Stochastic and Deterministic Models for Tropical Convection

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Outline

- Introduction
- The deterministic multicloud model for organized convection
- Stochastic interaction system for cloud area fractions
- Deterministic limit of stochastic model and effect congestus detrainment

Why a stochastic model for convection?

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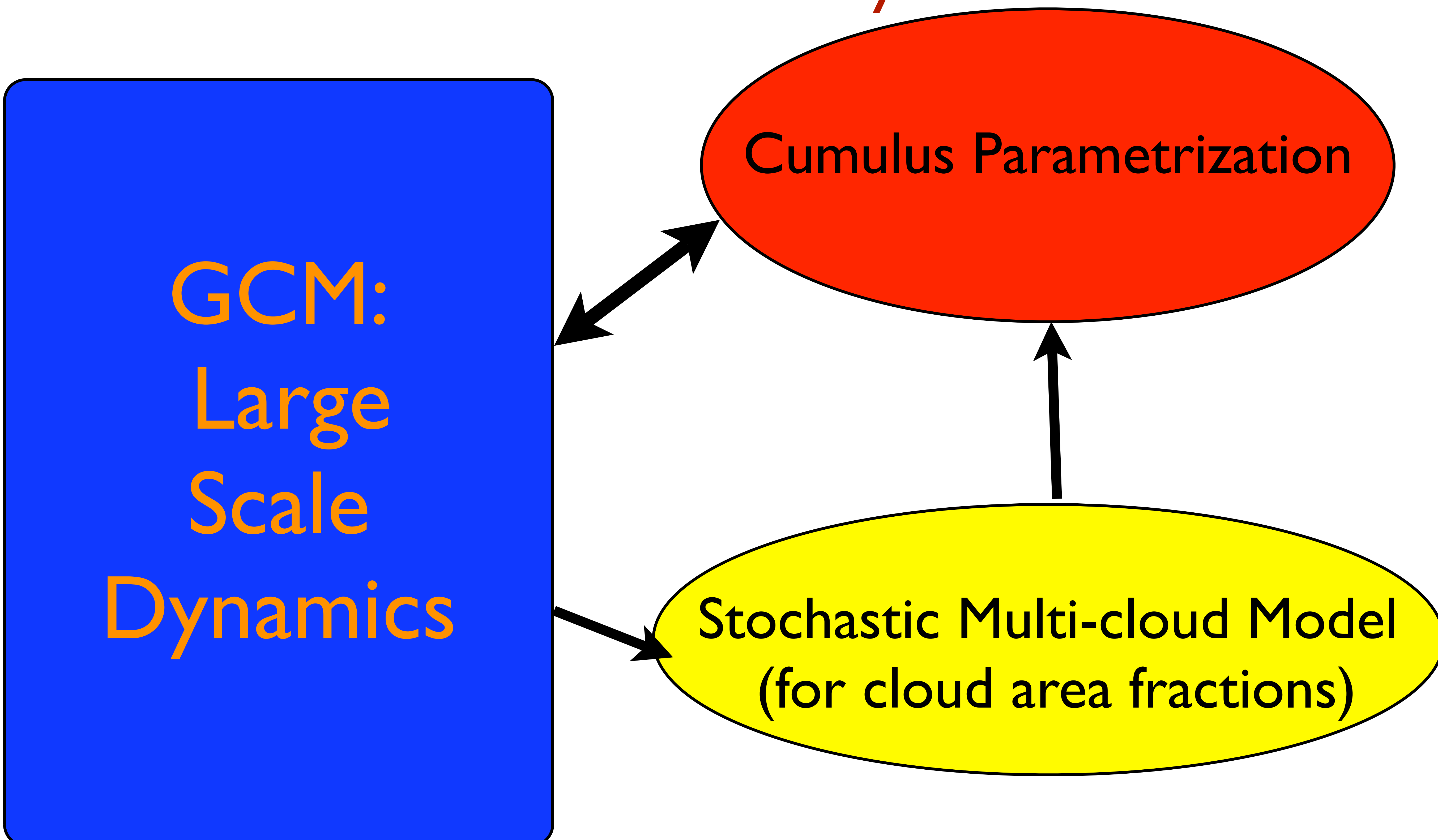
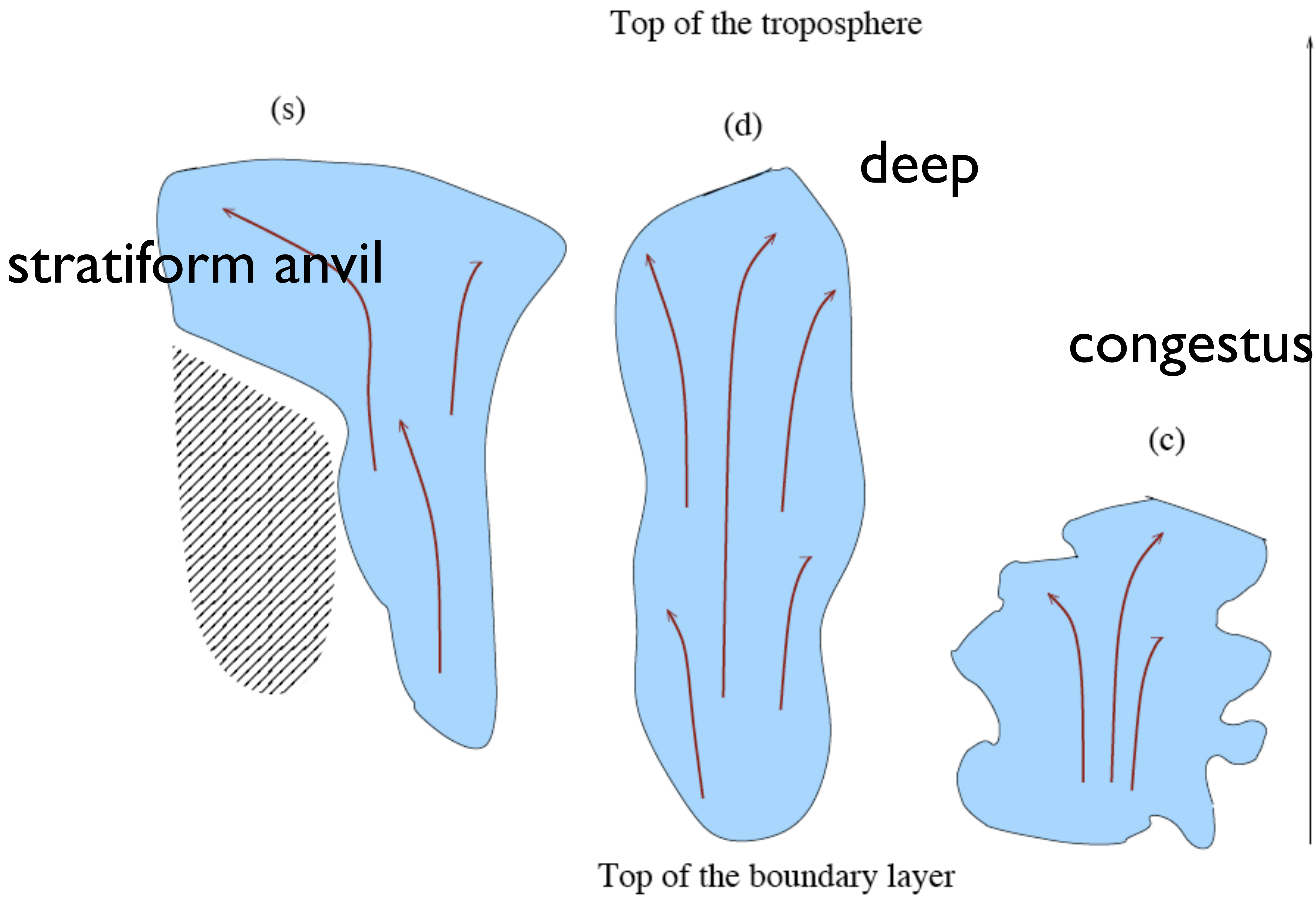
- How cloud systems interact with each other and with the environment?
- Adequate representation of sub-grid dynamics based on Statistical-self similarity across-scales of tropical convective systems
- Capture deviations from quasi-equilibrium paradigm
- Improve tropical variability in climate models ---> reduce model error
- We propose a stochastic model for cloud area-fractions ...

