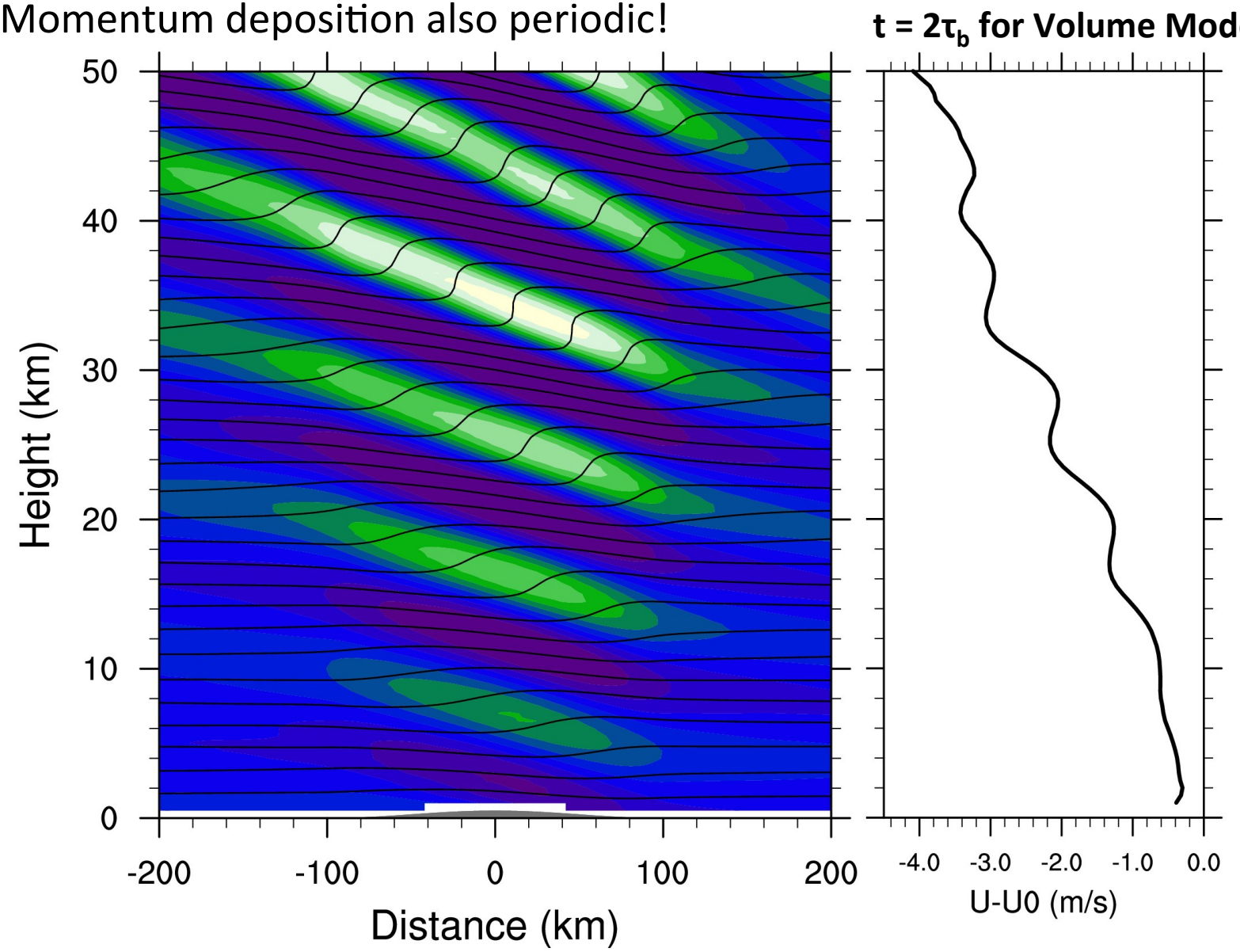
Positive Shear: **Fewer, spaced overturning levels**

Negative Shear: **Overturning confined to negative shear layer**

Color Shading: Zonal Wind
Contours: Isentropes

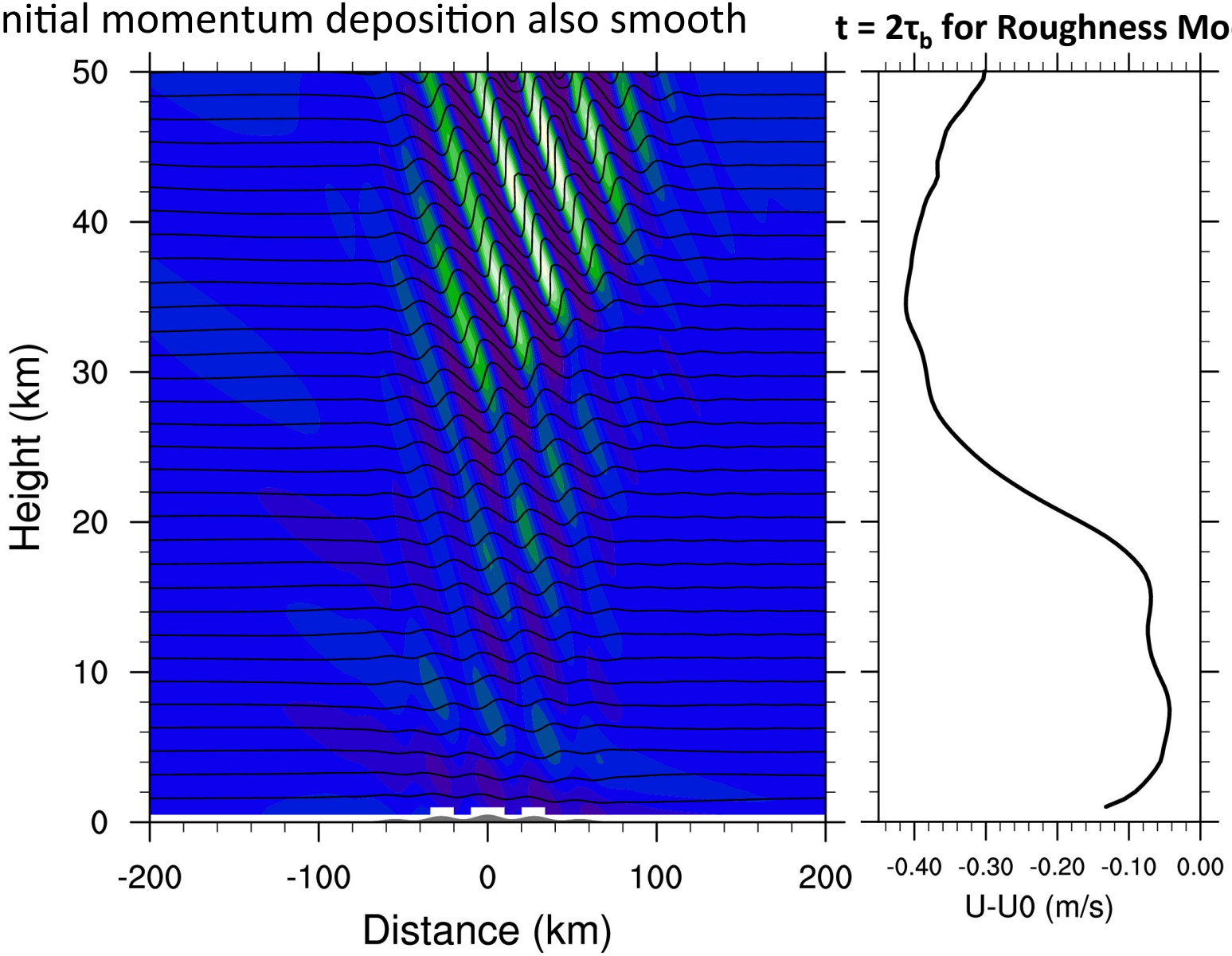
2. Attenuation Isolated in the Vertical?

- Overturning periodic in vertical for compact, smooth Volume Mode
  - Momentum deposition also periodic!



2. Attenuation Isolated in the Vertical?

- Overturning smooth in vertical for multiple mountain terrains
  - Initial momentum deposition also smooth



3. Simple Gravity Wave Drag Diagnostic

- Time Integrated Gravity Wave Drag per unit mass (GWD) gives the speed reduction of the mean flow:

$$\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x}\left(\rho u^2 + p\right) + \frac{\partial(\rho u w)}{\partial z} = 0$$

$$\overline{(\,.\,)} = \frac{1}{L}\int_0^L (\,.\,)\, dx$$

MF$_x$

$$\frac{\partial \overline{u}}{\partial t} = -\frac{1}{\overline{\rho}}\frac{\partial(\overline{\rho}\,\overline{u'w'})}{\partial z} = GWD$$

$$\Delta\overline{u}(z,t) = \int_0^t GWD(z,t')\,\partial t'$$

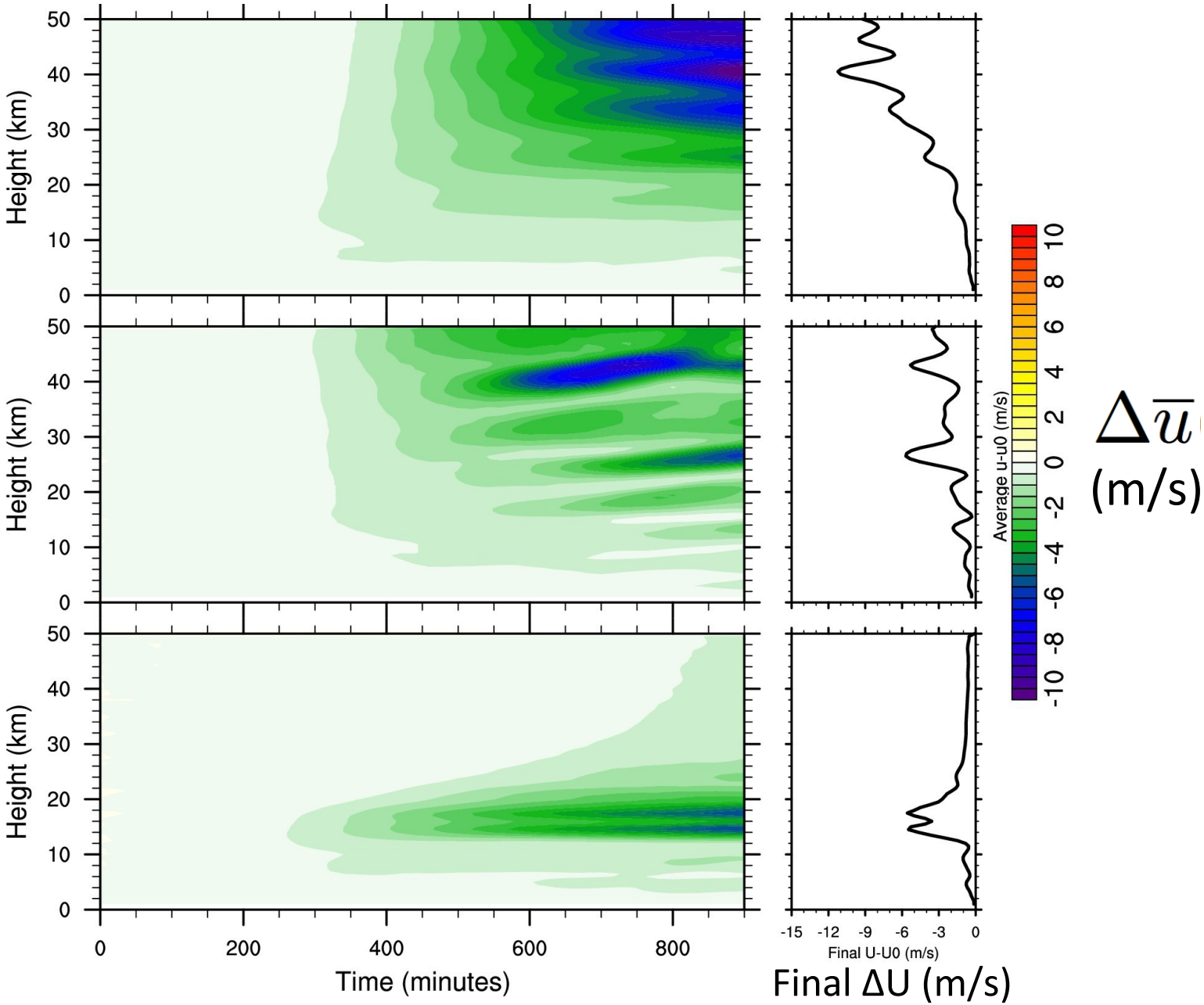
Wave-Induced Wind Speed Reduction

Volume Mode Only

No Shear

Positive Shear

Negative Shear



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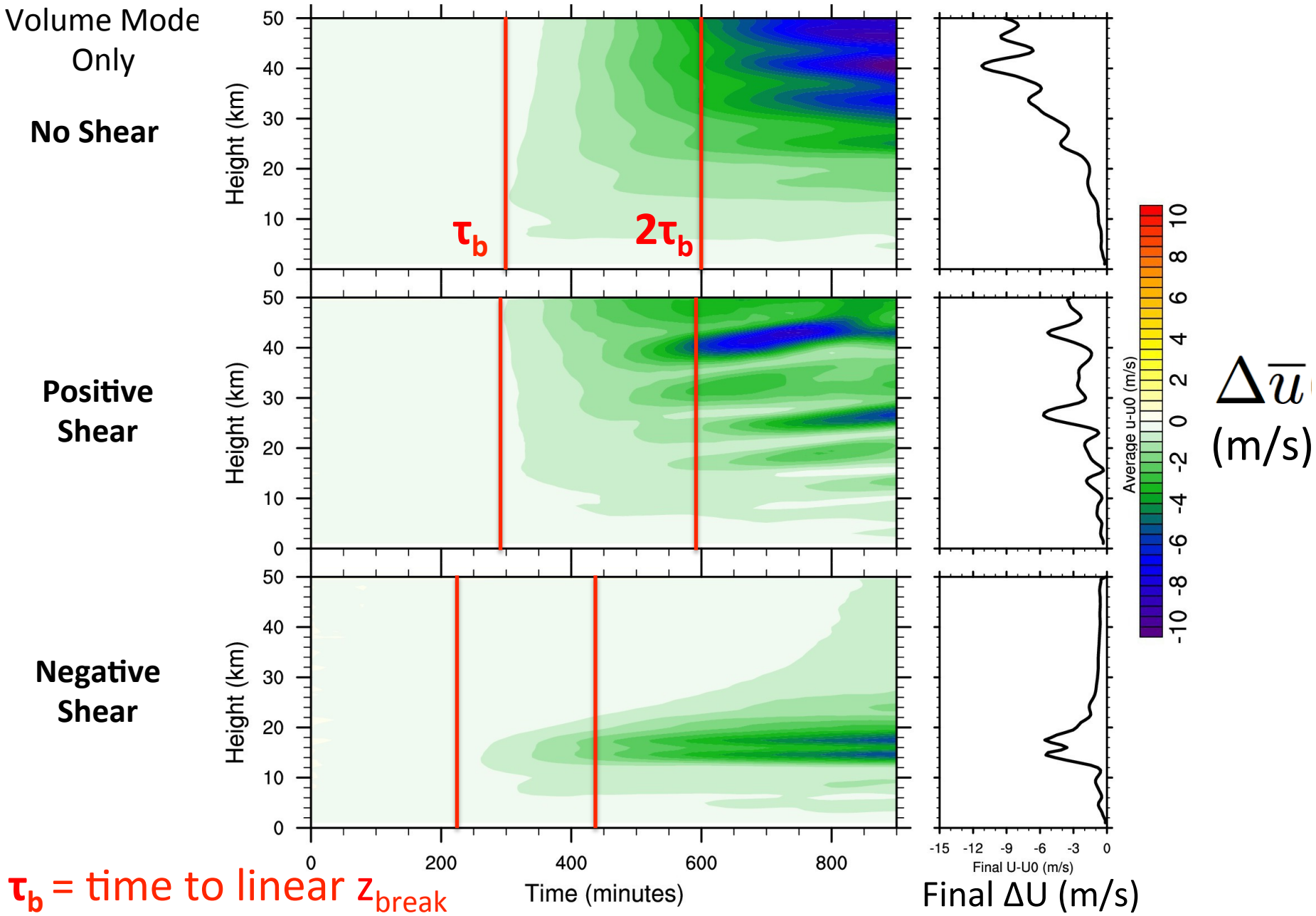


