Combining short-term moments for longer time periods

ISFF computes 5-minute average statistics as a standard product. This period is thought to be the shortest that most users would use to compute second-order moments, such as eddy-correlation fluxes. Usually, averages over much longer periods, 15-30 minutes, are used to include low-frequency variations associated with, for example, convection. To save our users the trouble of figuring out how to generate these longer-term averages, we offer the following cookbook.

For two time series $X(i)$ and $Y(i)$, the mean and covariance are defined by:

$$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X(i)$$

$$\overline{x'y'} = \frac{1}{N}\sum_{i=1}^{N}[X(i)-\overline{X}][Y(i)-\overline{Y}]$$

The above expands to 4 terms, but the last 3 collapse into one.

$$\overline{x'y'} = \frac{1}{N}\sum_{i=1}^{N} X(i)Y(i) - \overline{XY}$$

The above definitions are true for any averaging period.

Suppose we subdivide a period N into m segments of length N_j (for example, a 30-minute period into 6 5-minute periods). In order to find $\overline{x'y'}^N$ in terms of values computed from periods of length N_j, we need to find $\overline{X}^N$ and $\sum_{i=1}^{N} X(i)Y(i)$. The first is easy:

$$\begin{aligned}\overline{X}^N &= \frac{1}{N}\sum_{j=1}^{m}\sum_{i=1}^{N_j} X_j(i) \\ &= \frac{1}{N}\sum_{j=1}^{m} N_j\overline{X}^j\end{aligned}$$

Similary:

$$\begin{aligned}\sum_{i=1}^{N} X(i)Y(i) &= \sum_{j=1}^{m}\sum_{i=1}^{N_j} X_j(i)Y_j(i) \\ &= \sum_{j=1}^{m} N_j(\overline{x'y'}^j + \overline{X}^j\overline{Y}^j)\end{aligned}$$

Thus:

$$\overline{x'y'}^N = \frac{1}{N}\sum_{j=1}^{m} N_j(\overline{x'y'}^j + \overline{X}^j\overline{Y}^j) - \frac{1}{N}\sum_{j=1}^{m} N_j\overline{X}^j\,\frac{1}{N}\sum_{j=1}^{m} N_j\overline{Y}^j$$

(ISFF saves N_j for this purpose.) If all N_j are equal this simplifies to:

$$\overline{x'y'}^N = \frac{1}{m}\sum_{j=1}^{m}(\overline{x'y'}^j + \overline{X}^j\overline{Y}^j) - \frac{1}{m}\sum_{j=1}^{m}\overline{X}^j\,\frac{1}{m}\sum_{j=1}^{m}\overline{Y}^j$$

Similar expressions may be derived for higher-order moments.