Stochastic Multi-cloud Model to inform cumulus parametrization: represent the missing sub-grid scale variability



Main cloud types of tropical weather



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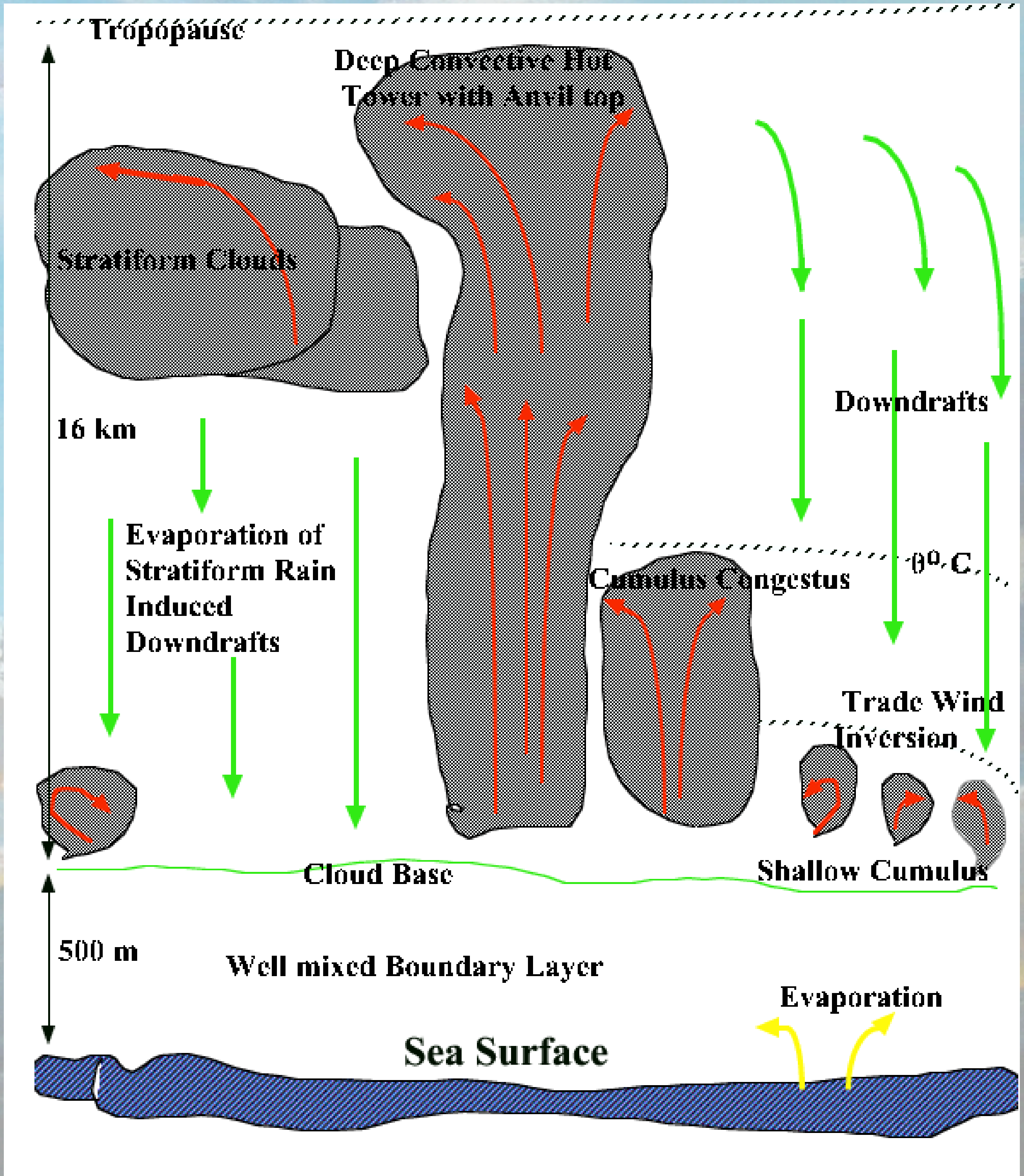
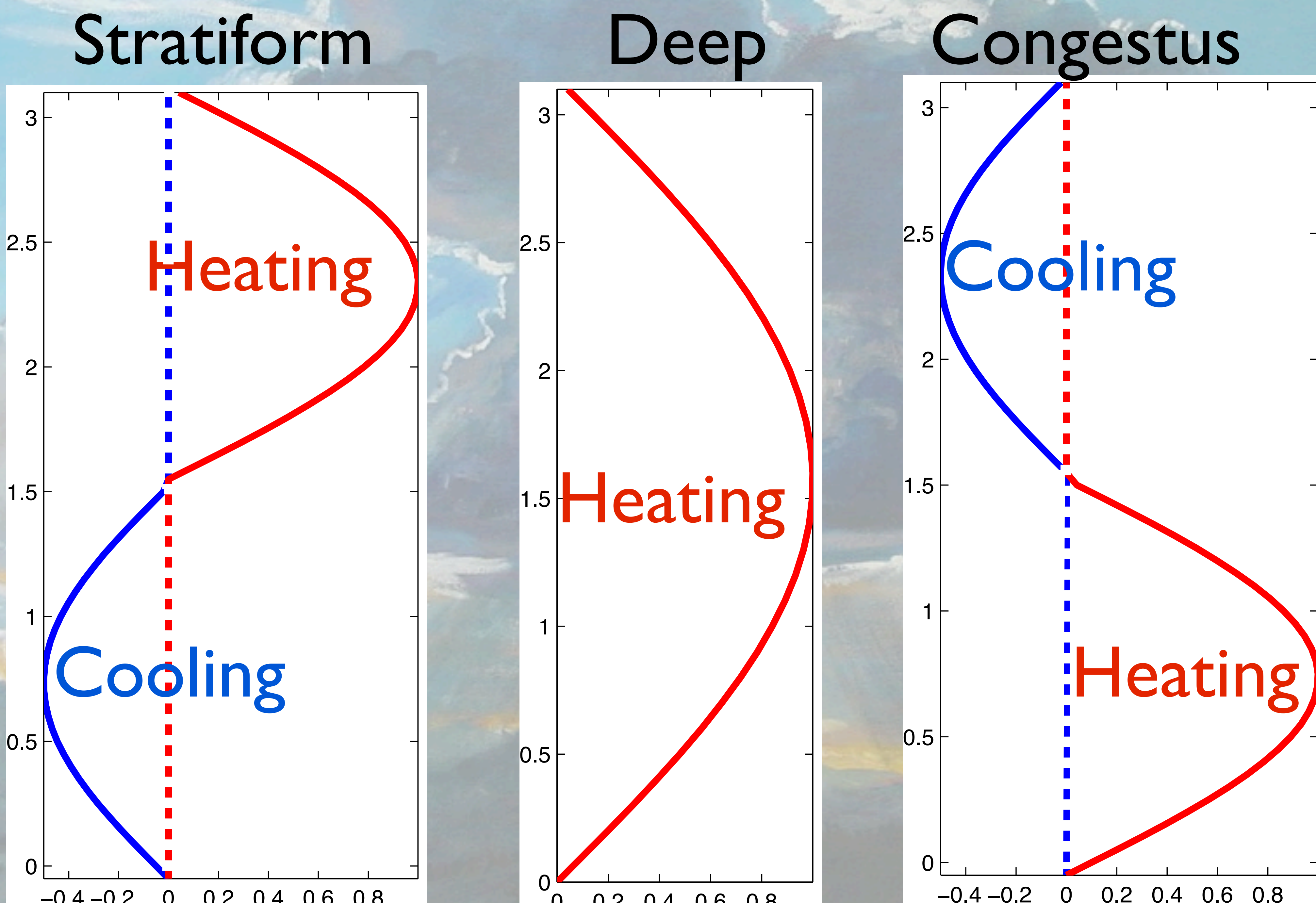
Deterministic Multicloud Model