Wave-Induced Wind Speed Reduction

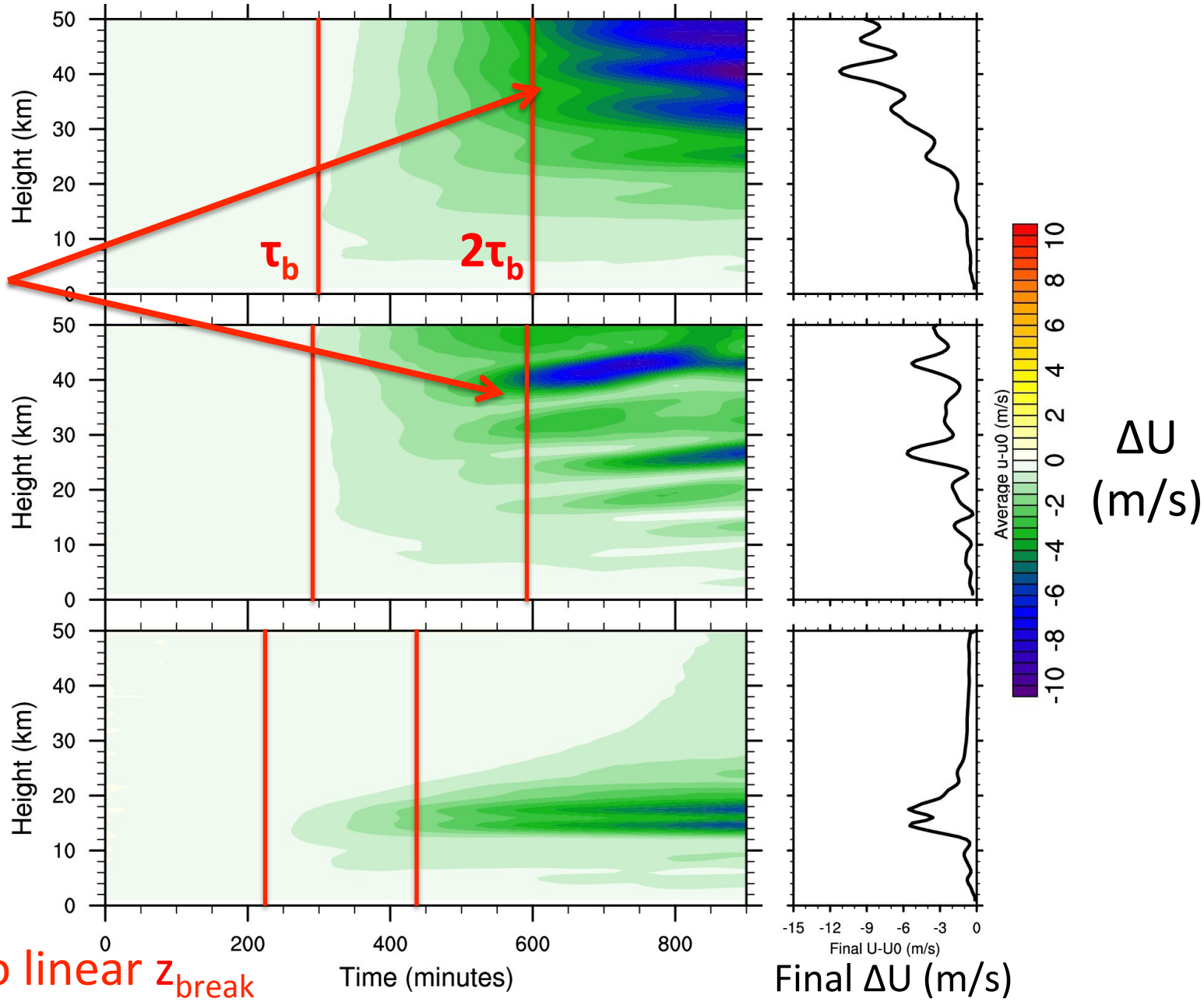
Volume Mode Only

No Shear

Momentum Deposition Periodic in z After Overturn

Positive Shear

Negative Shear



τ_b = time to linear z_{break}

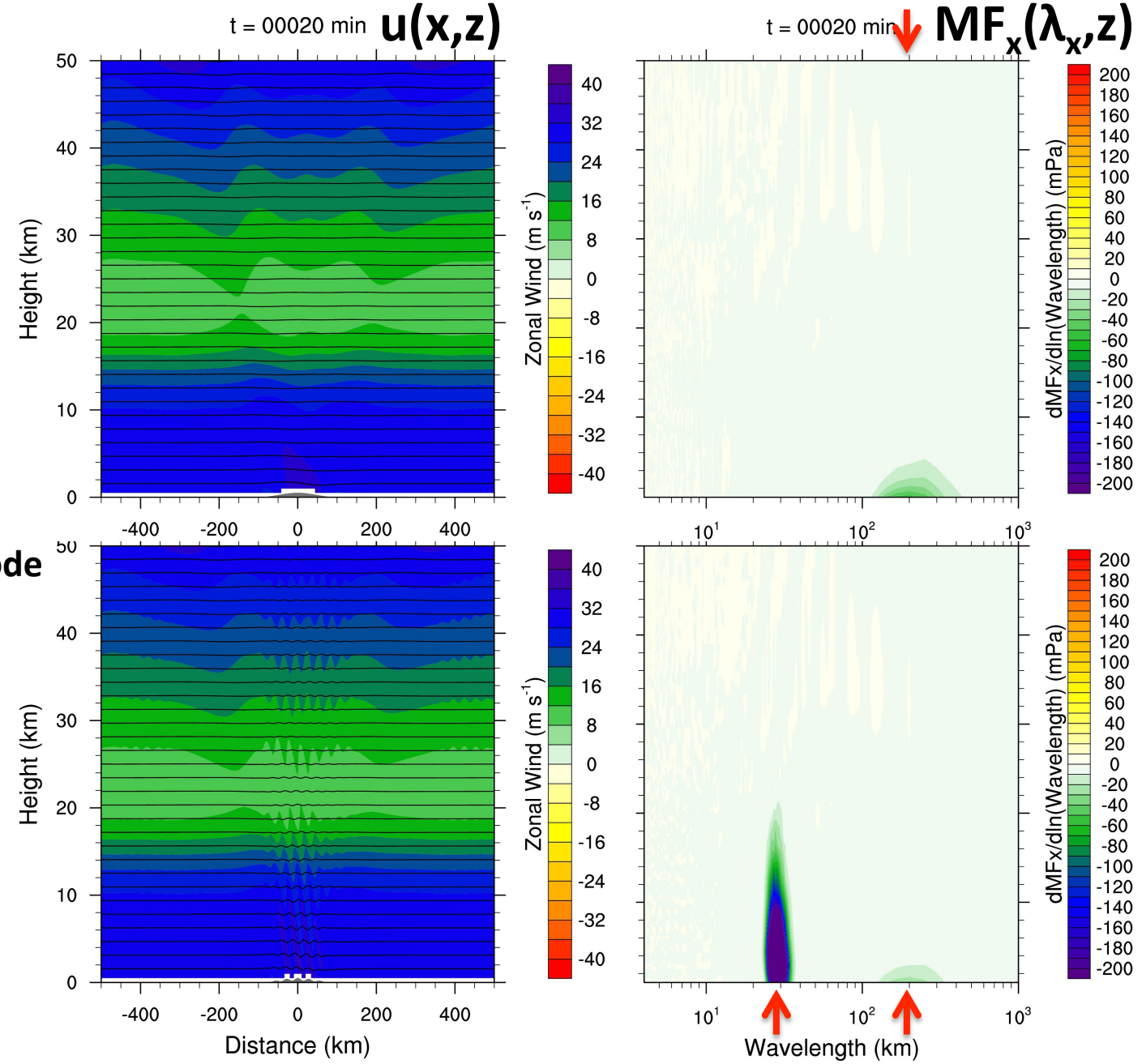
Spectral Evolution in Negative Shear

Volume Mode

λ_x = 200 km

Volume + Roughness Mode

λ_x = 28.5, 200 km



Spectral Evolution in Negative Shear

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Volume + Roughness Mode
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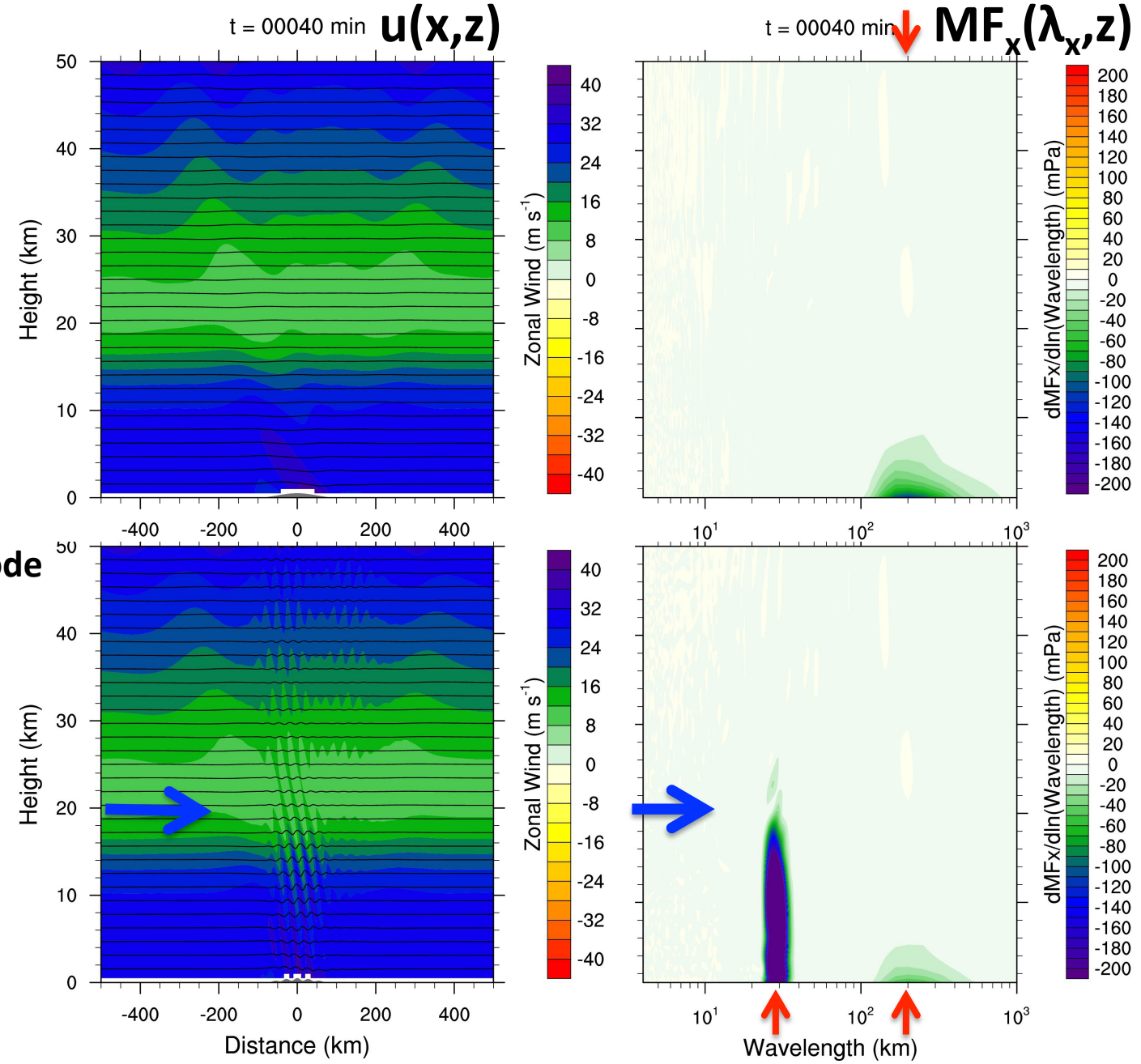
- Roughness mode propagates up very quickly
- Smallest scales most important at high altitudes initially



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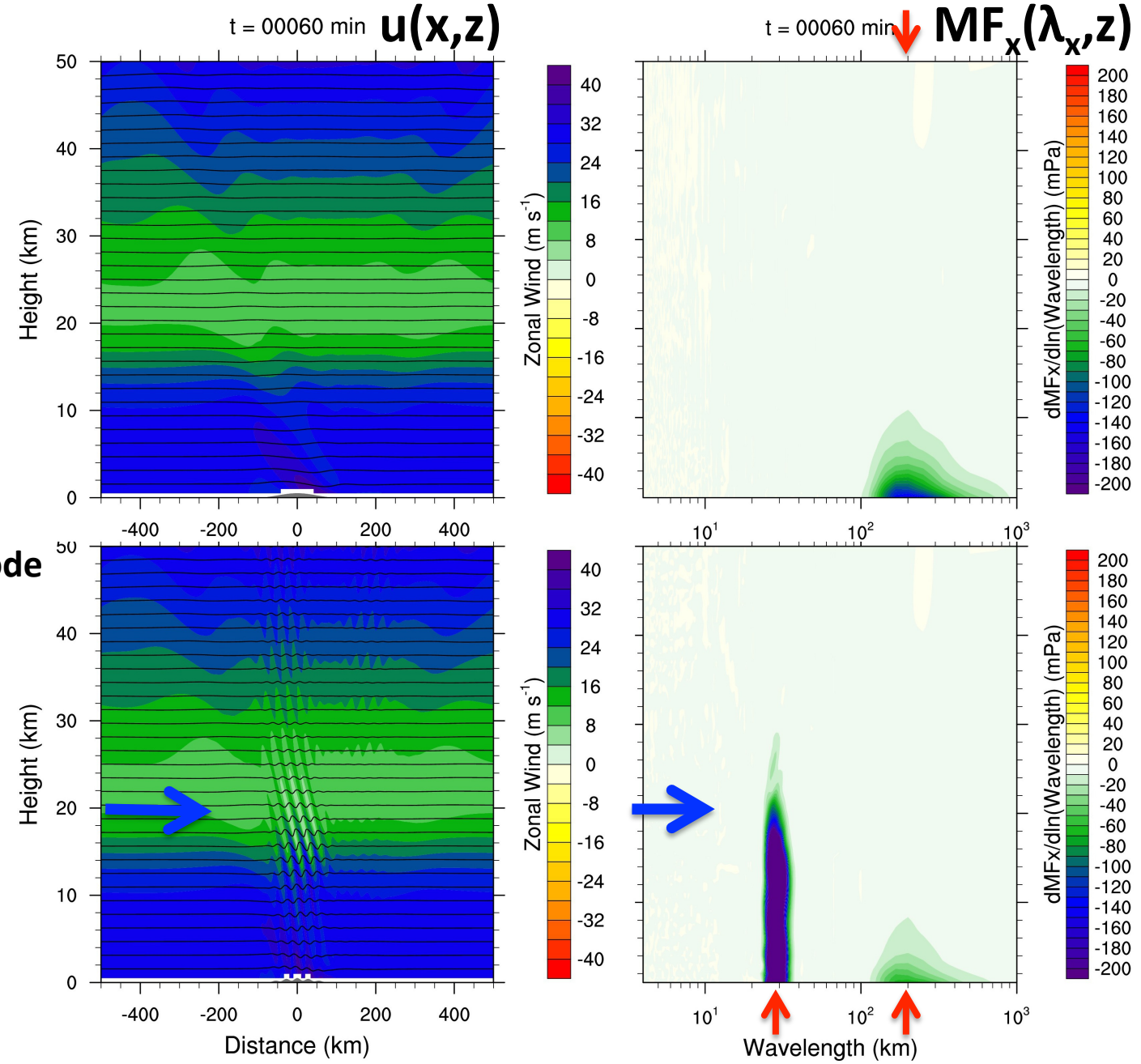

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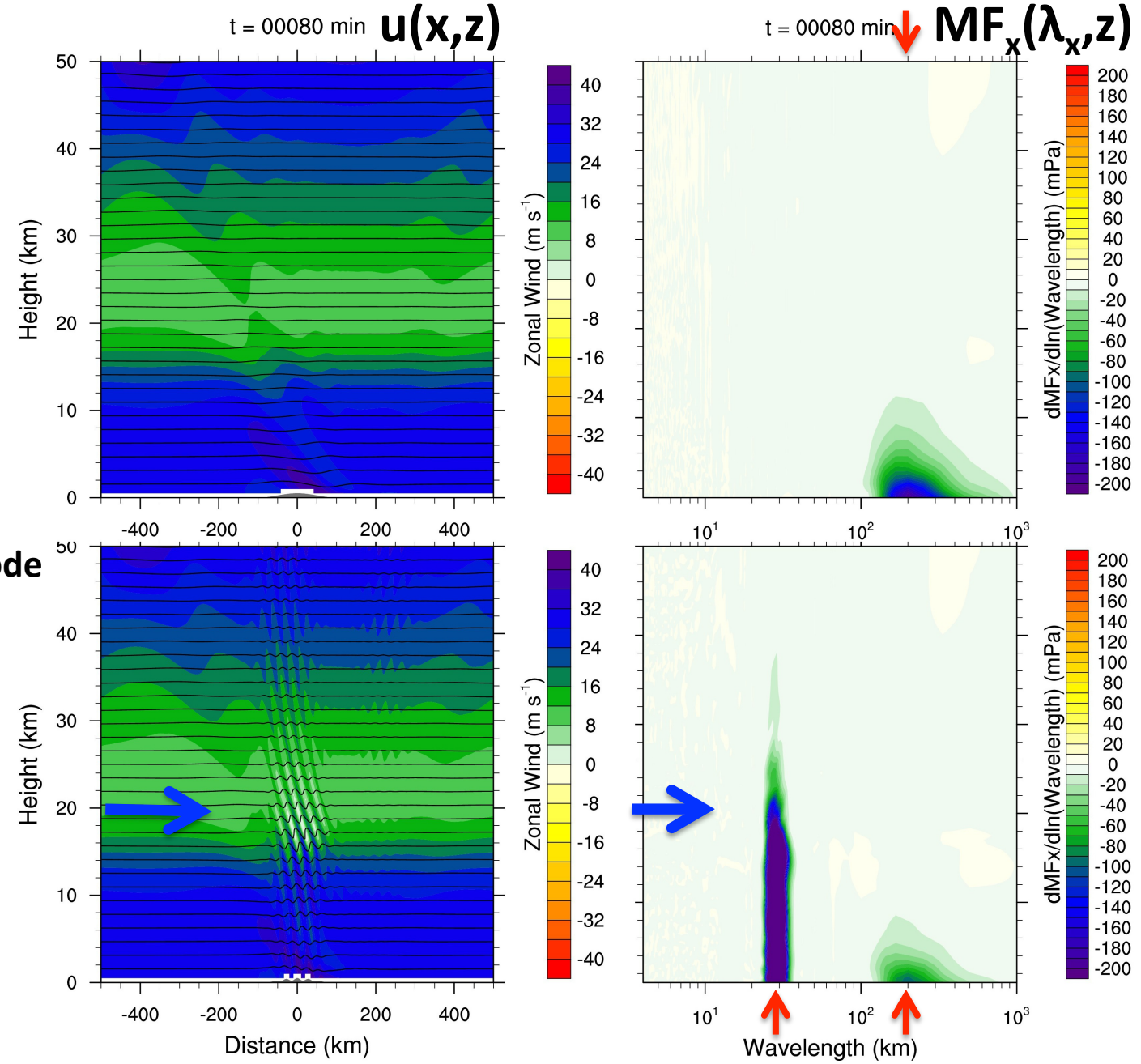

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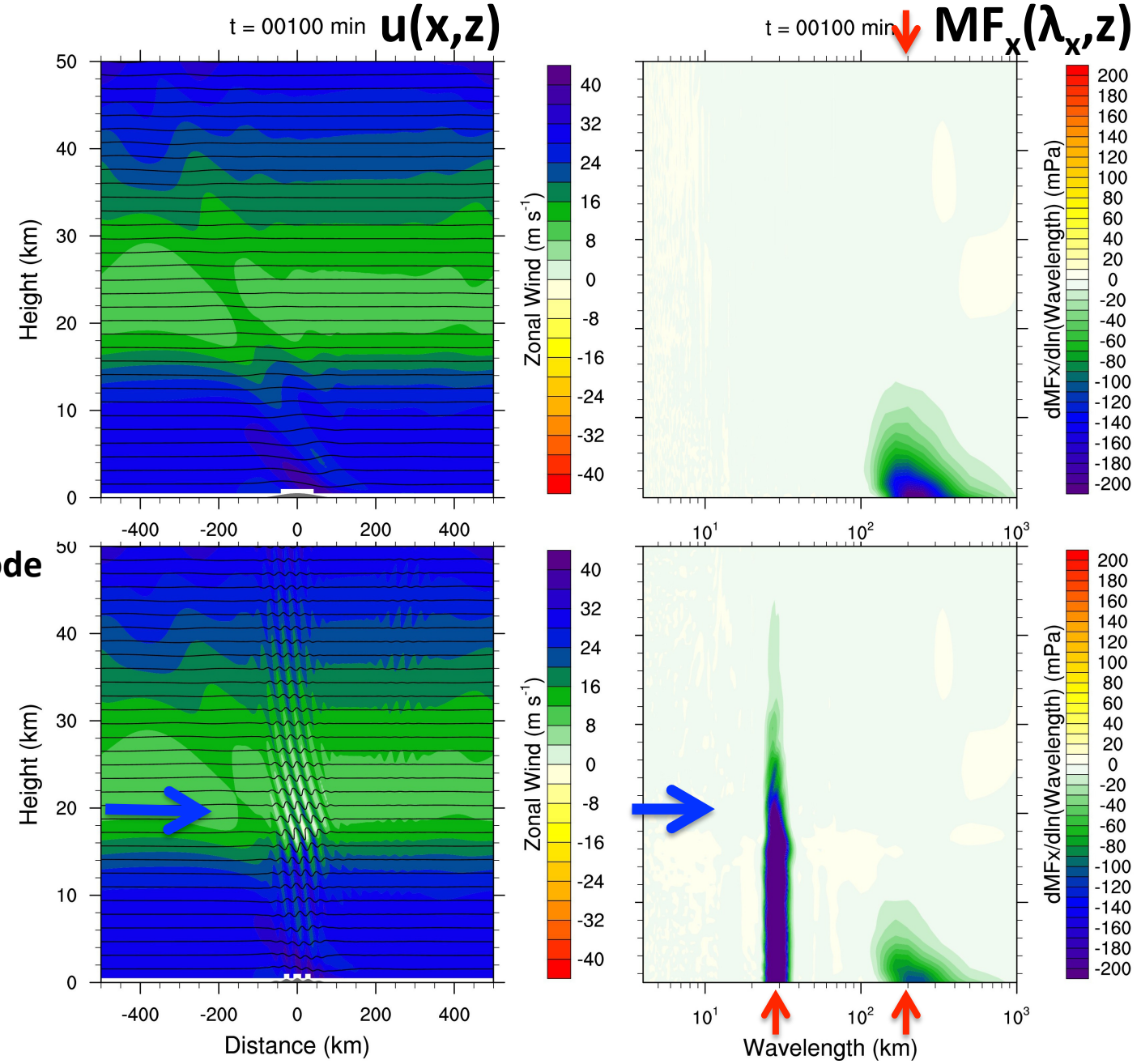

