Uncertainty Analysis of Ensemble Streamflow Prediction

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Uncertainty Analysis frequently uses ensemble prediction to obtain the probability distribution of the output.

Streamflow uncertainty comes from errors of input (precipitation), model parameters, and streamflow measurements.

Model calibration and data assimilation can reduce the uncertainty of streamflow prediction.

Particle filtering is a technique capable of incorporating ensemble forecast, model calibration, and data assimilation jointly to achieve an optimal probabilistic prediction.
Application for the VIC Model

- 2 small basins in California
  - Coffee
    - 7 cells at 1/8 degree
    - 5 years of data (1960-1964)
    - Area: 149 mi²
  - Lights
    - 11 cells at 1/16 degree
    - 5 years of data (1958-1962)
    - Area: 57.6 mi²
- 2 stages:
  - Calibration of UH
    - Approximation by Gamma distribution (Bunya 2003, Johnsen 2005, Ramirez 2000)
  - Calibration of soil parameters
    - Zou 2004, Liang 1994
Stream Flow Tracking

220 particles
Parameter Calibration

- **Variable infiltration curve parameter**
  - $b_{\text{inf}}$

- **Fraction of maximum baseflow velocity when non-linear baseflow begins**
  - $D_s$

- **Maximum baseflow velocity**
  - $D_m$ (mm/day)

- **Fraction of maximum baseflow velocity when non-linear baseflow begins**
  - $W_s$

- **Thickness of the 3rd soil layer**
  - $D_3$ (m)

Graphs showing trends over iterations for each parameter.
Particle Filtering can be applied to the nonlinear dynamic process with non-Gaussian uncertainty distribution.

**System state dynamics**

\[ x_k = f_{k-1}(x_{k-1}, w_{k-1}) \]

**Observation dynamics**

\[ z_k = h_k(x_k, v_k) \]

Particle Filtering is used to examine the NWS River Prediction System (Constructed by the Sacramento Soil Moisture Accounting model and the Unit Hydrograph):

1) **Under what conditions can the system be approximated as a step-wise linear system?** (Uncertainty propagation in a linear system can be studied systematically.)

\[ x_k = f_{k-1}(x_{k-1}, w_{k-1}) \Rightarrow x_k = F_{k-1}x_{k-1} + B_{k-1}u_k + w_k \]

2) **Display the system state variable’s posterior distributions** \( p(x_k | z_{1:k}) \) (as well as model parameter uncertainties). Are they Gaussian? (A non-Gaussian system will be limited from many effective uncertainty analysis and reduction approaches.)
Sacramento Soil Moisture Accounting Model

6 state variables (reservoir water contents) with 13 model parameters

[Burnash et al 1973]
State Variable Uncertainty Bound Tracking
Hydrograph Prediction with Uncertainty
System Step-Wise Linearity

\[ x_k = f_{k-1}(x_{k-1}, w_{k-1}) \Rightarrow x_k = F_{k-1}x_{k-1} + B_{k-1}u_k + w_k \]

Examine:
1) Regression Significance Test
   Linear Regression Coefficients in \( F_{k-1} \) should be greater than their 95% confidence intervals
2) Correlation Coefficient
   \[
   R^2 = 1 - \frac{\sum_{i=1}^{n} (x_{k,i}^{\text{mod}} - x_{k,i}^{\text{reg}})^2}{\sum_{i=1}^{n} (x_{k,i}^{\text{mod}} - \bar{x}_k^{\text{mod}})^2}
   \]

Leaf River Basin, Mississippi (3/12-4/30, 1978)
System Gaussianalinity

Are the posteriors of state variables \( p(x_k \mid z_{1:k}) \) Gaussian?

Typical distributions calculated from particles
Conclusions

1) The results of particle filtering show that the NWS river prediction system can be approximated as a step-wise linear dynamic system when the state variables vary relatively slow.

2) The posteriors of state variables of the NWS river prediction system in many cases are Non-Gaussian in particular, when a reservoir is drying out.

3) These non-linear and non-Gaussian processes could affect the results of many data assimilation and uncertainty analysis approaches.

4) Particle Filtering is computation expensive.

5) Particle Filtering will also confront convergence problems in particular, when the number of model parameters are large and correlated.
Particle Filtering employs the Sequential Importance Sampling to retrieves the Posterior Distribution.

\[ w(x_i) \text{ retrieves the shape} \]

\[ w(x_i) = \frac{p(x_i)}{q(x_i)} \]

\[ q(x) = U[0, 2] \]

\[ p(x) \text{ Difficult for sampling} \]

\[ w(x_1), w(x_2), w(x_3), w(x_4), w(x_5), w(x_6), w(x_7), w(x_8), w(x_9), w(x_{10}), w(x_{11}) \]