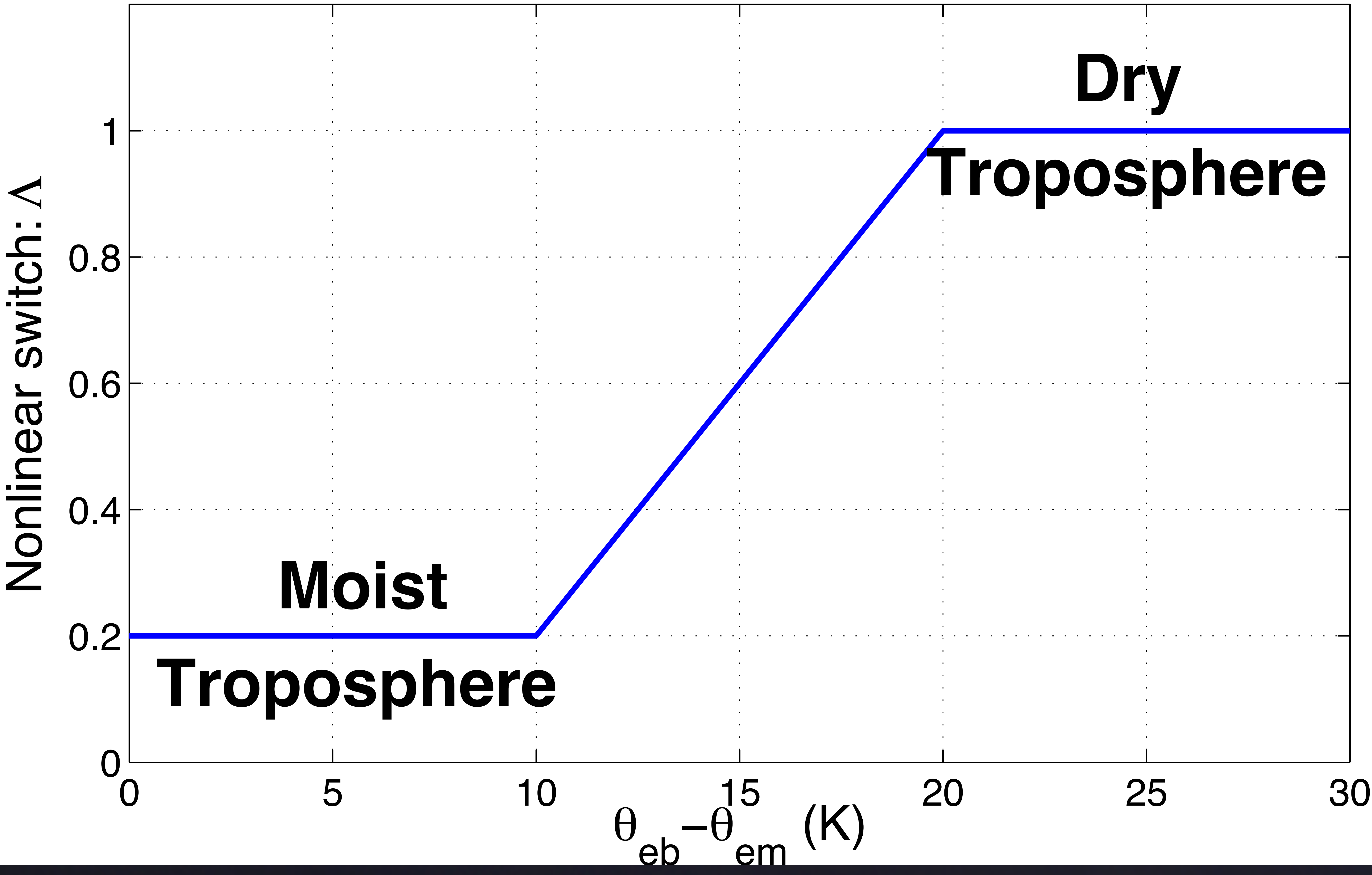


K. and Majda (JAS, 2006, 2008, etc.)

Imposed heating profiles



Moisture Switch to make transition from one type of convection to another



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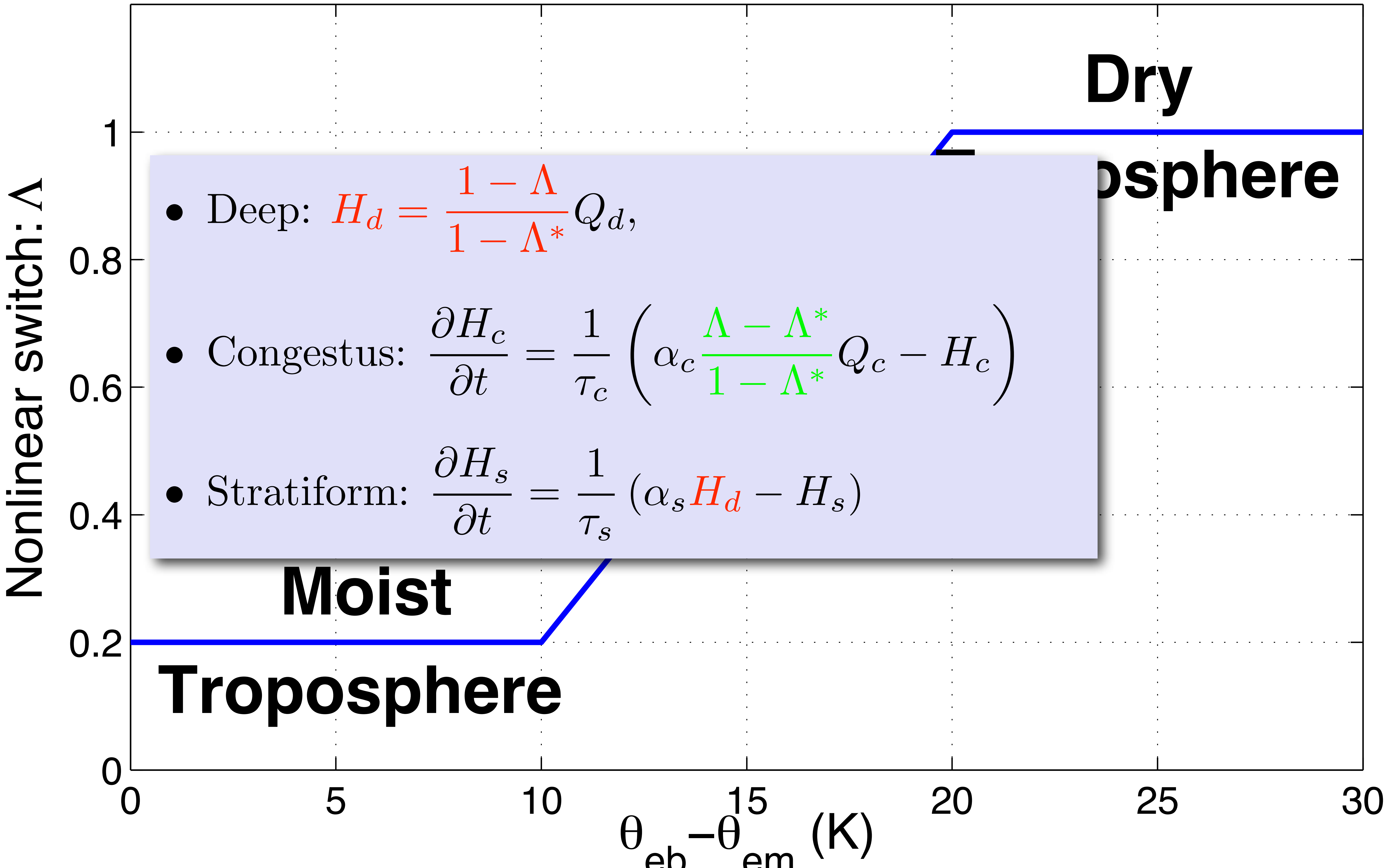


- Deep: $H_d = \frac{1-\Lambda}{1-\Lambda^*} Q_d,$

- Congestus: $\frac{\partial H_c}{\partial t} = \frac{1}{\tau_c}\left(\alpha_c \frac{\Lambda-\Lambda^*}{1-\Lambda^*} Q_c - H_c\right)$