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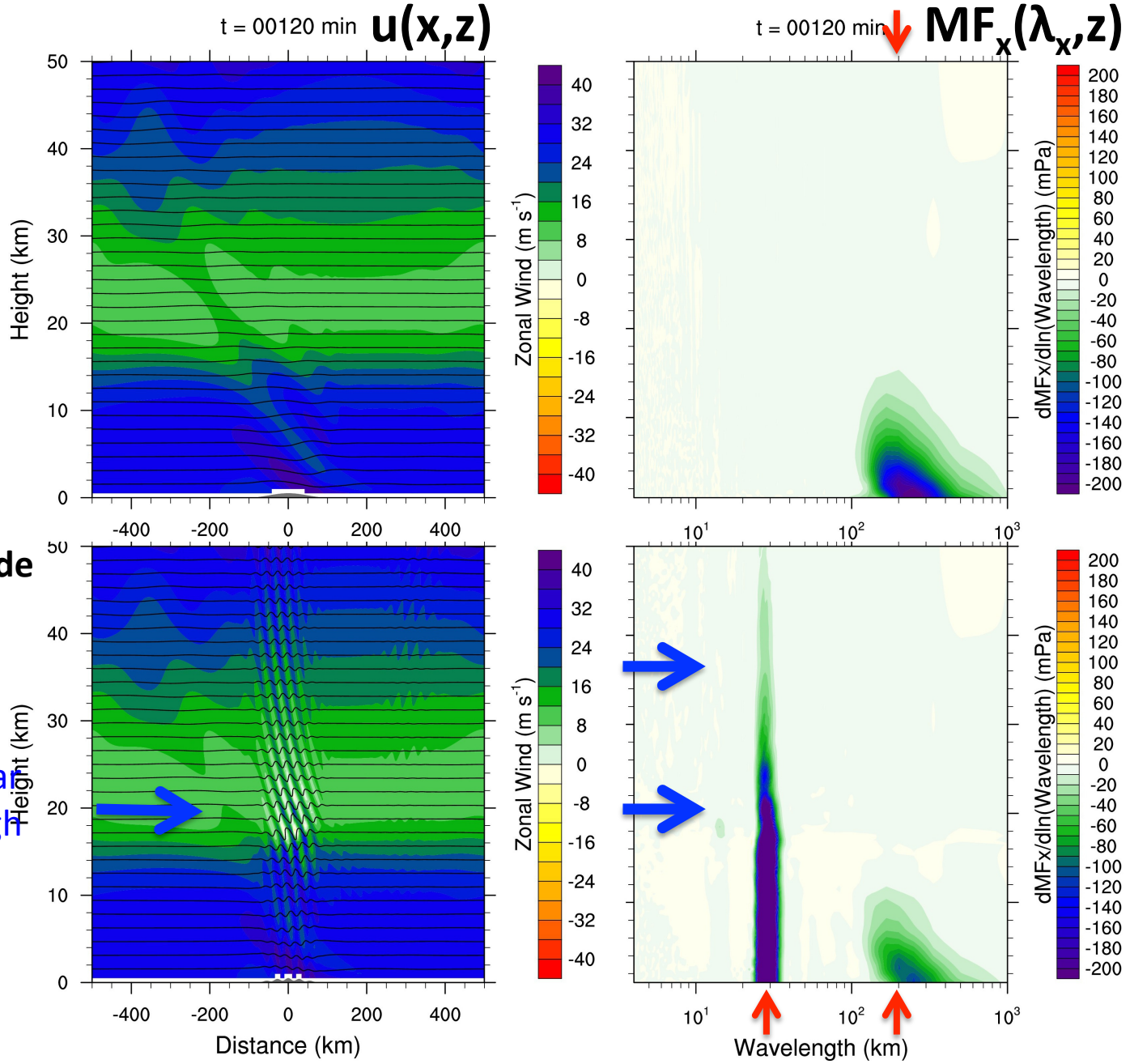
- Roughness mode breaks in negative shear
- Still propagates through despite overturning in negative shear



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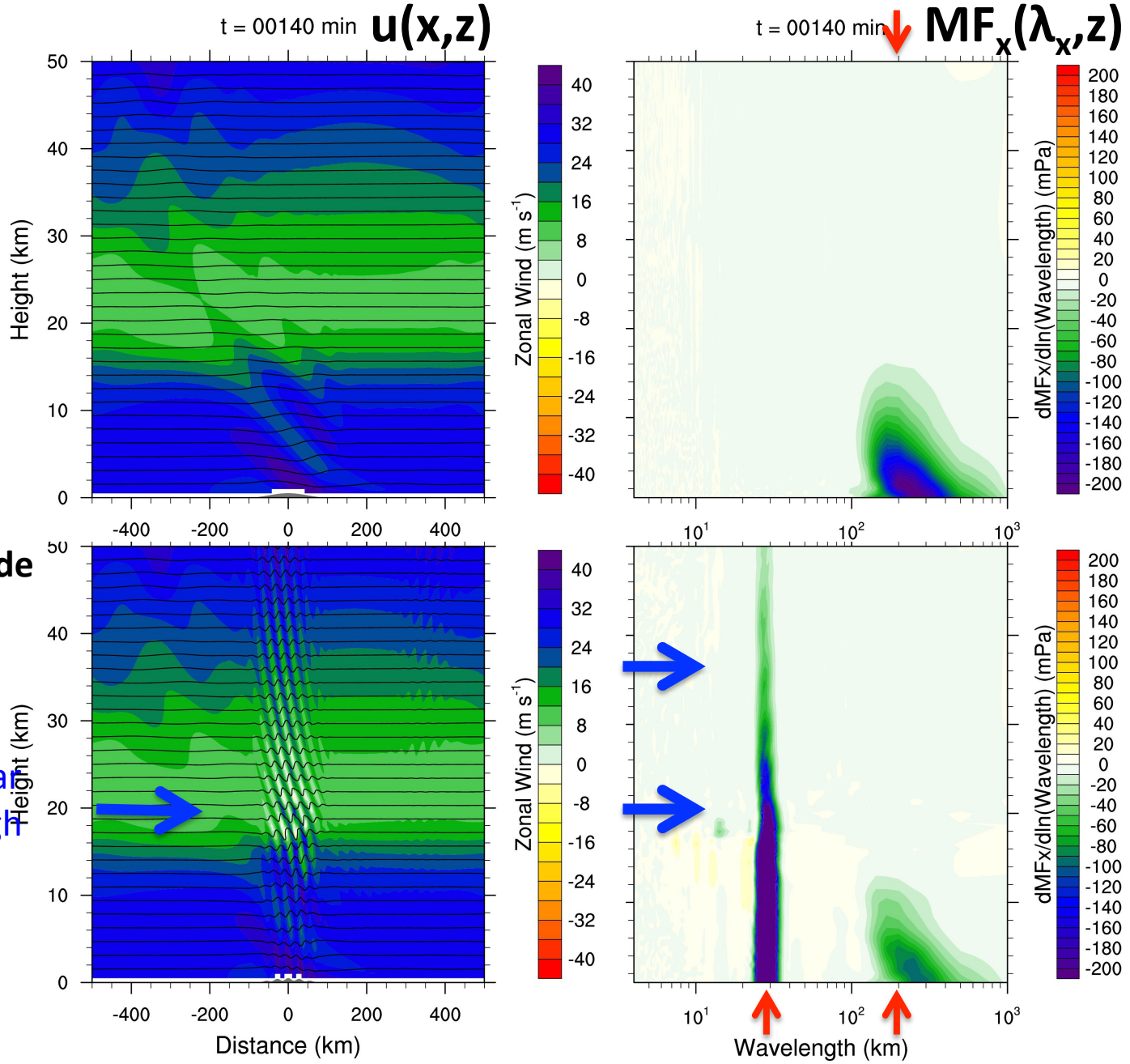

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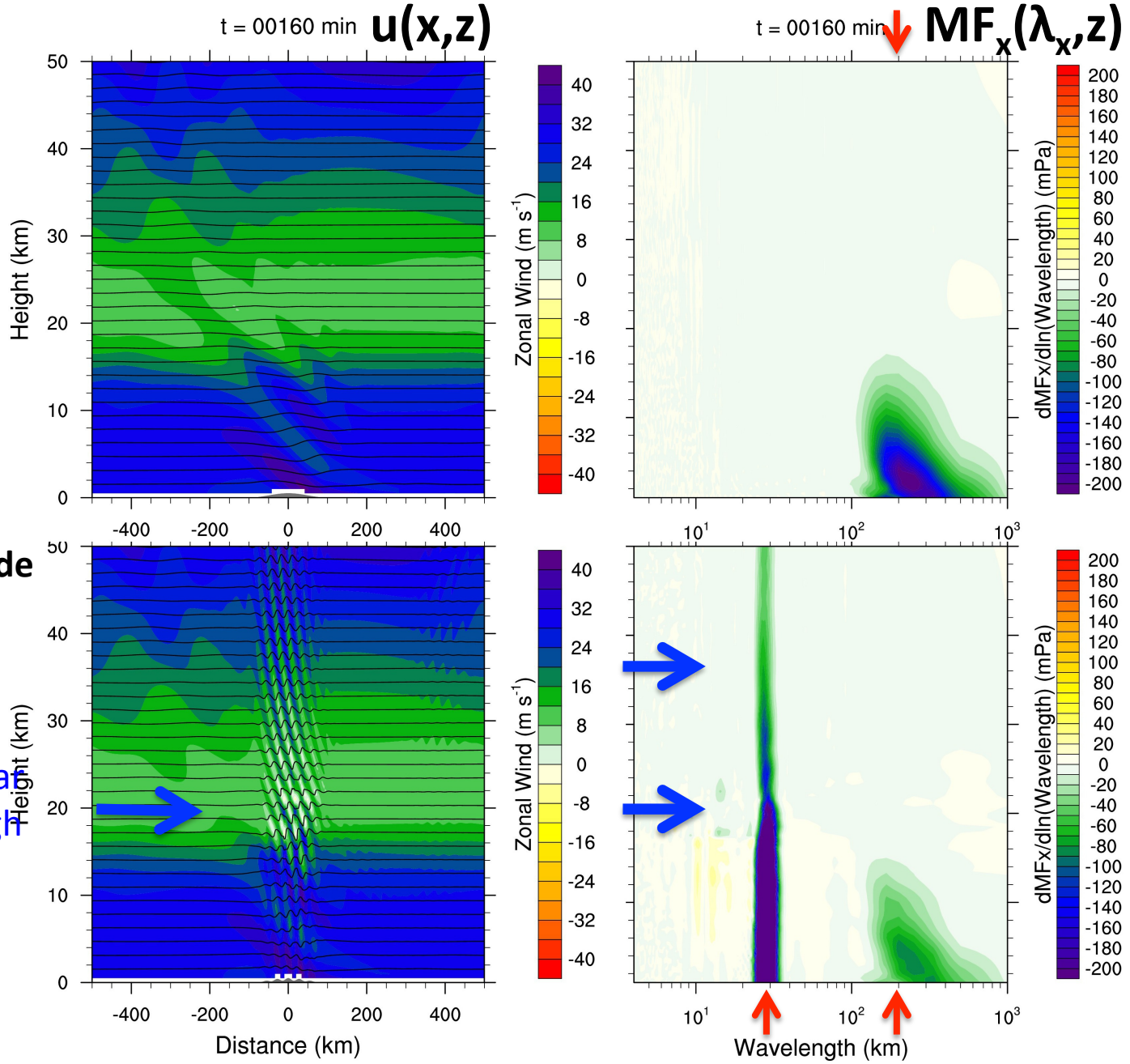

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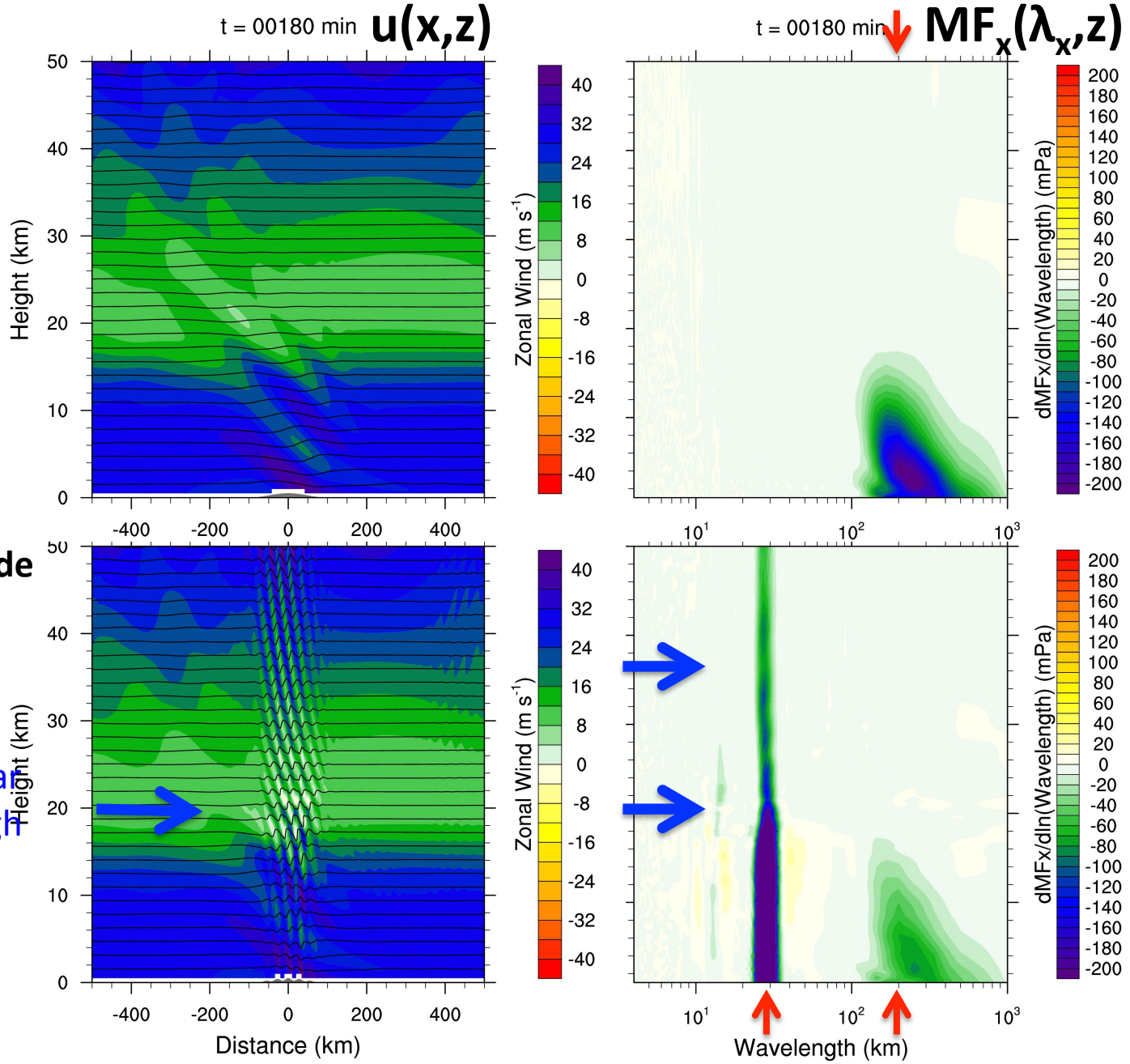

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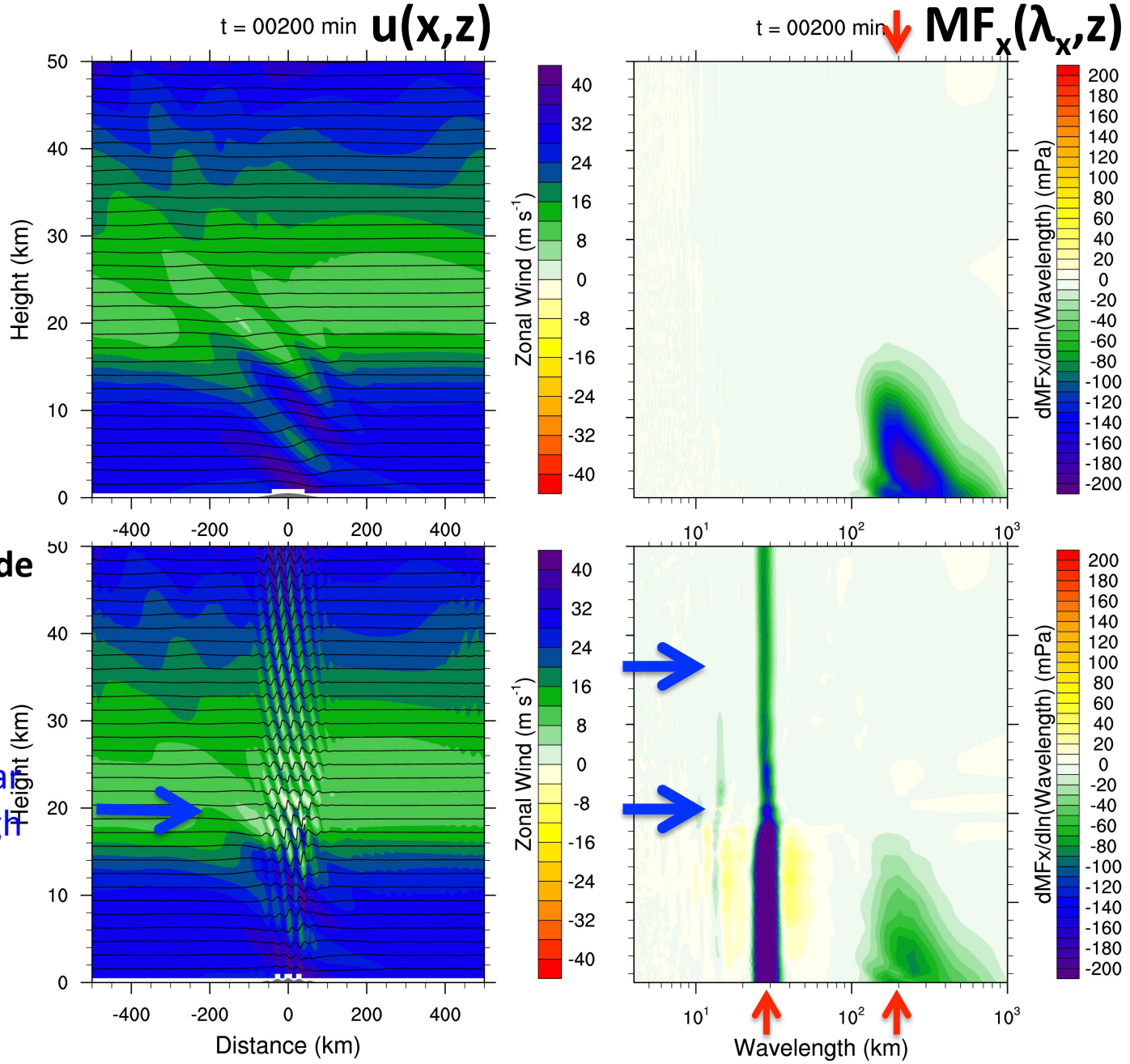