- Stratiform: $\frac{\partial H_s}{\partial t} = \frac{1}{\tau_s}\left(\alpha_s H_d - H_s\right)$

- Tropospheric dynamics

Fst Mode

$$\begin{cases} \frac{\bar{d}\mathbf{v}_1}{dt} + \beta y \mathbf{v}_1^{\perp} - \nabla\theta_1 = -C_d(u_0)\mathbf{v}_1 - \frac{1}{\tau_R}\mathbf{v}_1 \\ \frac{\bar{d}\theta_1}{dt} - \operatorname{div}\mathbf{v}_1 = H_d + \xi_s H_s + \xi_c H_c + S_1 \end{cases}$$

Snd Mode

$$\begin{cases} \frac{\bar{d}\mathbf{v}_2}{dt} + \beta y \mathbf{v}_2^{\perp} - \nabla\theta_2 = -C_d(u_0)\mathbf{v}_2 - \frac{1}{\tau_R}\mathbf{v}_2 \\ \frac{\bar{d}\theta_2}{dt} - \frac{1}{4}\operatorname{div}\mathbf{v}_2 = (-H_s + H_c) + S_2 \end{cases}$$

- Moisture Eqn: $P = \frac{2\sqrt{2}}{\pi}(H_d + \xi_s H_s + \xi_c H_c)$

$$\frac{\bar{d}q}{dt} + \operatorname{div}\left[(\mathbf{v}_1 + \tilde{\alpha}\mathbf{v}_2)q + \tilde{Q}(\mathbf{v}_1 + \tilde{\lambda}\mathbf{v}_2)\right] = -P + \frac{D}{H_T}$$

- Boundary layer: $\frac{\partial\theta_{eb}}{\partial t} = \frac{1}{h_b}(E - D)$

Closures

- Congestus heating prop. to low level CAPE

$$Q_c \propto \sigma_c \int_0^{z_m} (\theta_{eb} - \theta_e^*(z))dz \approx$$

$$Q_c = Q_c^0 + \frac{1}{\tau_{conv}}(\theta_{eb} - a_0'(\theta_1 + \gamma_2'\theta_2)$$

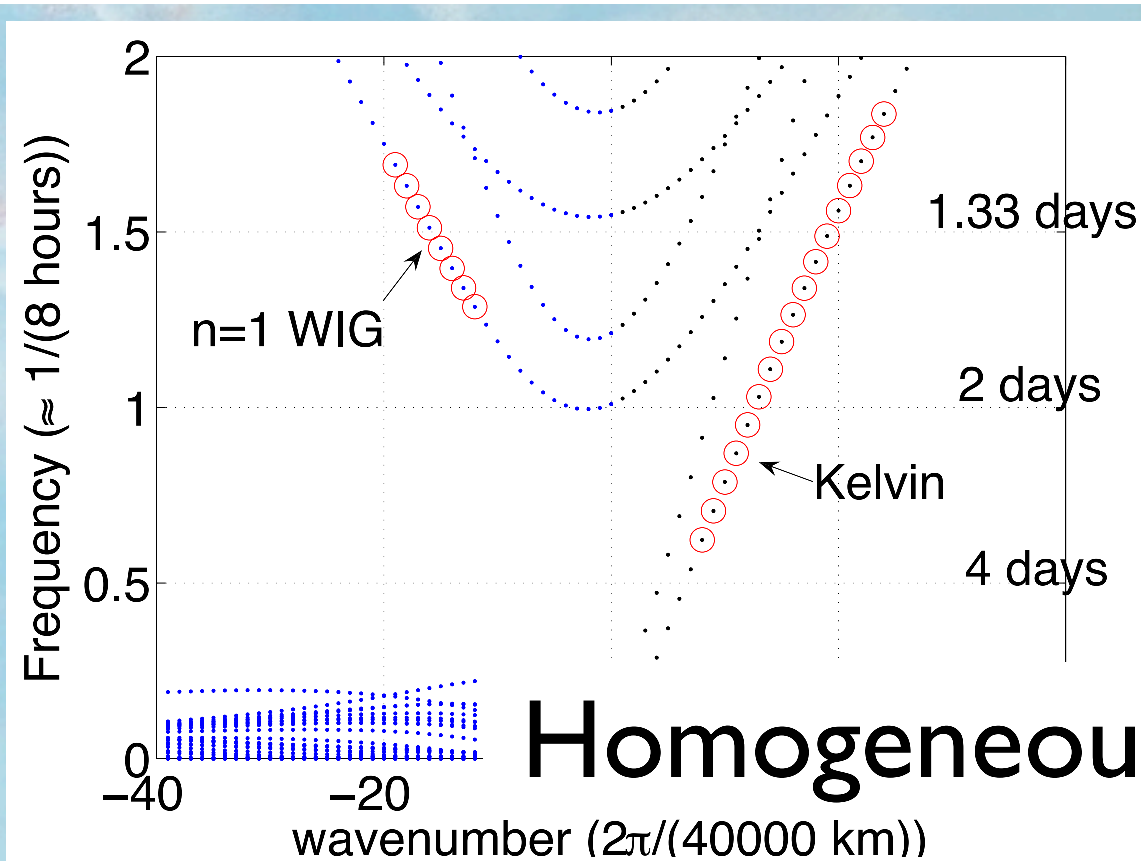
- Deep convection: CAPE & Betts-Miller

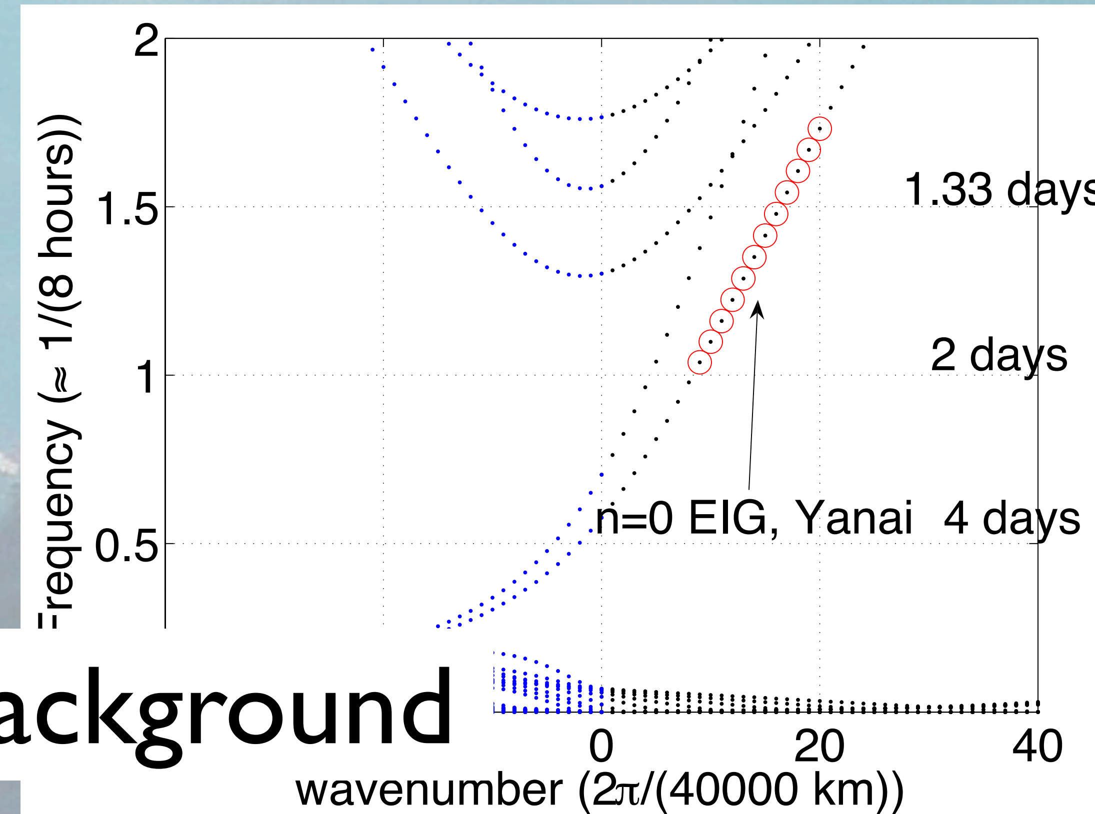
$$Q_d = Q_c^0 + \frac{1}{\tau_{conv}}(a_1\theta_{eb} + a_2 q - a_0(\theta_1 + \gamma_2\theta_2))$$

- Downdrafts

$$D = m_0(1 + \mu(H_s - H_c))^+(\theta_{eb} - \theta_{em})$$

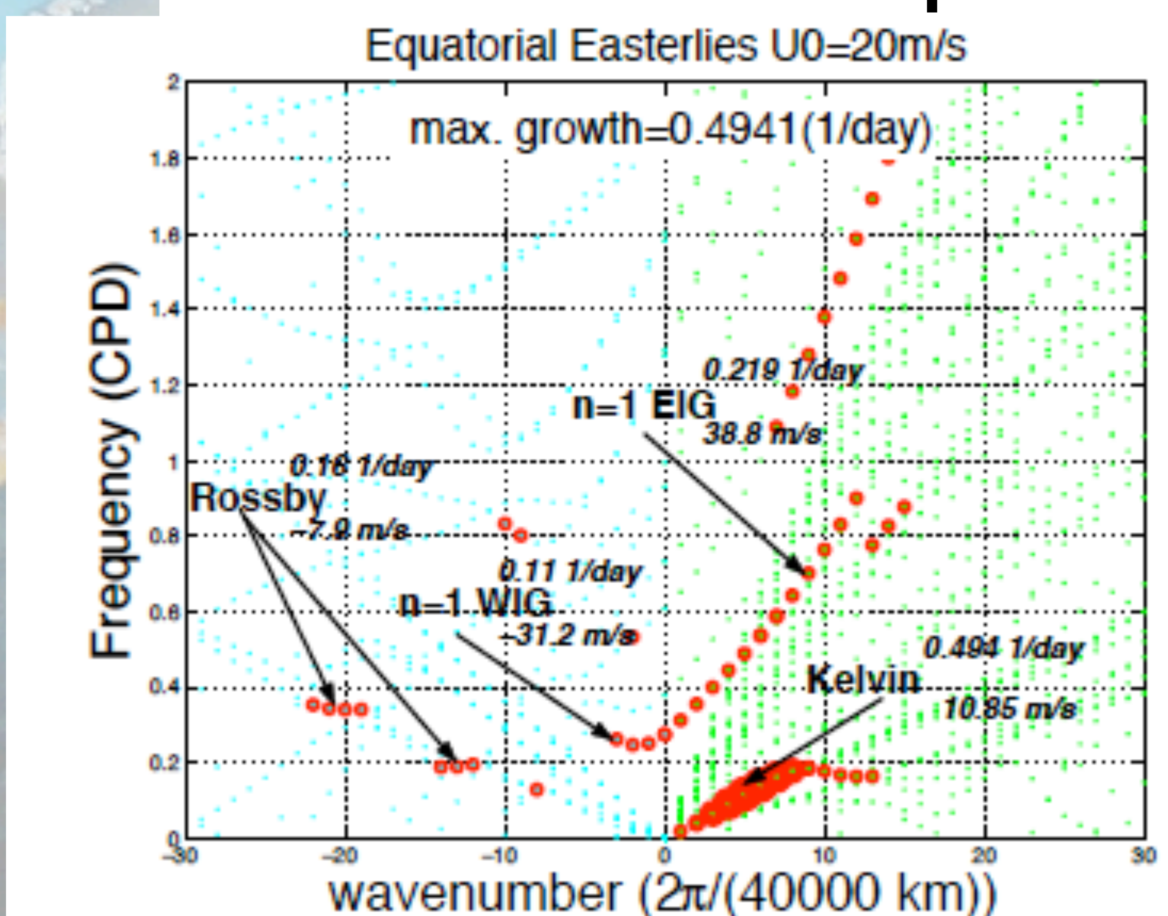
Linear Theory captures main convectively coupled waves