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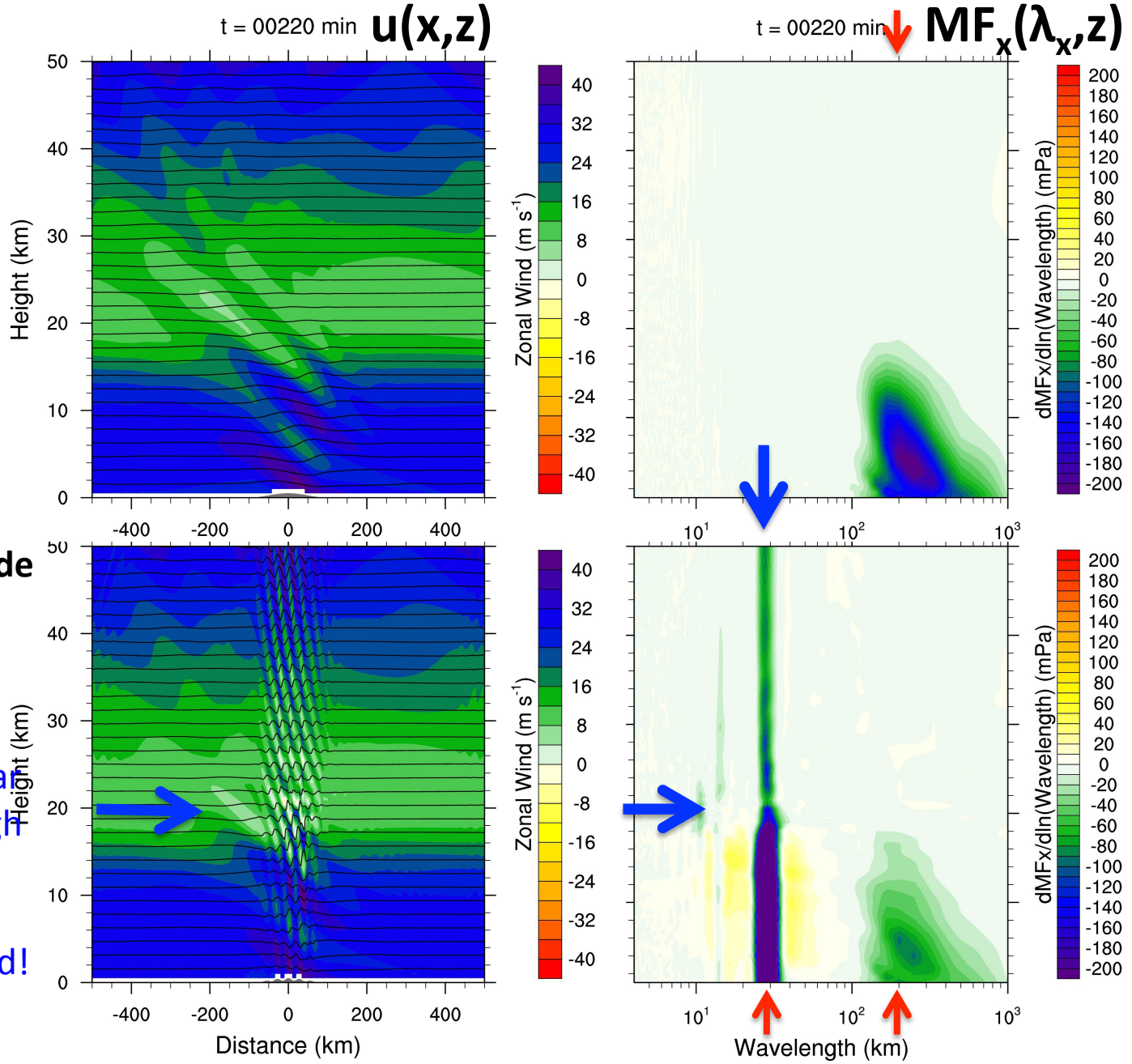
- Still propagates through despite overturning in negative shear
- Wavelength unchanged!



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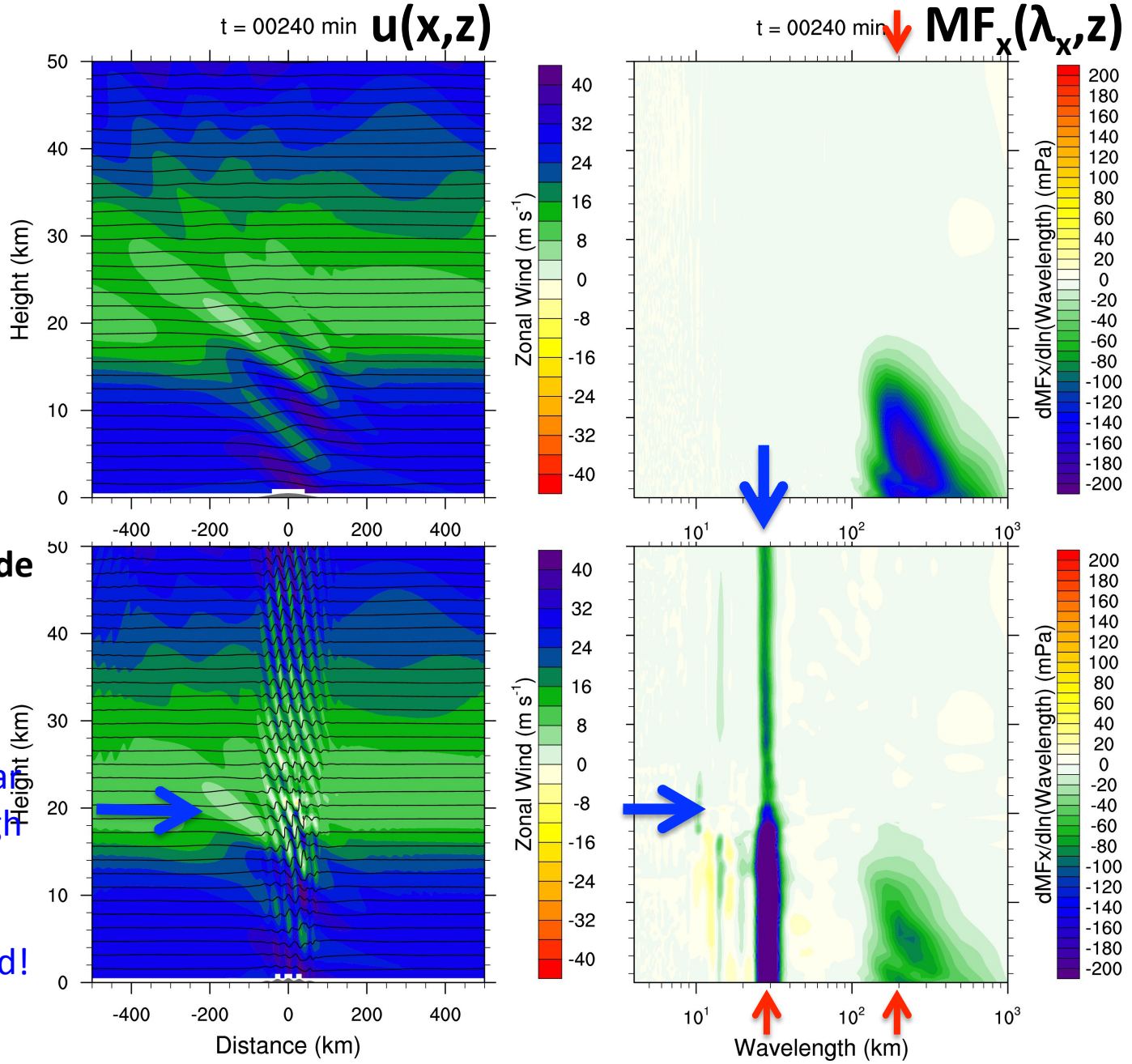

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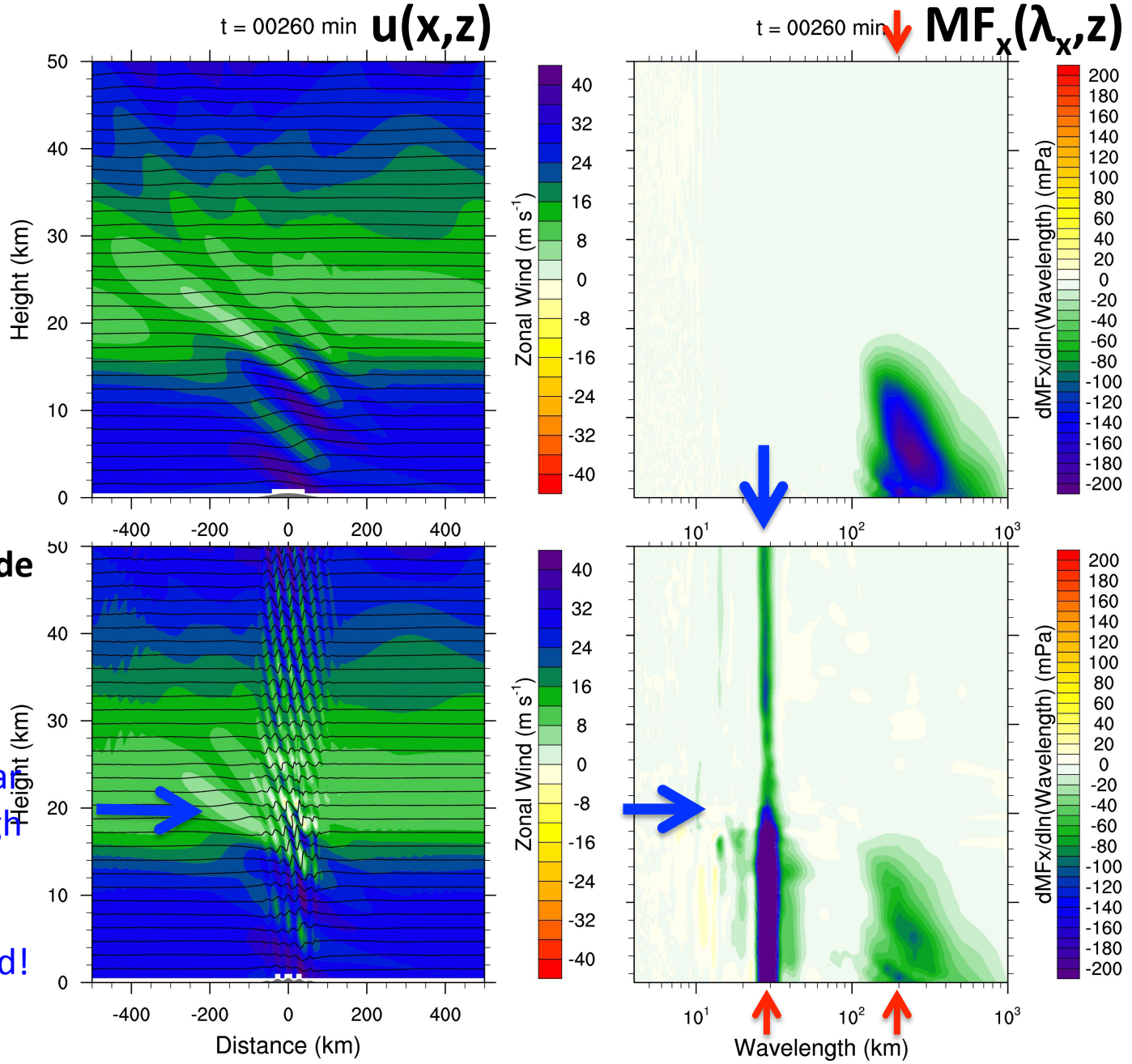

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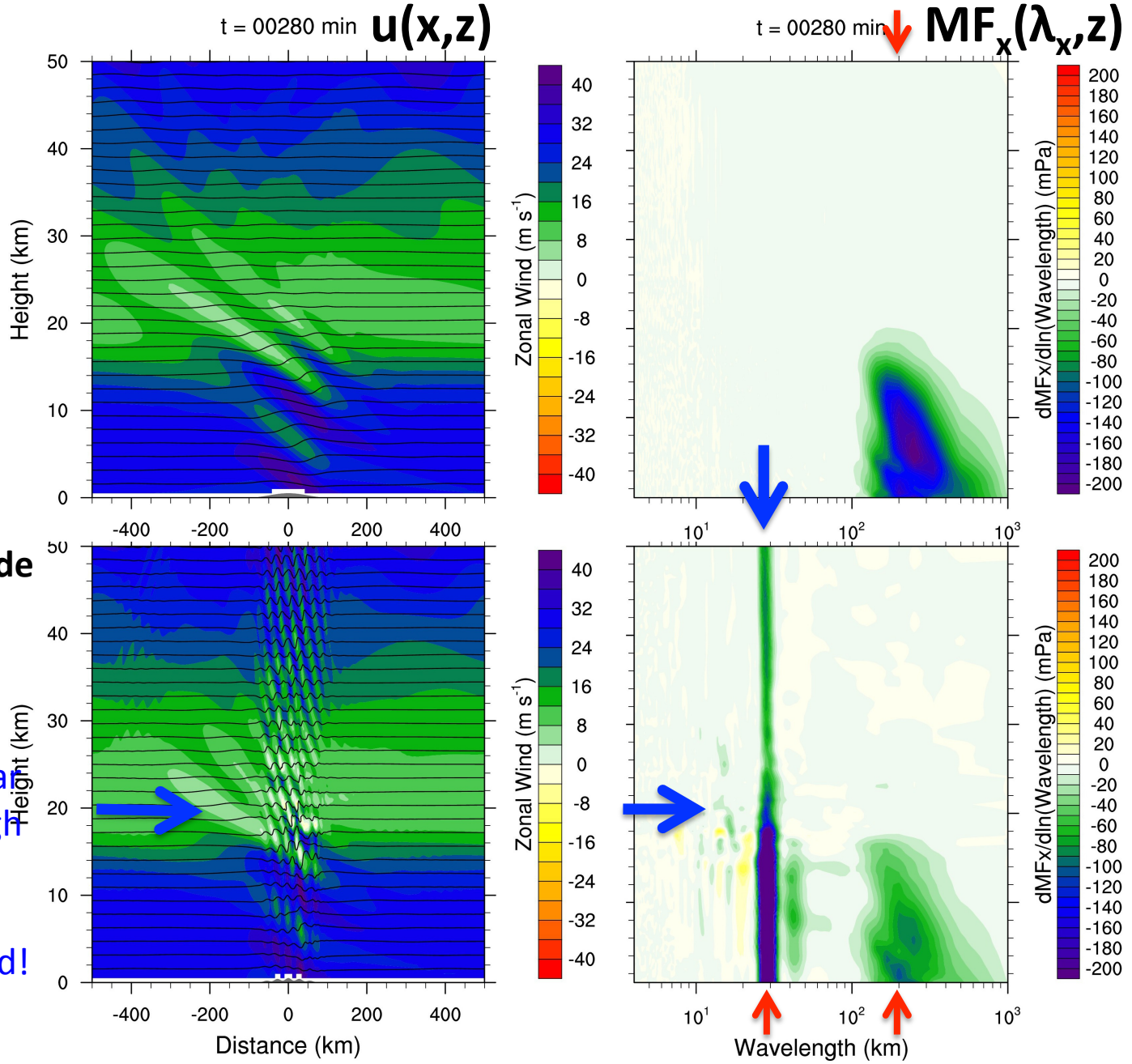

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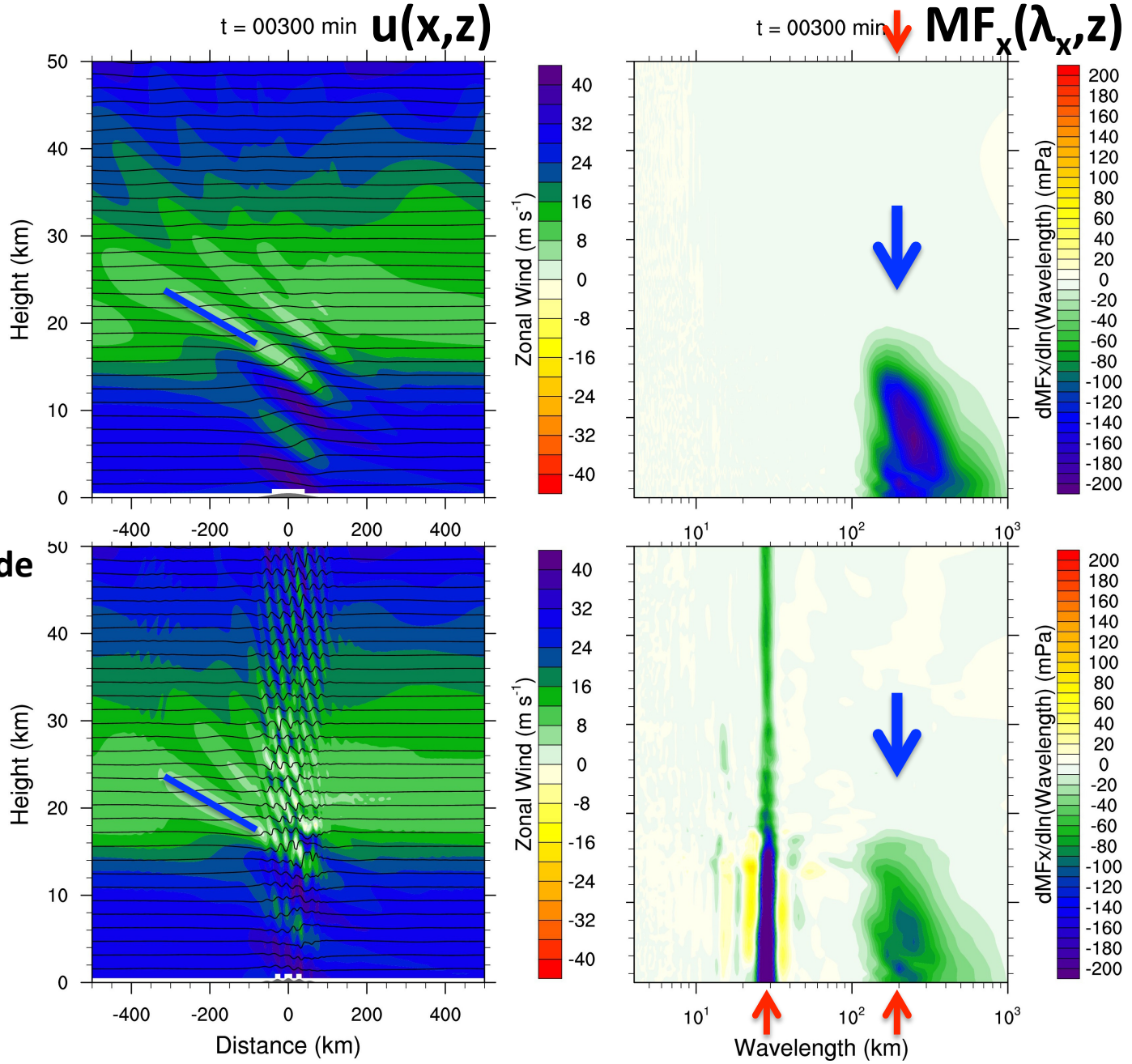
- Volume mode reaches negative shear



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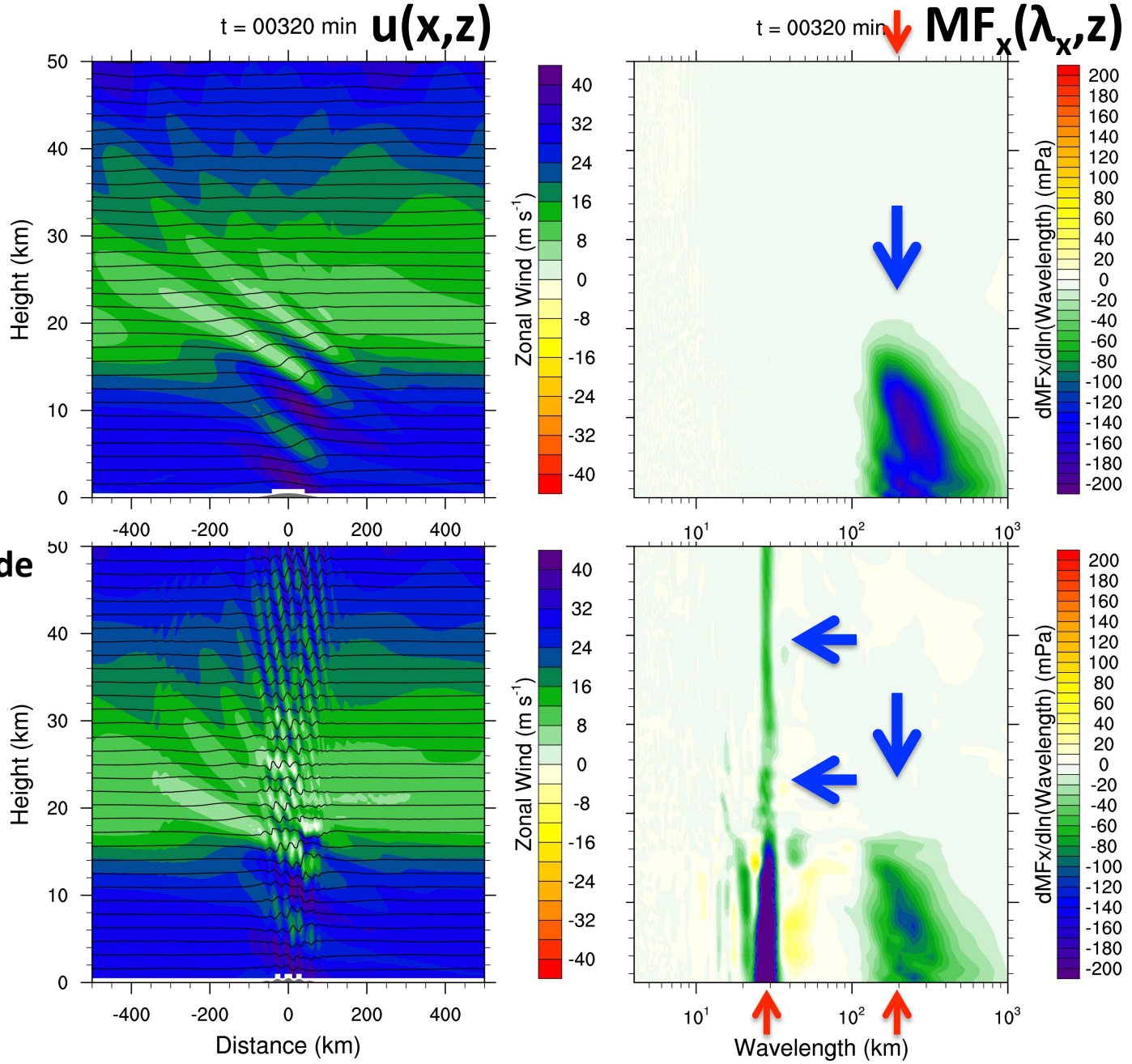
- Volume mode reaches negative shear
- Large scale -u' by volume mode causes further attenuation of roughness mode!