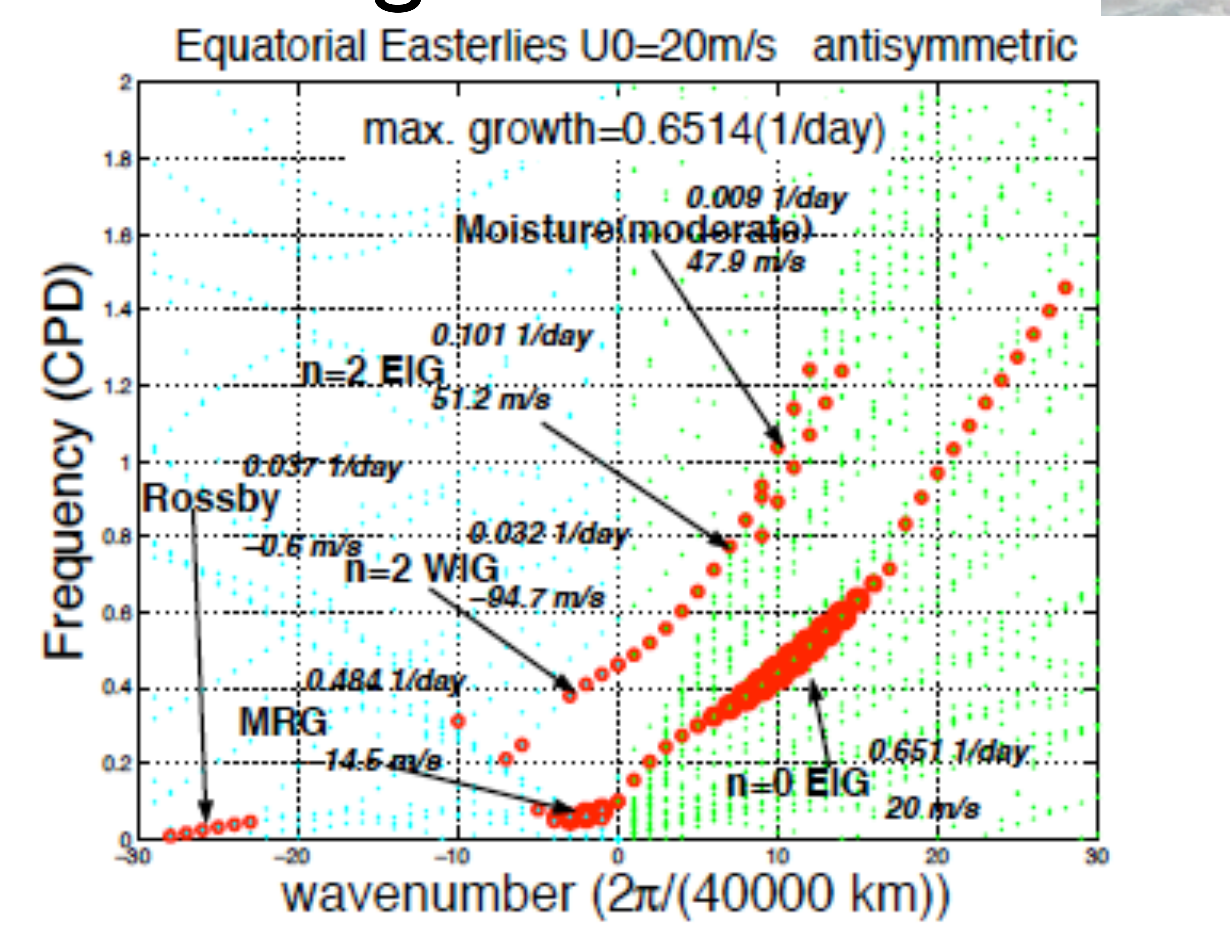




Homogeneous Background

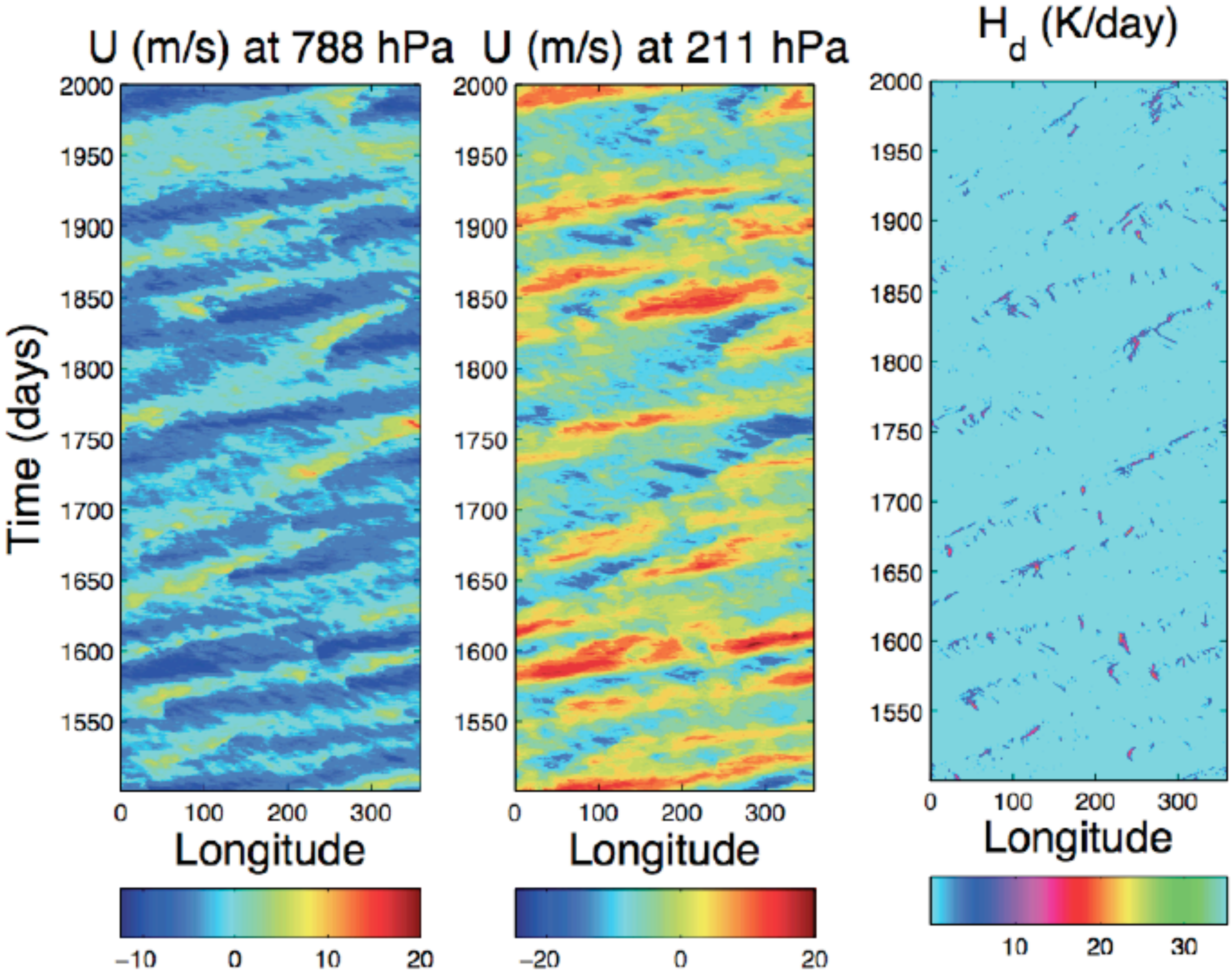
Barotropic Shear Background





Instability bands in dispersion relation curves

MJO in MC-GCM (HOMME)



Stochastic Multicloud Model

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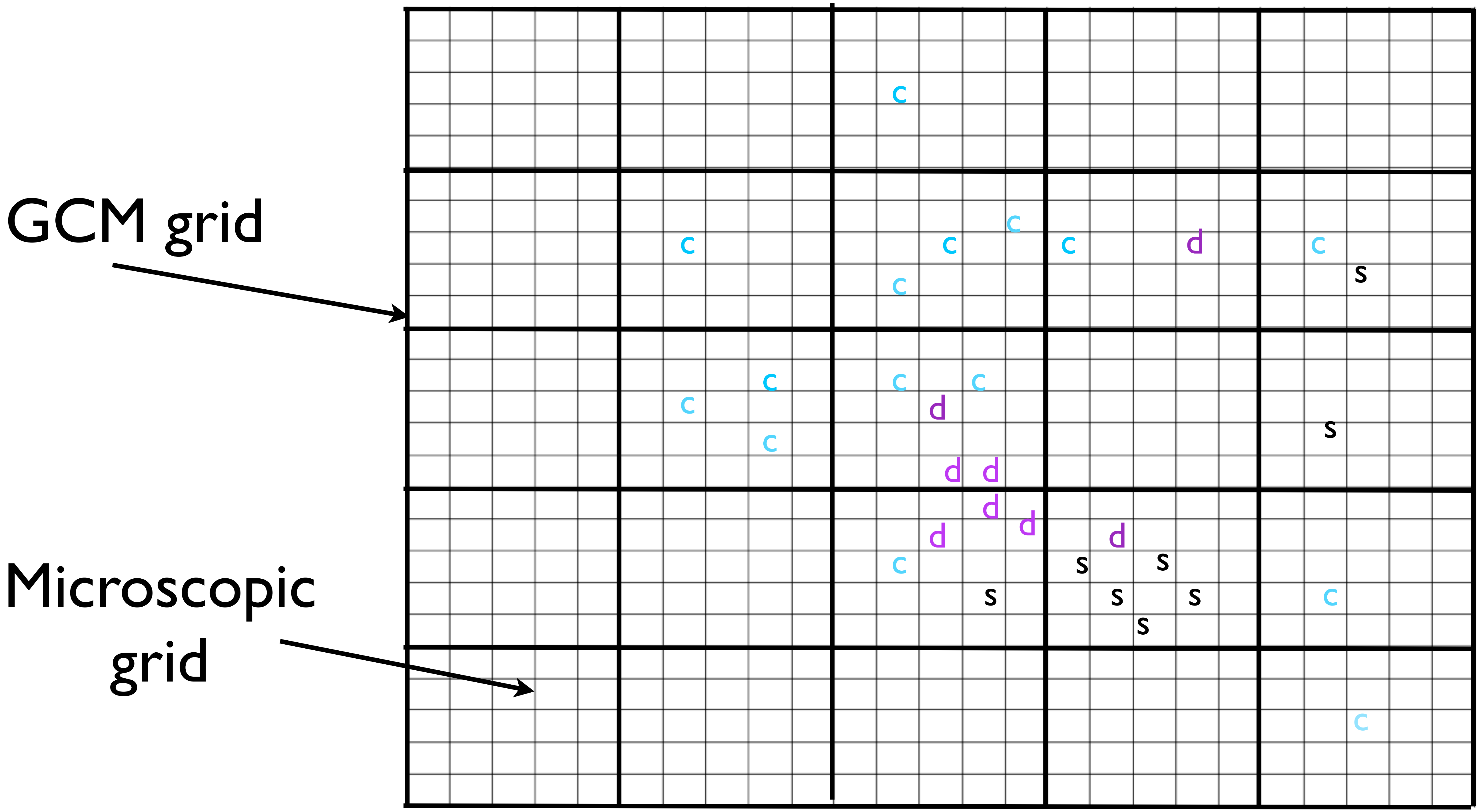

- Lattice points 1-10 km apart.
- Lattice site is occupied by a certain cloud type or is empty site

Intuitive transition rules

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- A clear sky site turns into a congestus site with high probability if CAPE>0 and middle troposphere is dry.
- A congestus or clear sky site turns into a deep site with high probability if CAPE>0 and middle troposphere is moist.
- A deep site turns into a stratiform site with high probability.
- All three cloud types decay naturally according to prescribed decay rates.

Particle Interacting System

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- Four state Markov process (at given site):

$$X_t = \begin{cases} 0 & \text{at clear sky site} \\ 1 & \text{at congestus site} \\ 2 & \text{at deep site} \\ 3 & \text{at statiform site} \end{cases}$$

- State Space:

$$\Sigma = \{0,1,2,3\}^N, \ N = \text{ total number of lattice sites}$$

- $X \in \Sigma$ is called a configuration

Spin flips or infinitesimal transitions

- Configuration waits an "exponential" time before it makes a transition at a random site
- A transition occurs at site j in $(t, t+\Delta t)$, if

$$X^i_{t+\Delta t} = \begin{cases} X^i_t & \text{if } i \neq j \\ X^j_t + \eta, & \text{if } i = j; \end{cases} \qquad \eta \in \{-3, -2, -1, 1, 2\}.$$

$$\eta = 1 : \text{clear} \longrightarrow \text{congestus or}$$

$$\text{congestus} \longrightarrow \text{deep or deep} \longrightarrow \text{stratiform}$$

$$\eta = -1 : \text{congestus} \longrightarrow \text{clear}$$

Special Case: No Local interactions

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- C = CAPE/low-level CAPE,
- D = mid-tropospheric dryness
- tau_kl transition time scales (parameters)
- Transition rates depend only on large-scale/external factors

$$R_{01} = \frac{1}{\tau_{01}}\Gamma(C)\Gamma(D) \qquad R_{02} = \frac{1}{\tau_{02}}\Gamma(C)(1-\Gamma(D))$$

$$R_{10} = \frac{1}{\tau_{10}}\Gamma(D) \qquad R_{12} = \frac{1}{\tau_{12}}\Gamma(C)(1-\Gamma(D))$$

$$R_{20} = \frac{1}{\tau_{20}}[1-\Gamma(C)] \qquad R_{23} = \frac{1}{\tau_{23}} \text{ or } \frac{\Gamma(C)}{\tau_{23}}$$

$$\Gamma(x) = 1 - e^{-x} \text{ if } x > 0$$

$$\Gamma(x) = 0 \text{ if } x \leq 0$$

$$R_{30} = 1/\tau_{30}$$

Cloud area fraction and Equilibrium measure

- When local interactions are ignored, X_t^i, are N independent four state Markov chains with the common equilibrium measure

$$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1, \pi_1 = \frac{R_{01}}{R_{10} + R_{12}} \pi_0,$$

$$\pi_2 = \frac{R_{02}\pi_0 + \pi_1 R_{12}}{R_{20} + R_{23}}, \pi_3 = \frac{R_{23}}{R_{30}} \pi_2$$

- Cloud area fractions on coarse mesh (e.g. congestus)

$$N_c^j(t) = \sum_{j \in D_i} \mathbb{I}_{\{X_t^i = 1\}}, \quad \sigma_c^j(t) = \frac{1}{Q} N_c^j(t)$$

$$0 \leq N_c \leq Q$$

$$E\sigma_c^j(t) = \pi_1(U_j) \text{ at equilibrium}$$

Time evolution and statistics of filling fraction

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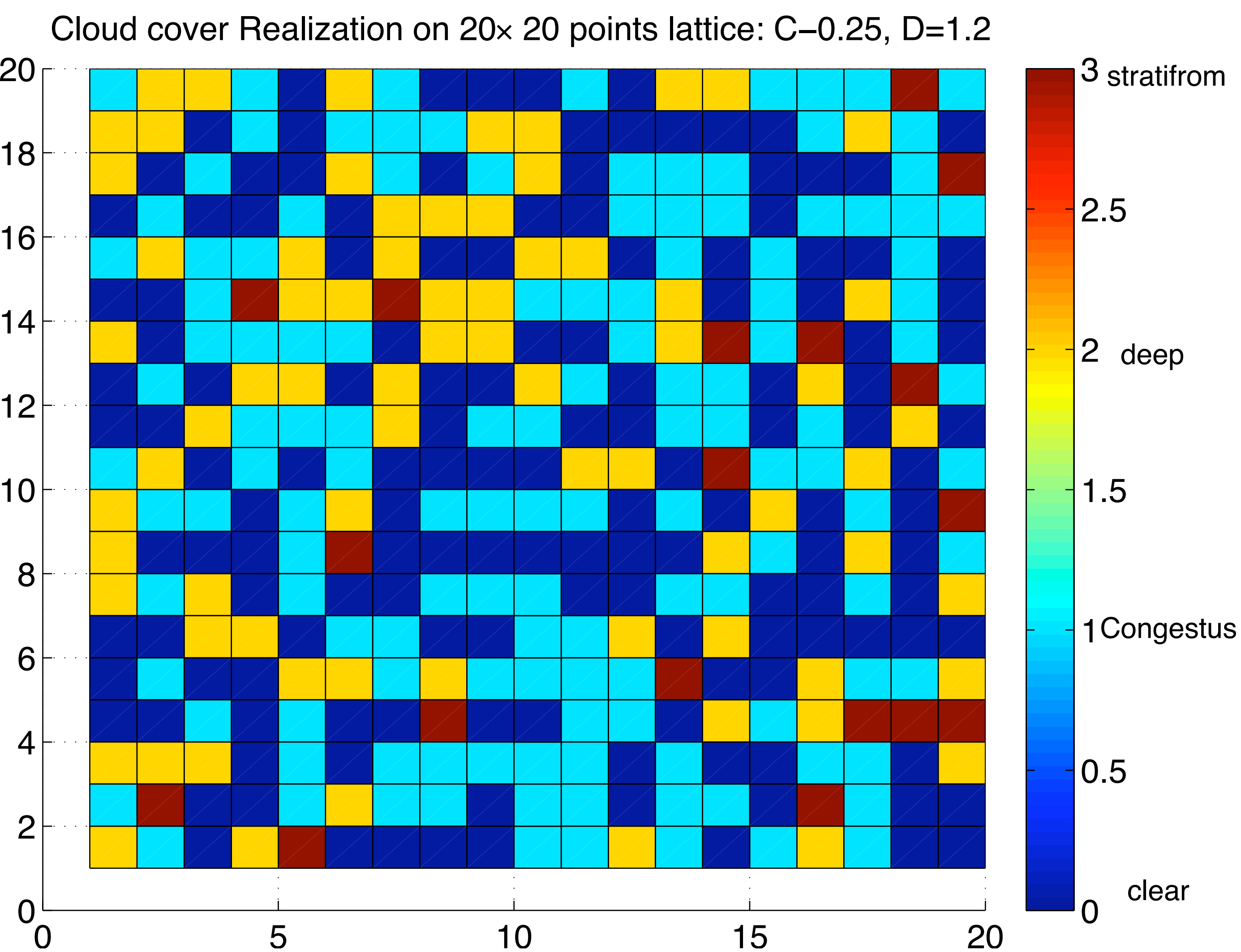
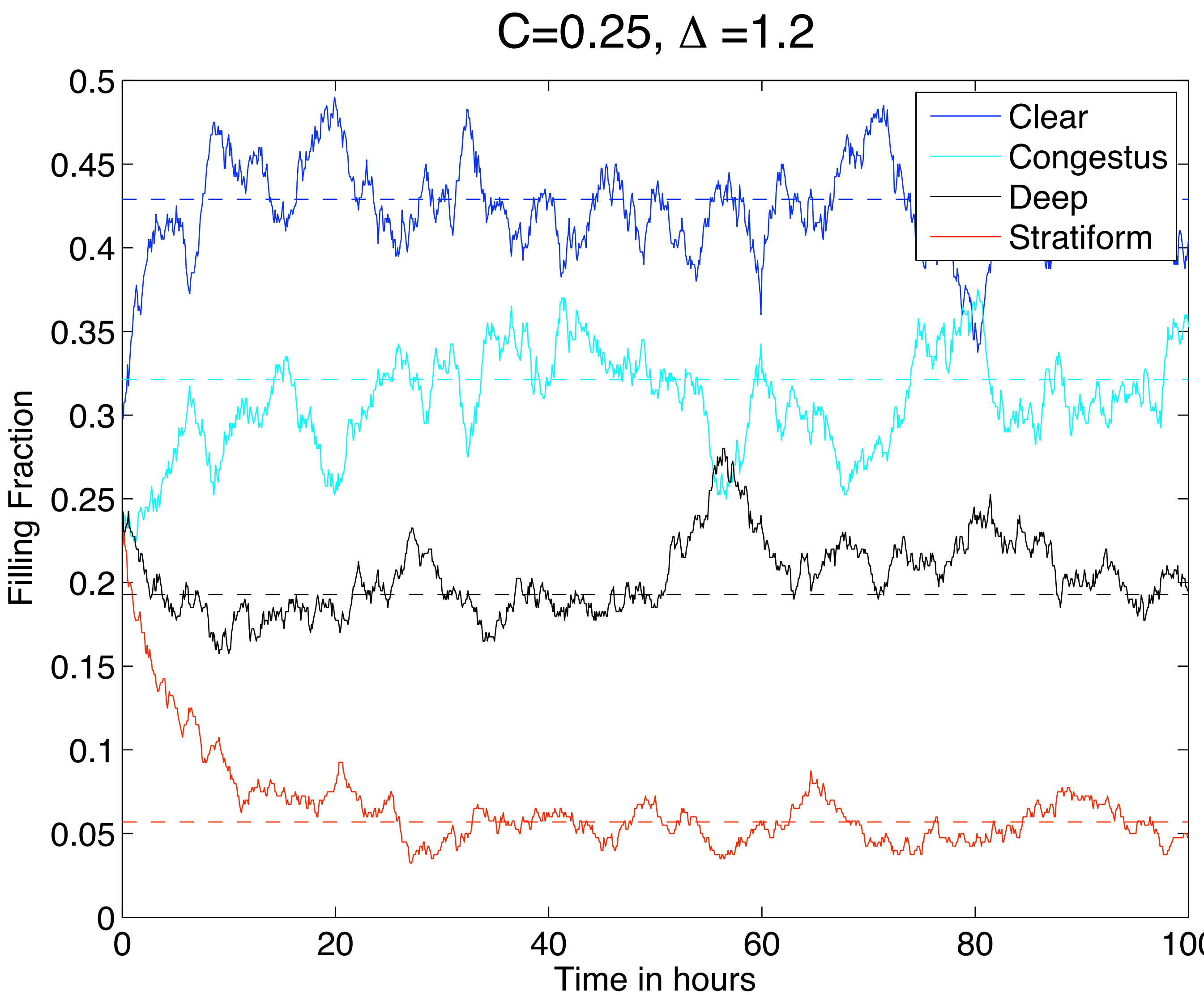