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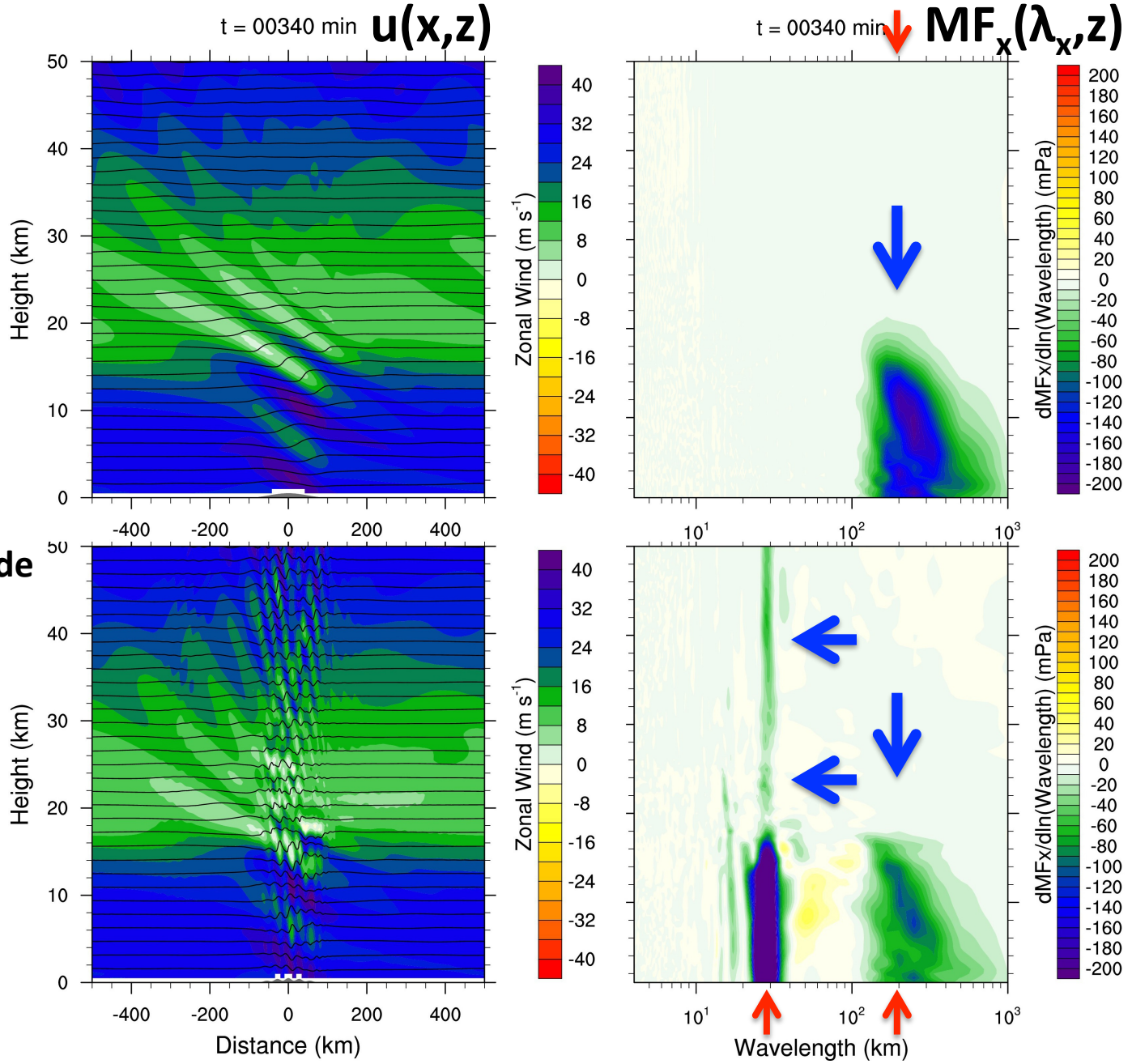

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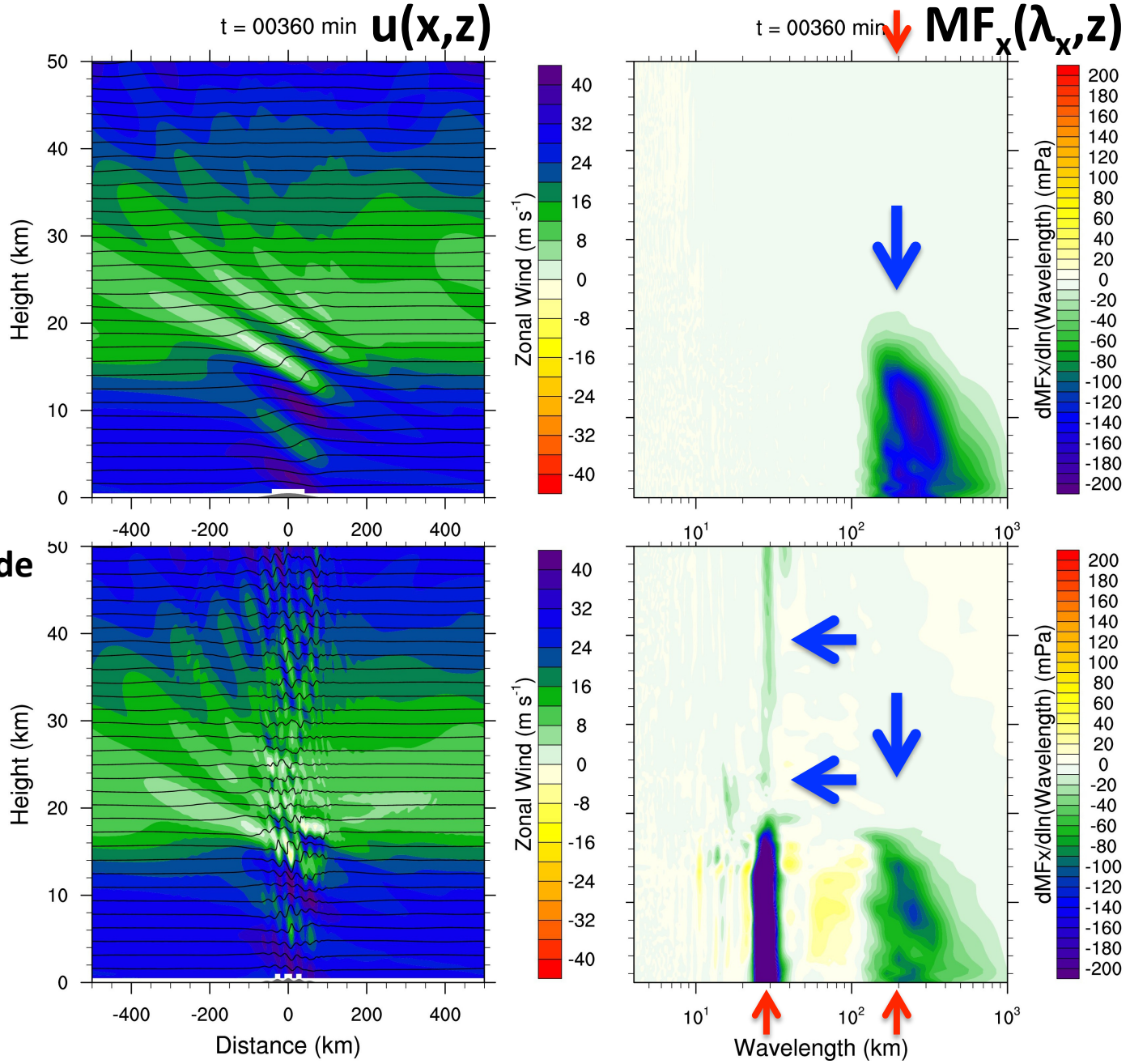

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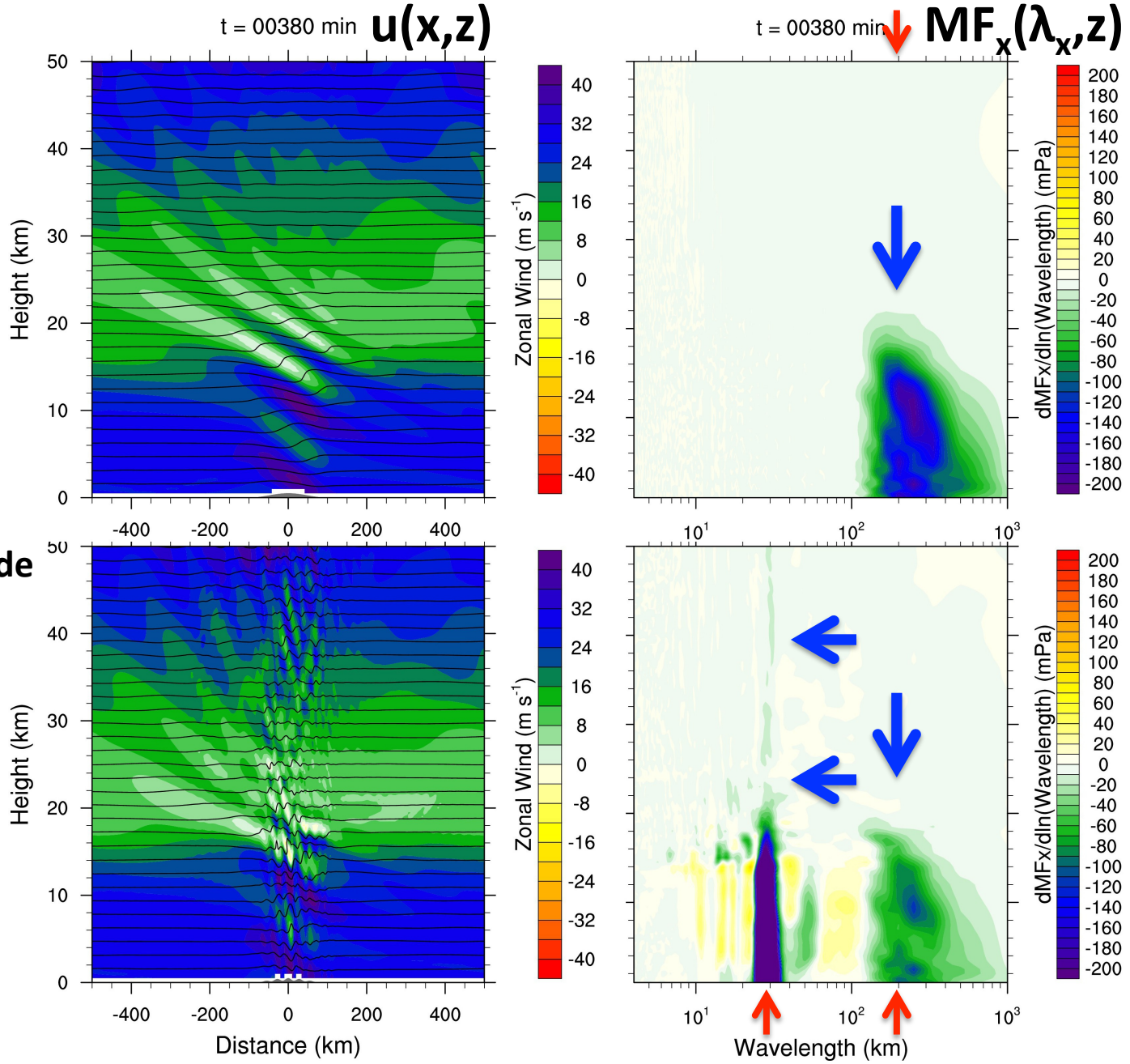

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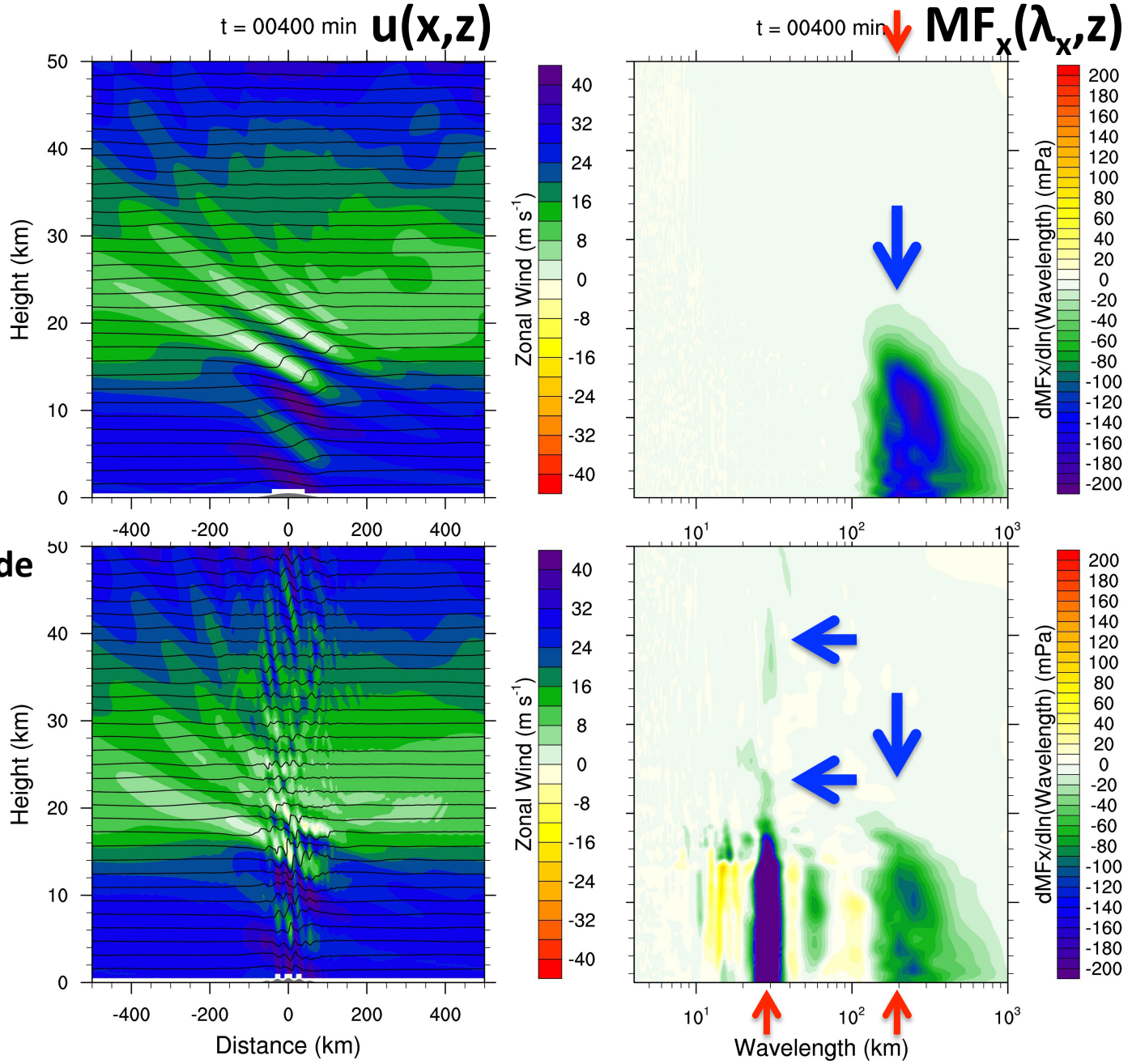

Spectral Evolution in Negative Shear

Volume Mode
λ_x = 200 km

Volume + Roughness Mode
λ_x = 28.5, 200 km

- Volume mode reaches negative shear
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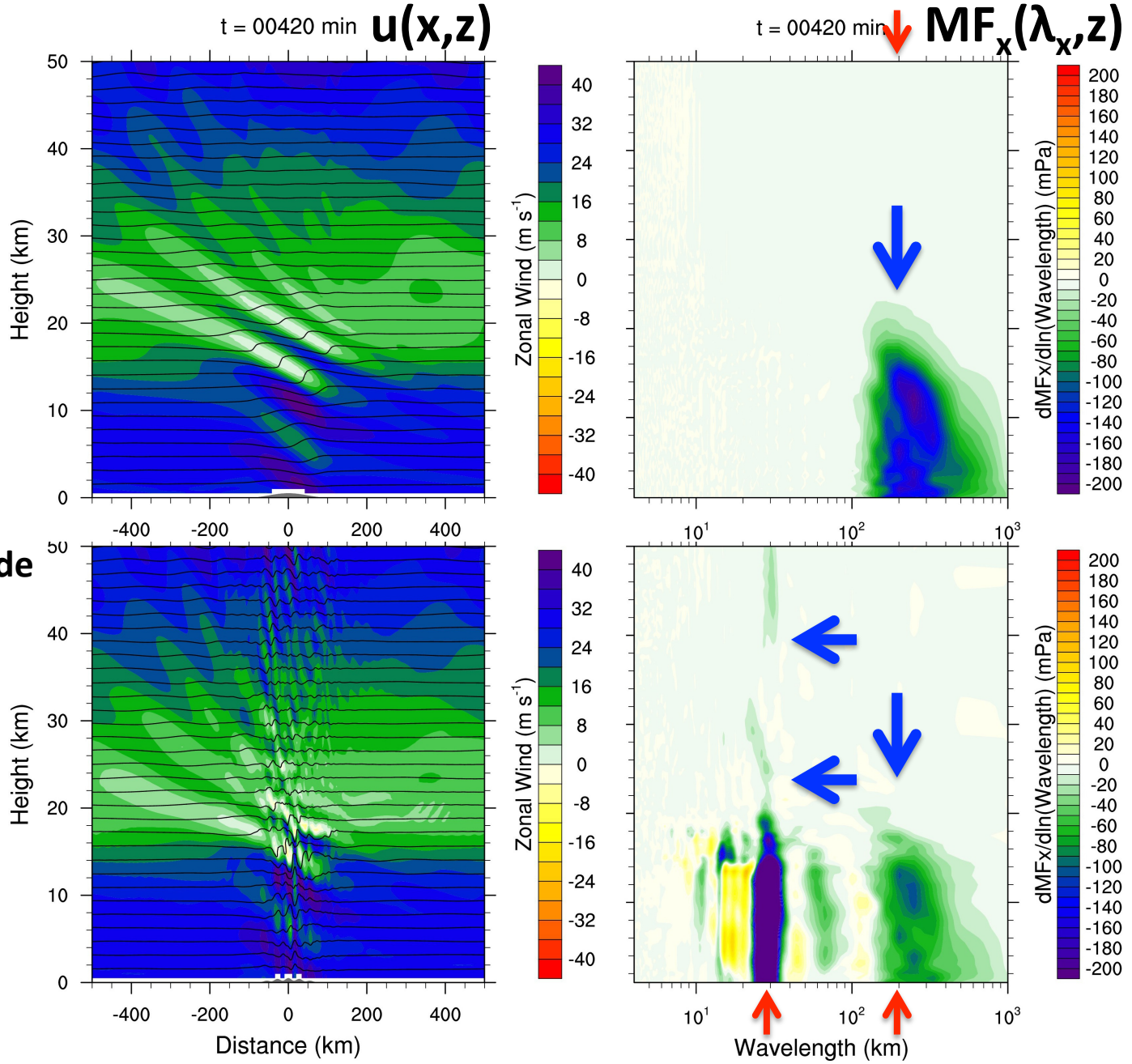

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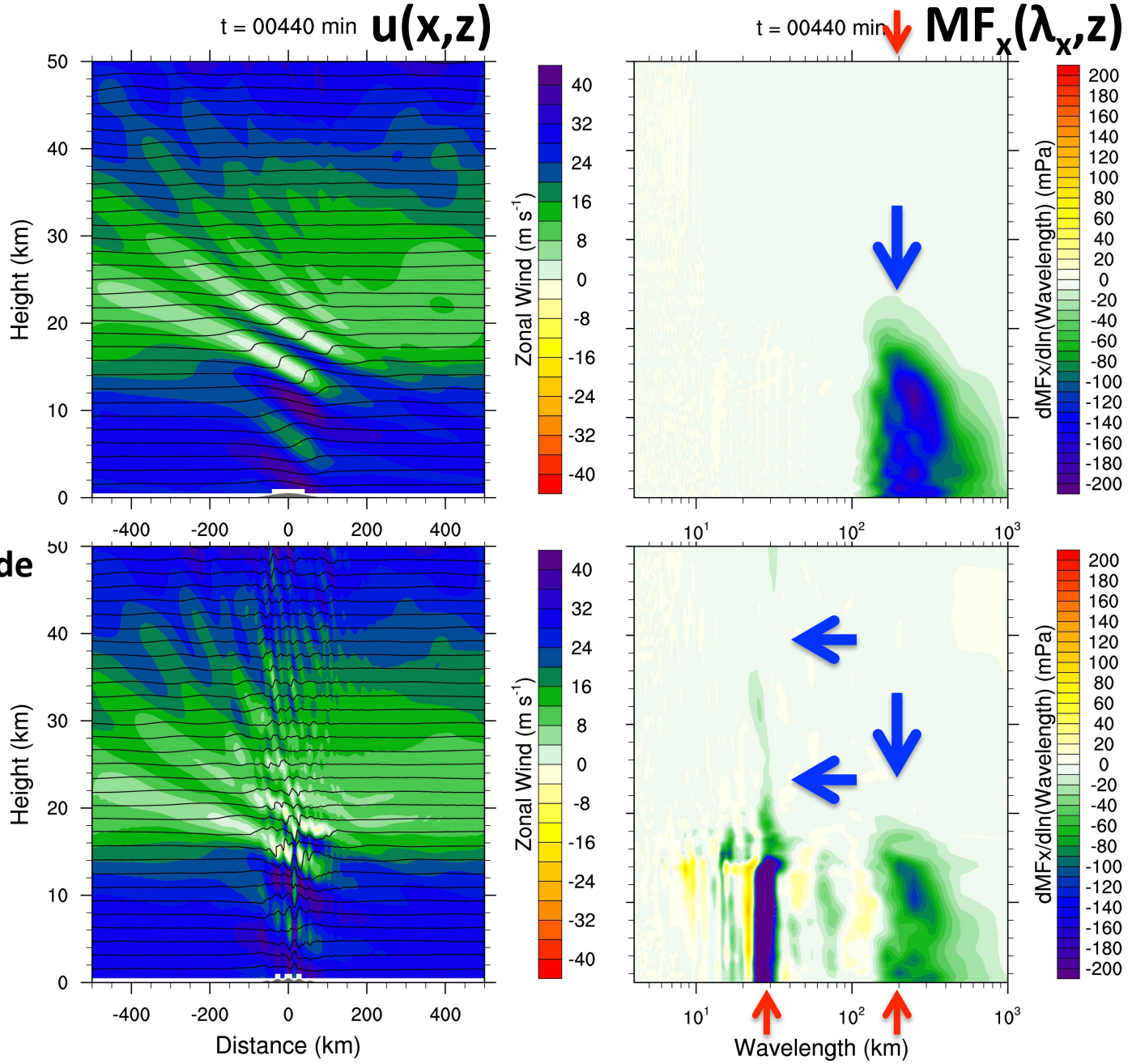
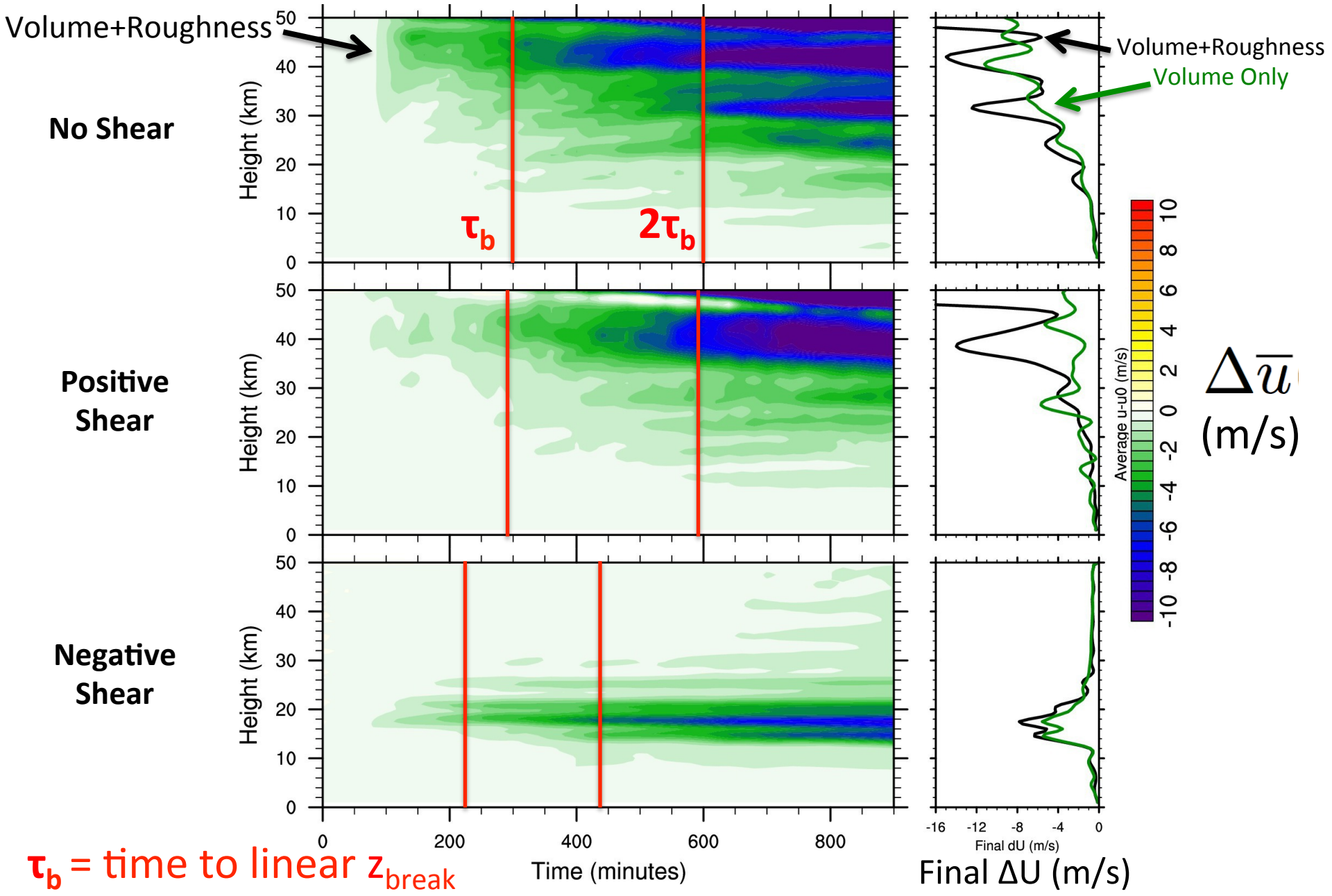


Wave-Induced Wind Speed Reduction



τ_b = time to linear z_{break}

Influences of Vertical Shear

- Attenuation/Momentum Deposition periodic in z

- Negative Shear - attenuation confined to region of negative shear
- No Shear - attenuation periodic in z
- Positive Shear - more spaced out regions of overturning aloft

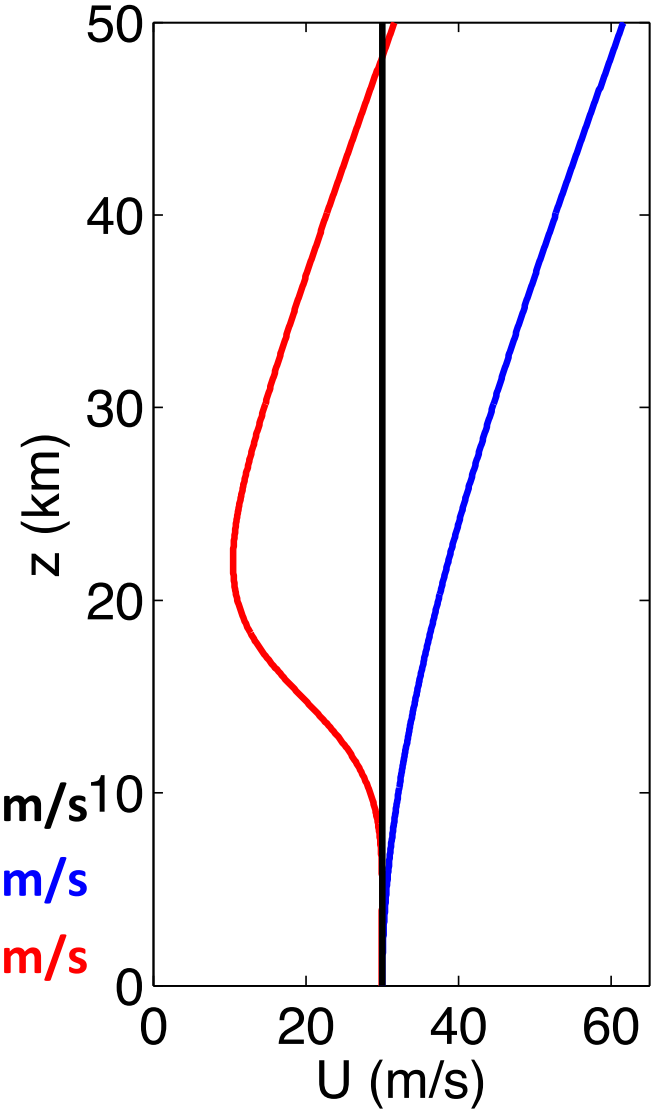
Scale Interaction

- Roughness Mode (λ_x=28.5km)
  - Carries more momentum flux than the Volume Mode (λ_x=200km)
  - Propagates up ≈7x faster than Volume Mode
    - Smaller scales more important initially aloft in MW events
  - Attenuation initially smoothly distributed in vertical
- Eventually, Volume Mode controls attenuation for both modes
  - Larger scale u' from Volume Mode influences background for smaller scale Roughness Mode
  - Attenuation periodic in vertical for both modes

Idealized Wind Profiles

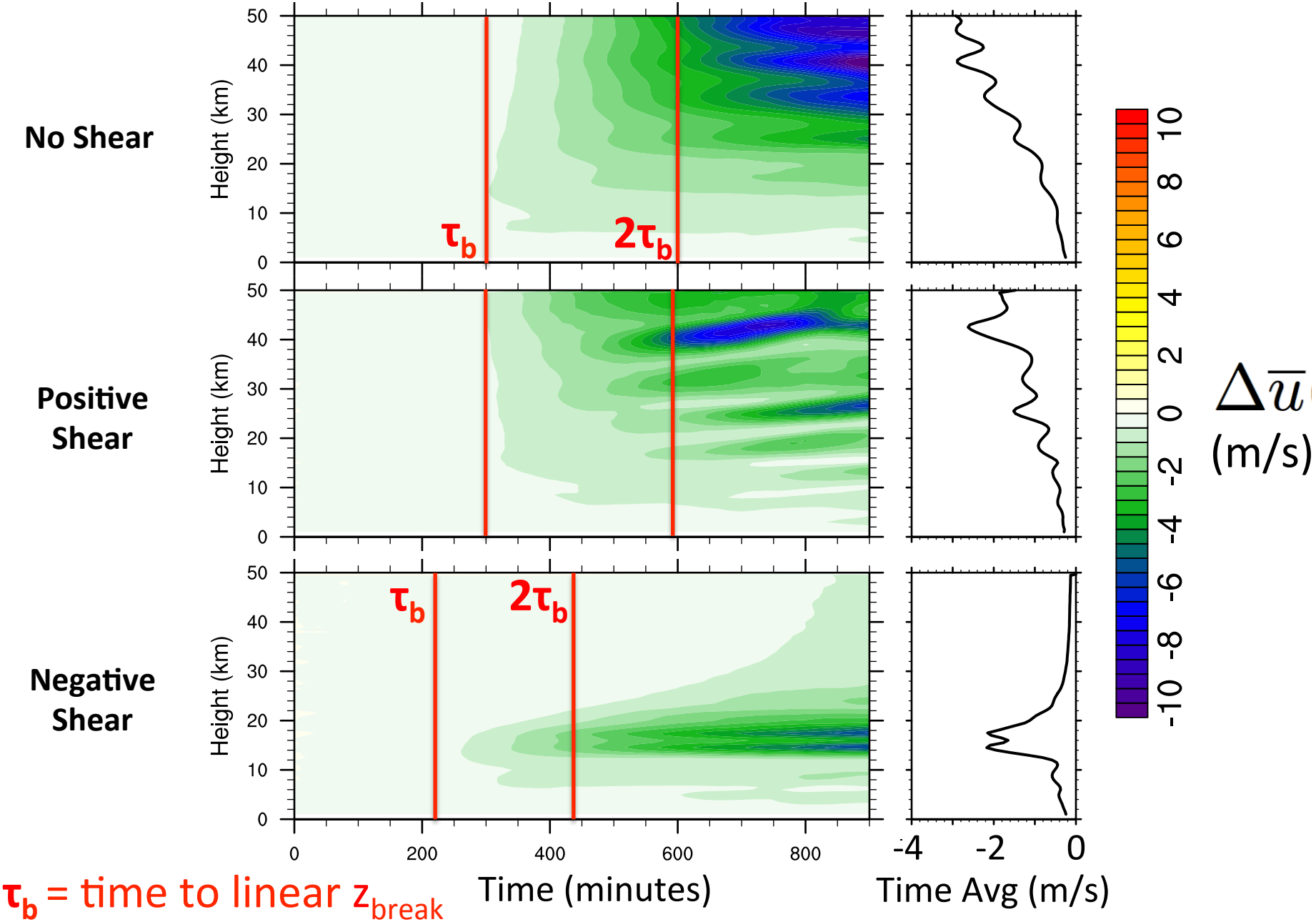
$$U(z) = U_{z_0}\left(z + Z_1 e^{-\left(\frac{z}{Z_1}\right)} - Z_1\right) - \frac{\Delta U}{2}\left(1 + \tanh\left(\frac{\pi}{2}\frac{(z - z_i)}{Z_2}\right)\right) + U_0$$

- Term 1 – Positive Shear Term
  - **U_{zo}** = 1 m s^{-1} km^{-1} – Upper Shear
- Term 2 – Negative Shear Term
  - **ΔU** – total wind speed reduction
- Term 3 – Surface Wind: **U_0**



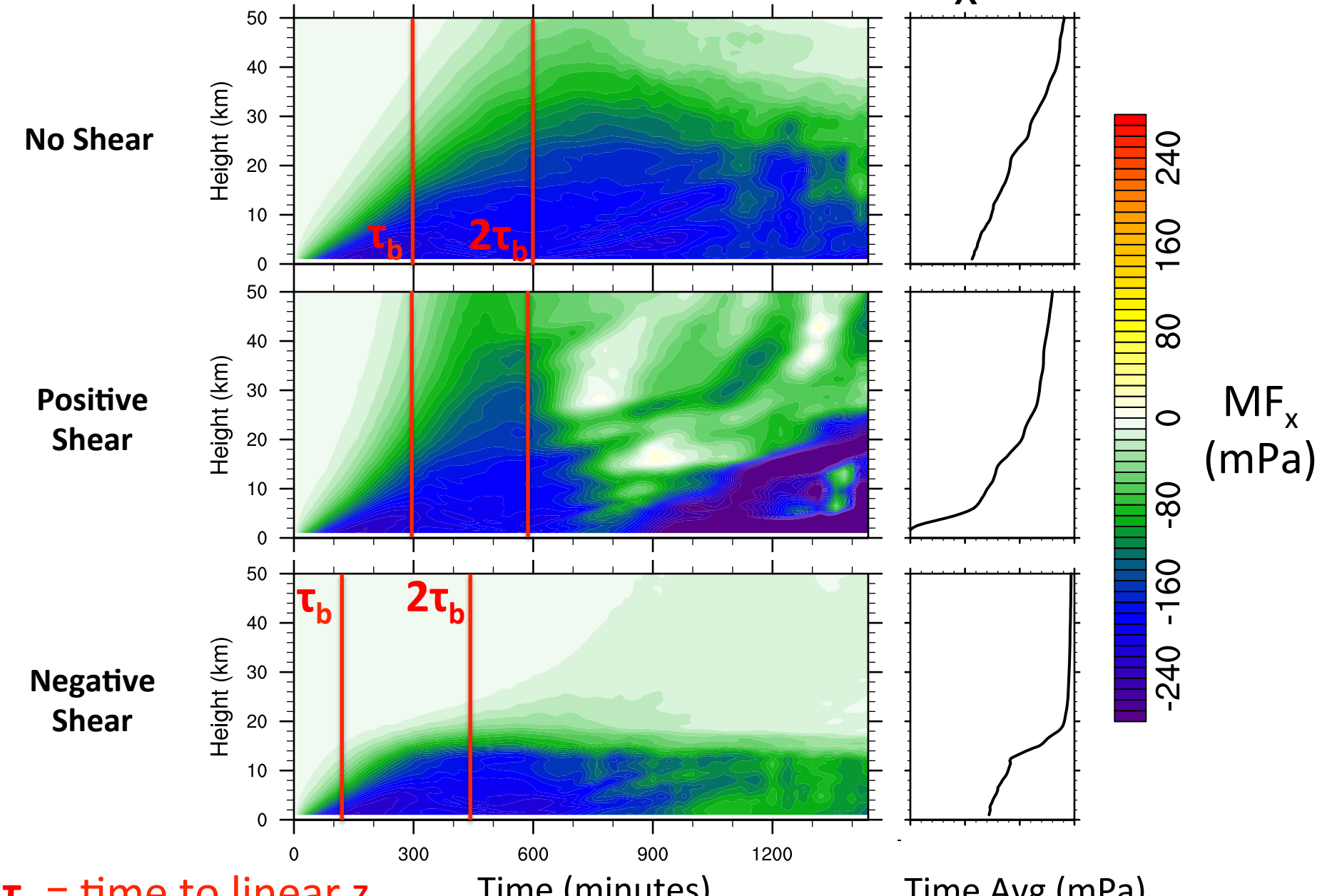
No Shear:	**U_{zo} = 0,**	**ΔU = 0 m/s**	**U_0 = 30 m/s**
Positive Shear:	**U_{zo} = 1 m/s/km**	**ΔU = 0**	**U_0 = 30 m/s**
Negative Shear:	**U_{zo} = 1 m/s/km**	**ΔU = 30 m/s**	**U_0 = 30 m/s**

Volume Mode ΔU



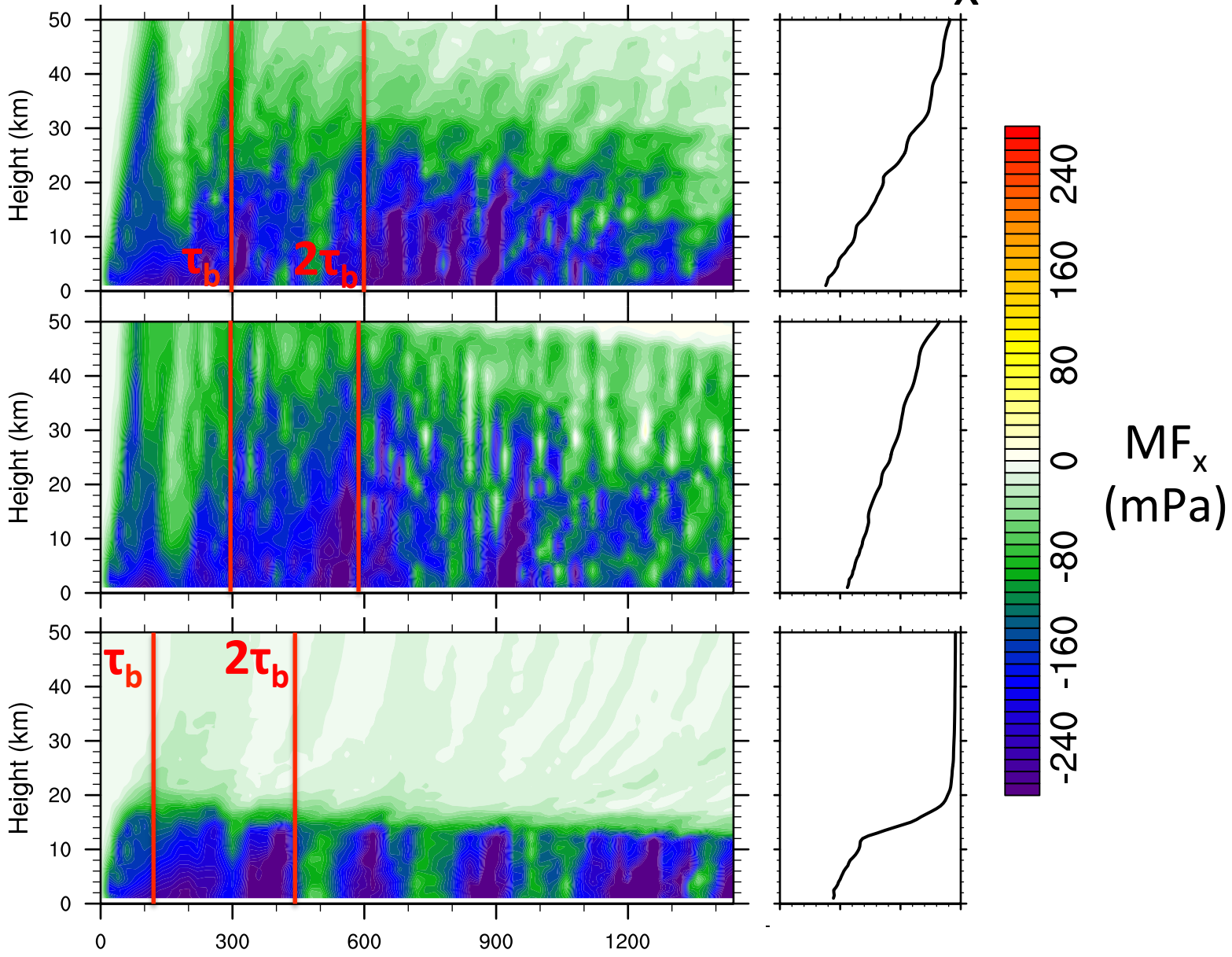
τ_b = time to linear z_{break}

Volume Mode MF_x



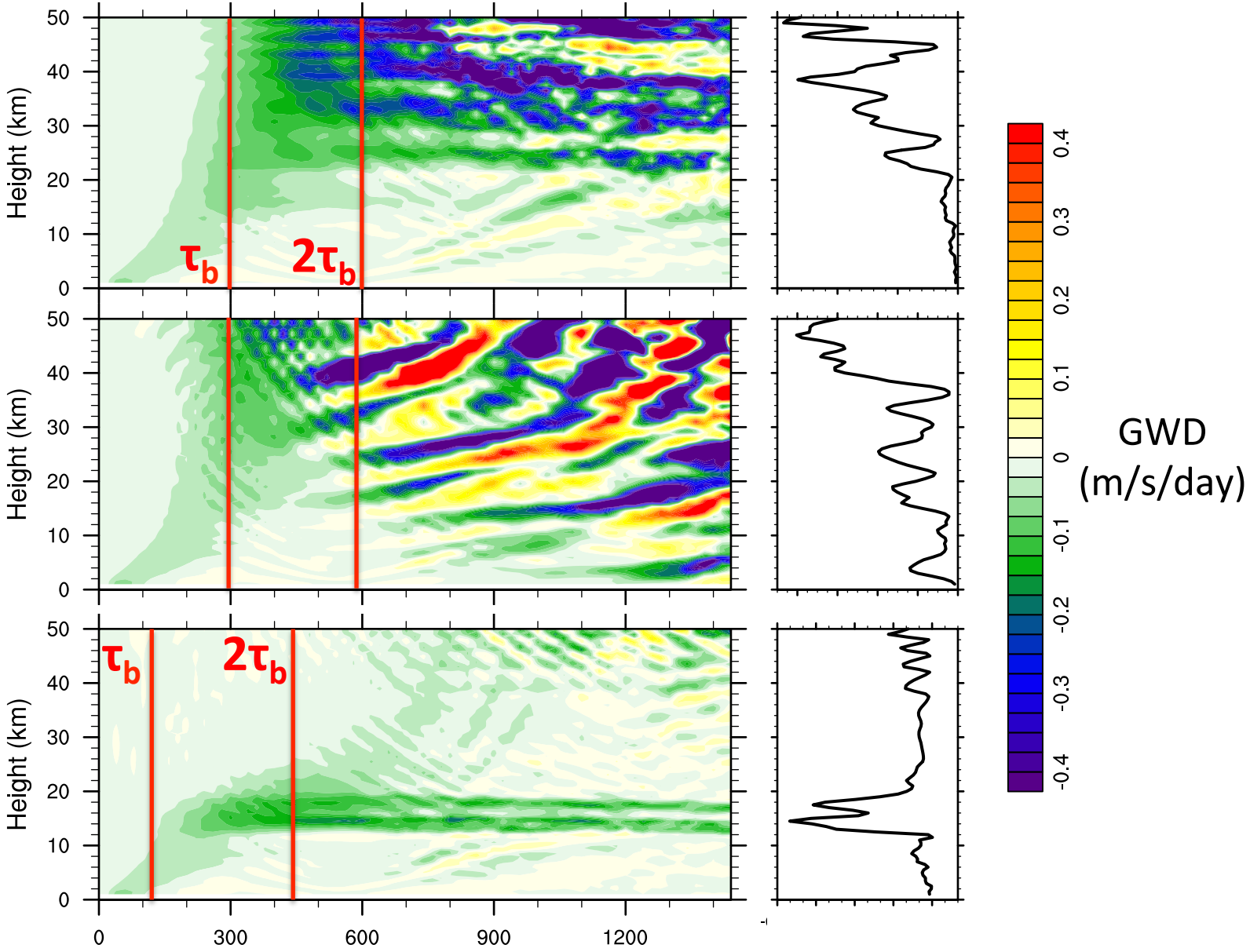
τ_b = time to linear z_{break}

Volume + Roughness MF_x



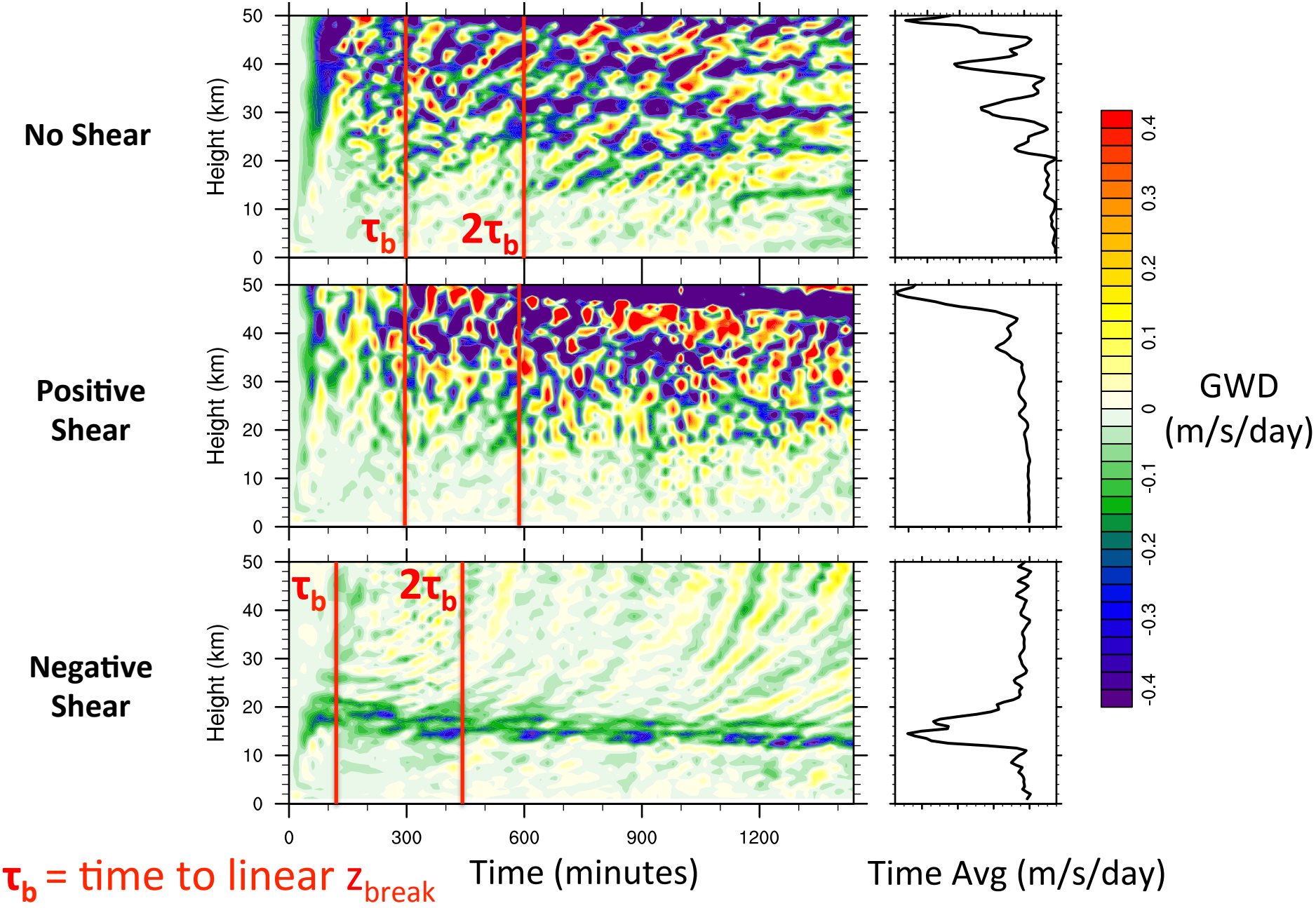
τ_b = time to linear z_{break}

Volume Mode GWD



τ_b = time to linear z_{break}

Volume + Roughness Mode GWD



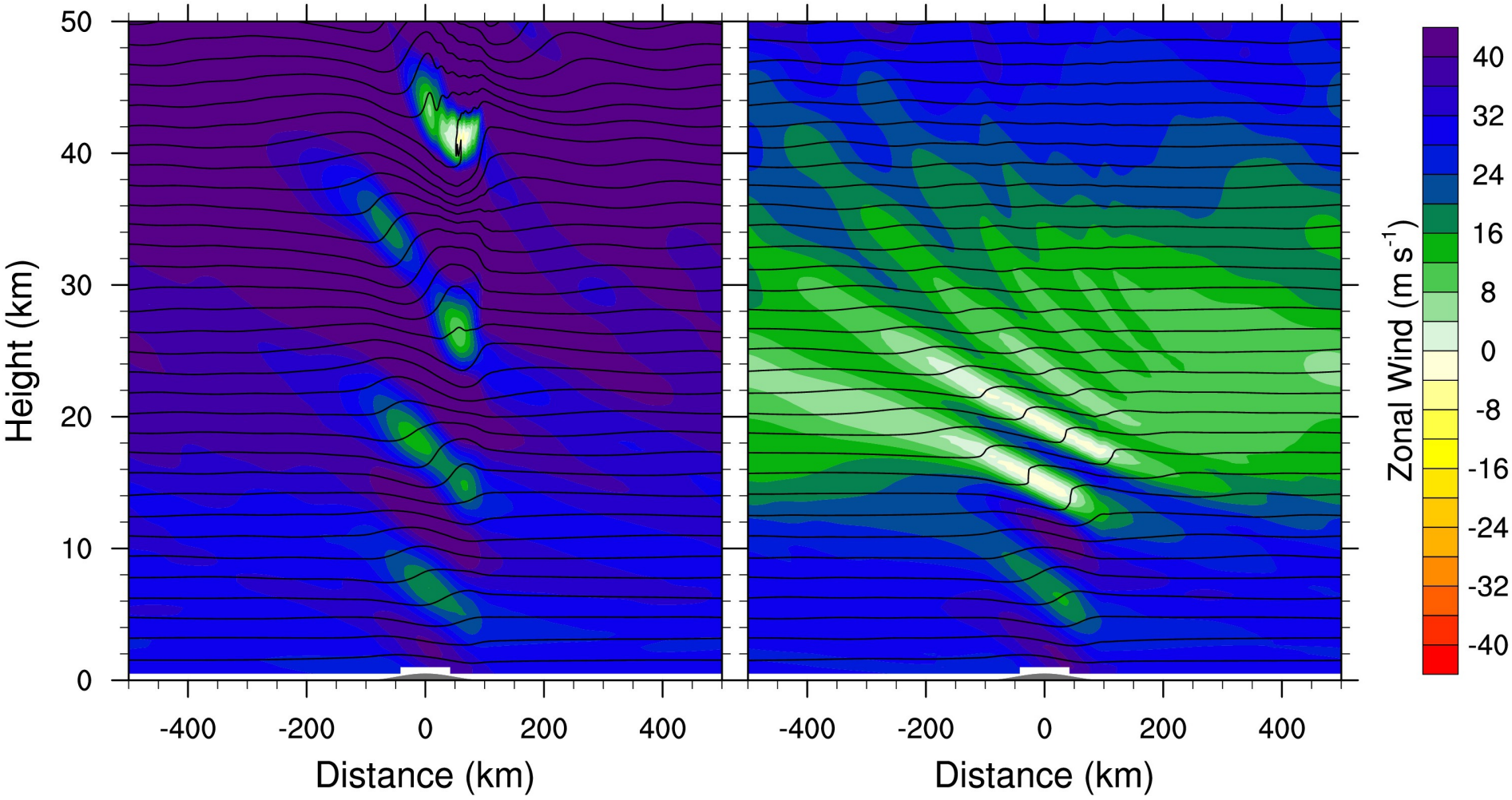
τ_b = time to linear z_{break}

Linear Theory Predicts z_b?

- Waves overturn where u' most negative
- Most negative u' at x = 0, φ = ¾2π + 2πn

WKB phase at x = 0:

$$\phi(z) = \int_0^z \frac{N(z')}{U(z')}\, dz'$$



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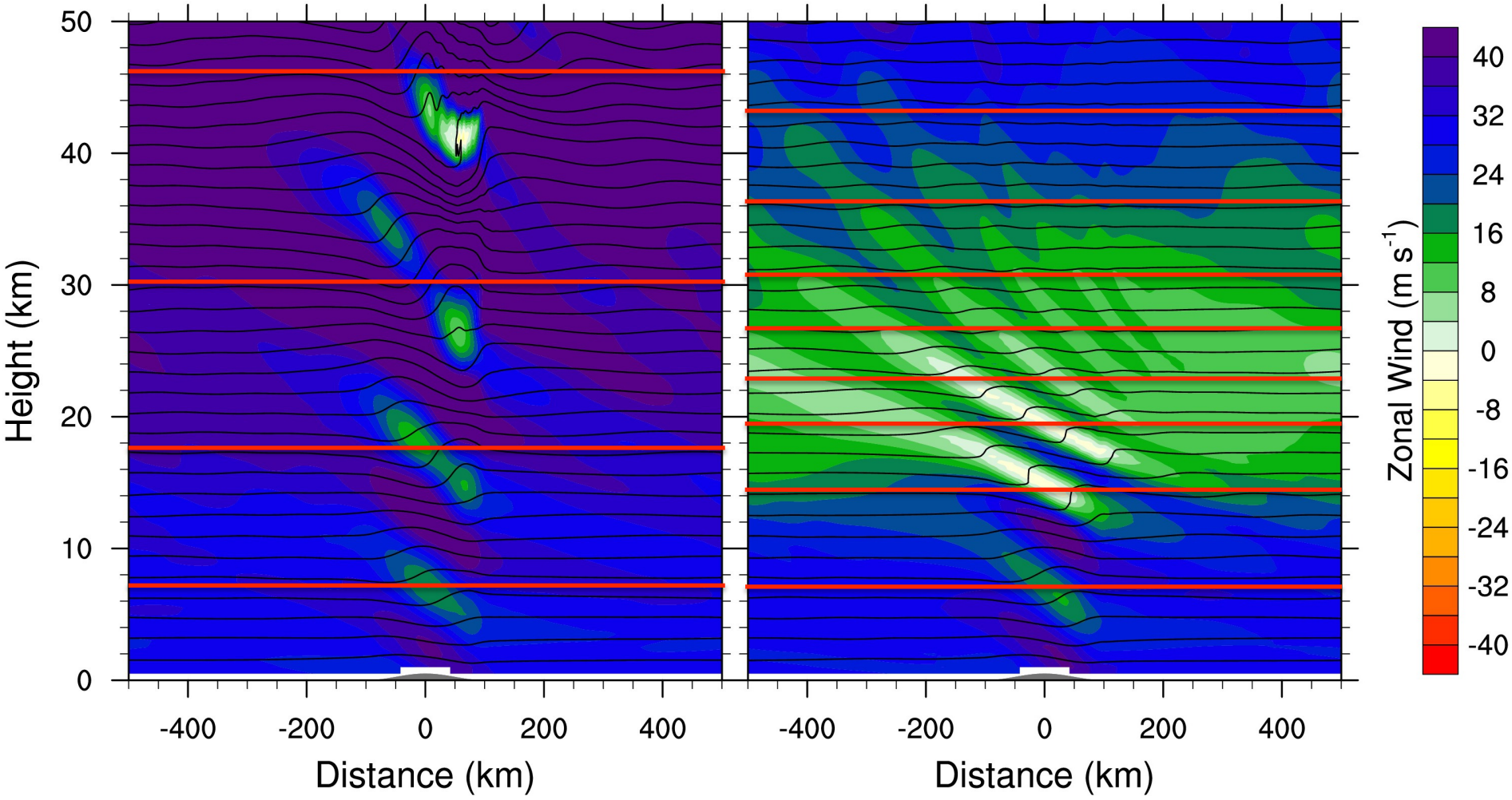
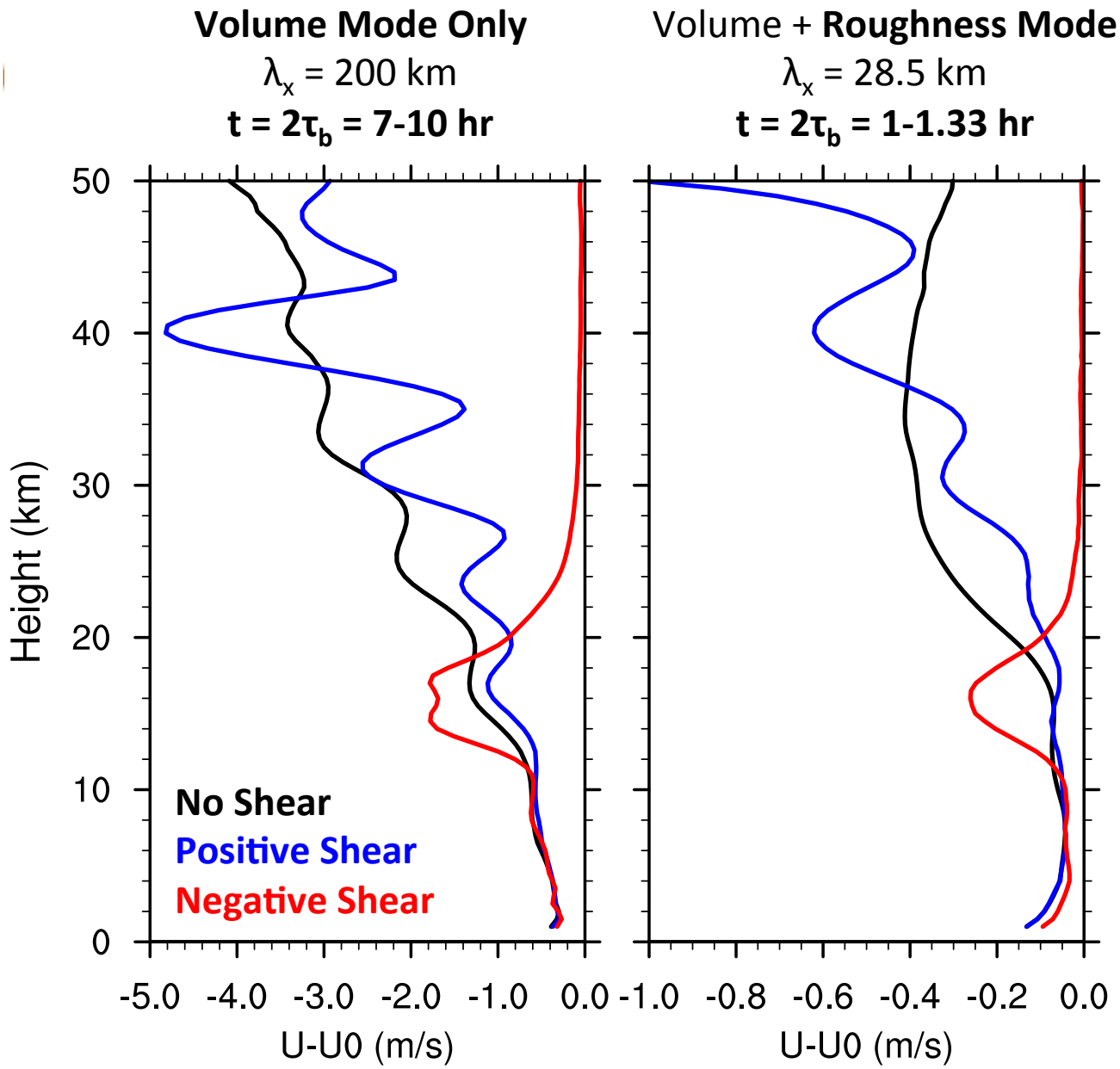


Momentum Deposition by Mode

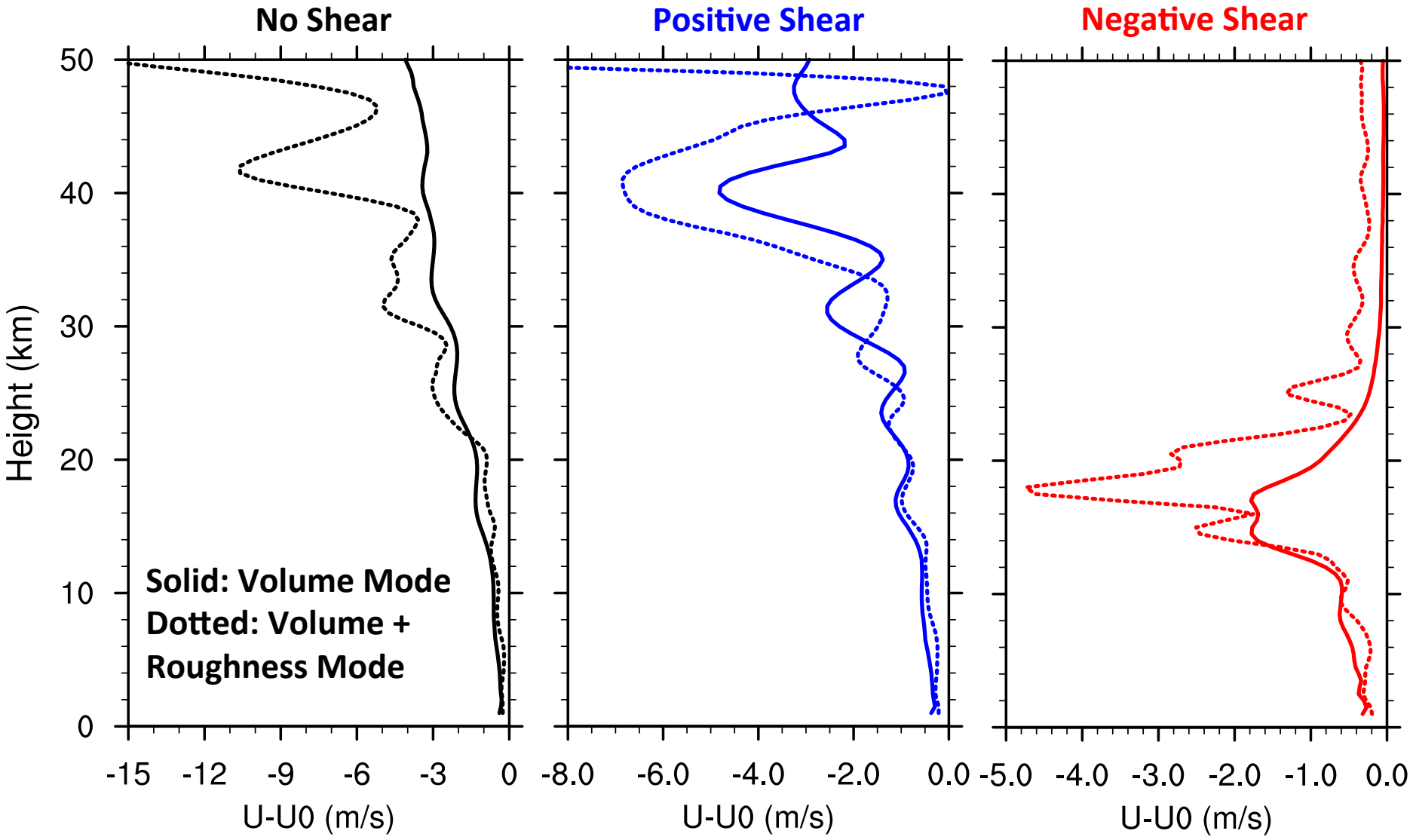
$\Delta\overline{u}$



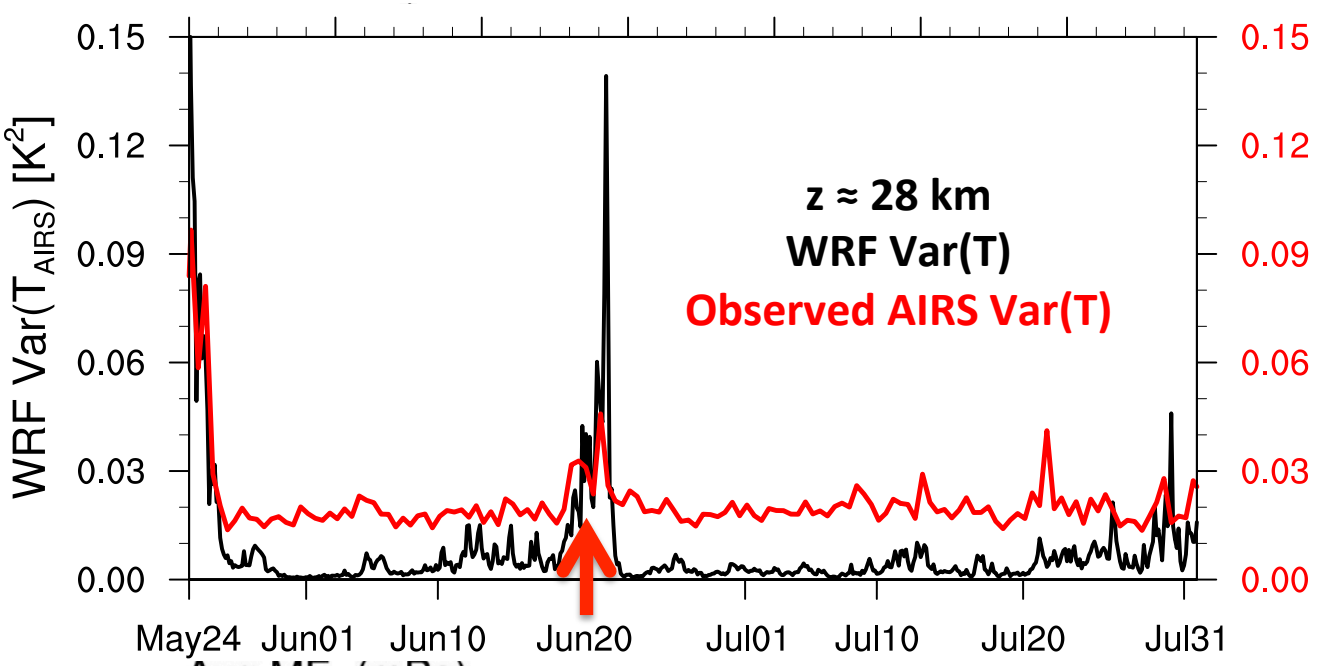
Momentum Deposition by Mode

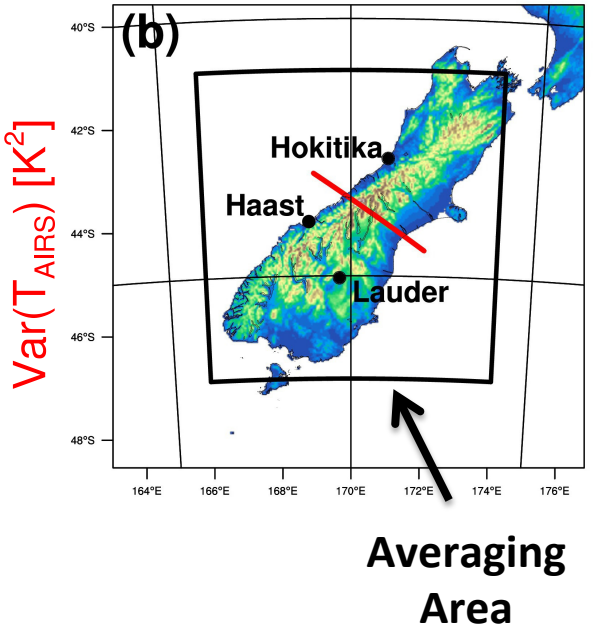
@ Volume Mode Overturning Time:
$t = 2\tau_b$ = 7-10 hr

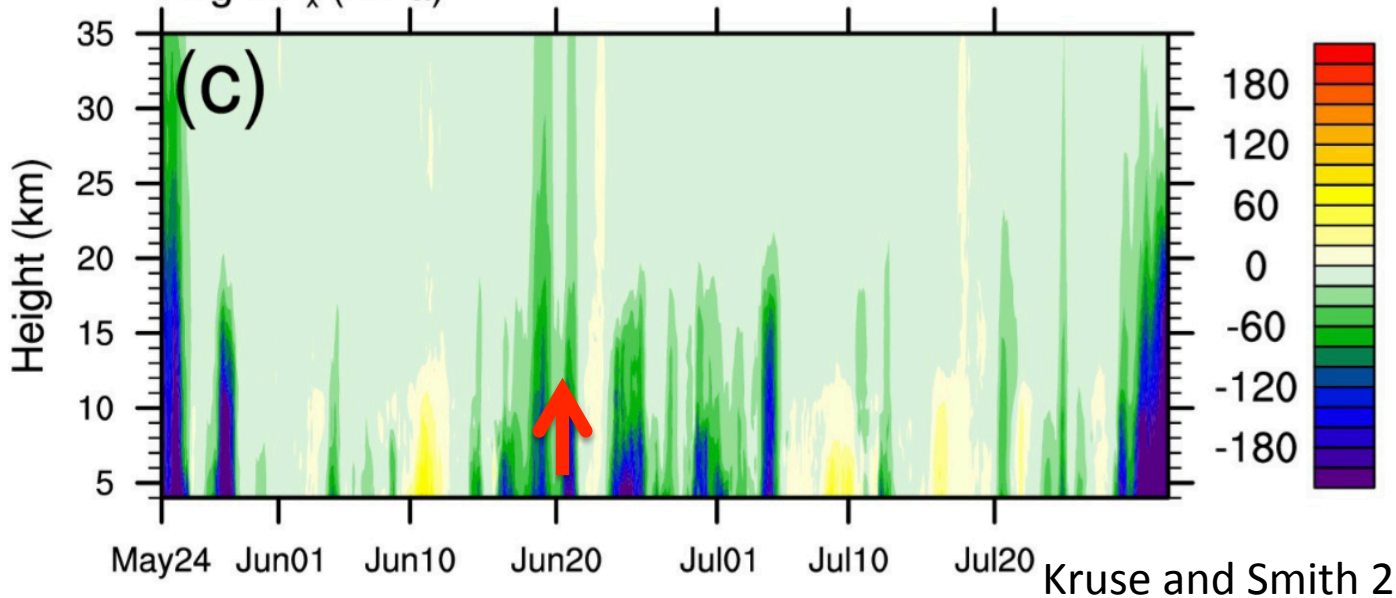
$\Delta\overline{u}$



Background/Motivation





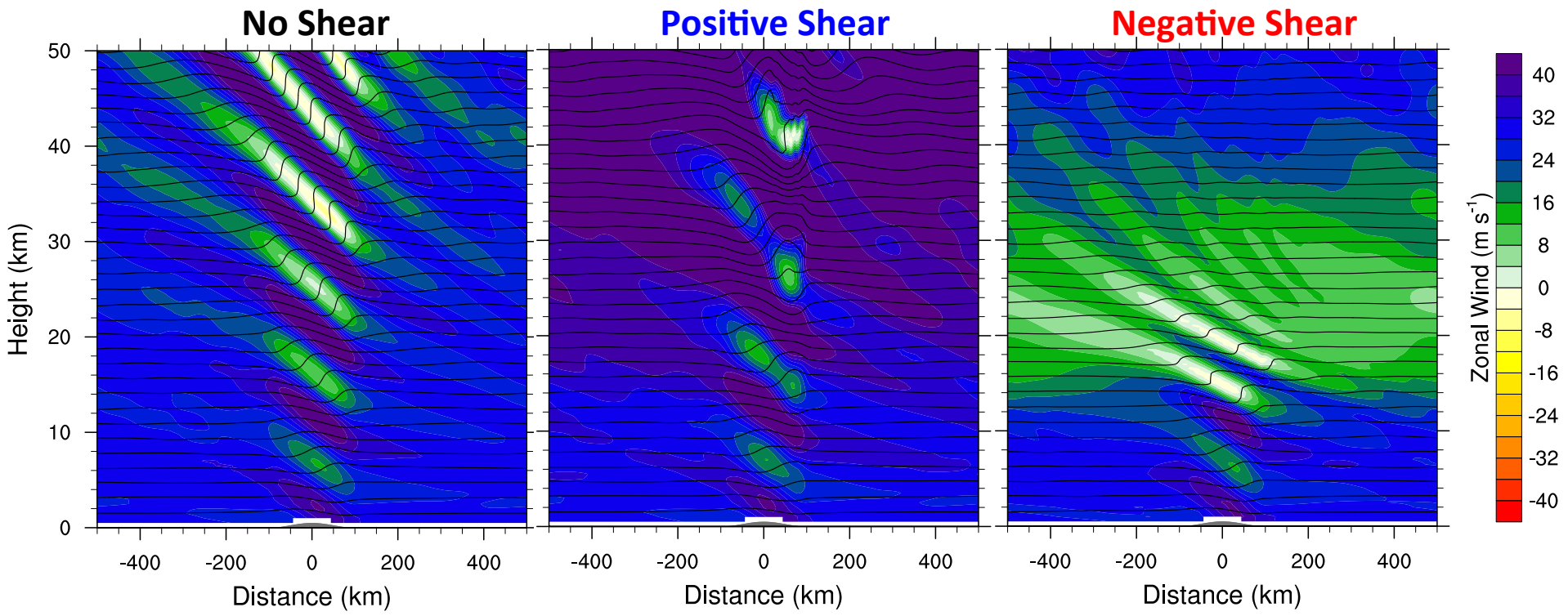


Deep propagation observed as well!

Kruse and Smith 2016

Mountain Wave Overturning

- **$t = 2\tau_b + 1$ hr**
- $2\tau_b \approx$ 7-10 hr, 1 hr ≈ 12 buoyancy periods, t ≈ 8-11 hr



No Shear: **Many overturning levels**

Positive Shear: **Fewer, spaced overturning levels**

Negative Shear: **Overturning confined to negative shear layer**

Color Shading: Zonal Wind
Contours: Isentropes

Linear Theory Predicts z_b?

- Waves overturn where u' most negative
- Most negative u' at x = 0, φ = ¾2π + 2πn

WKB Phase @ x=0: $\phi(z) = \int_0^z \frac{N(z')}{U(z')}\, dz'$

Number of Vertical Wavelengths:

$$\hat{z} = \frac{\phi(z)}{2\pi} = \frac{1}{2\pi} \int_0^z \frac{N(z')}{U(z')}\, dz'$$

Should break @ ž = ¾ + n