Case When Local Interactions are Ignored

- Coarse grained Process: Nx = number of sites filled with cloud type x within GCM grid box. e.g.
- NG talk (Thursday) will discuss local interactions
- Transition rates depend only on large scale variables
- $X_t^i, i = 1, 2, \cdots, N$ are i.i.d random variables
- Exact Dynamics for Coarse Grained process: Birth-death process with immigration:

$$Prob\{N_c^{t+\Delta t} = k + 1/N_c^t = k\} = N_{cs} R_{01} \Delta t + o(\Delta t)$$

$$Prob\{N_c^{t+\Delta t} = k - 1/N_c^t = k\} = N_c (R_{10} + R_{12}) \Delta t + o(\Delta t)$$

$$Prob\{N_d^{t+\Delta t} = k + 1/N_d^t = k\} = (N_{cs} R_{01} + N_c R_{12}) \Delta t + o(\Delta t)$$

clear sky $\quad N_{cs} = N - (N_c + N_d + N_s)$

Linking the stochastic model to the cumulus parameterization

$$H_c = \sigma_c \frac{\alpha_c \bar{\alpha}}{H_m} \sqrt{CAPE_l^+},$$

$$H_d = \sigma_d [\bar{Q} + \frac{1}{\tau_c^0 \bar{\sigma}_d} (a_1 \theta_{eb} + a_2 q - a_0(\theta_1 + \gamma_2 \theta_2))]^+,$$

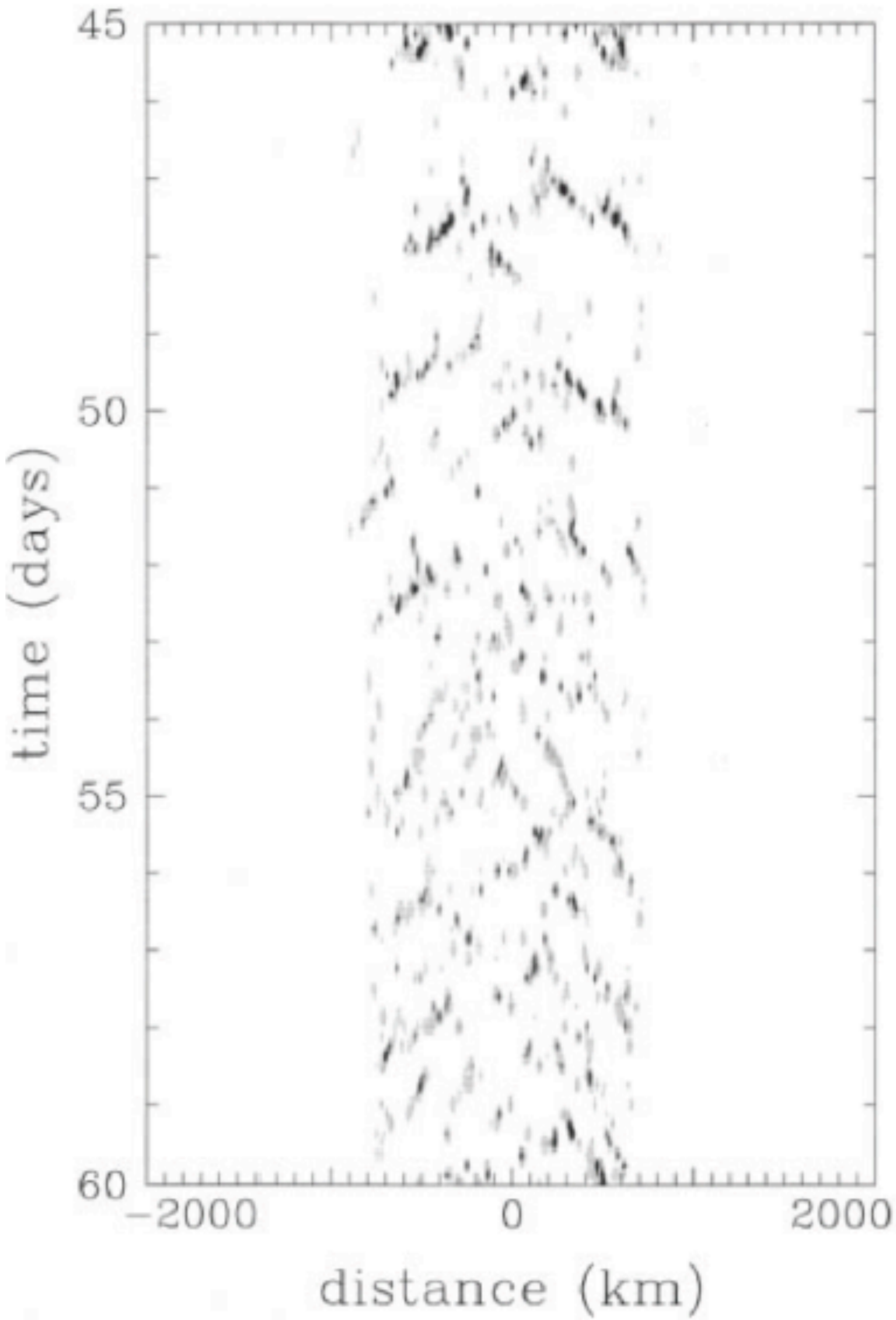
$$H_s = \sigma_s \alpha_s [\bar{Q} + \frac{1}{\tau_c^0 \bar{\sigma}_s} (a_1 \theta_{eb} + a_2 q - a_0(\theta_1 + \gamma_2 \theta_2))]^+$$

or

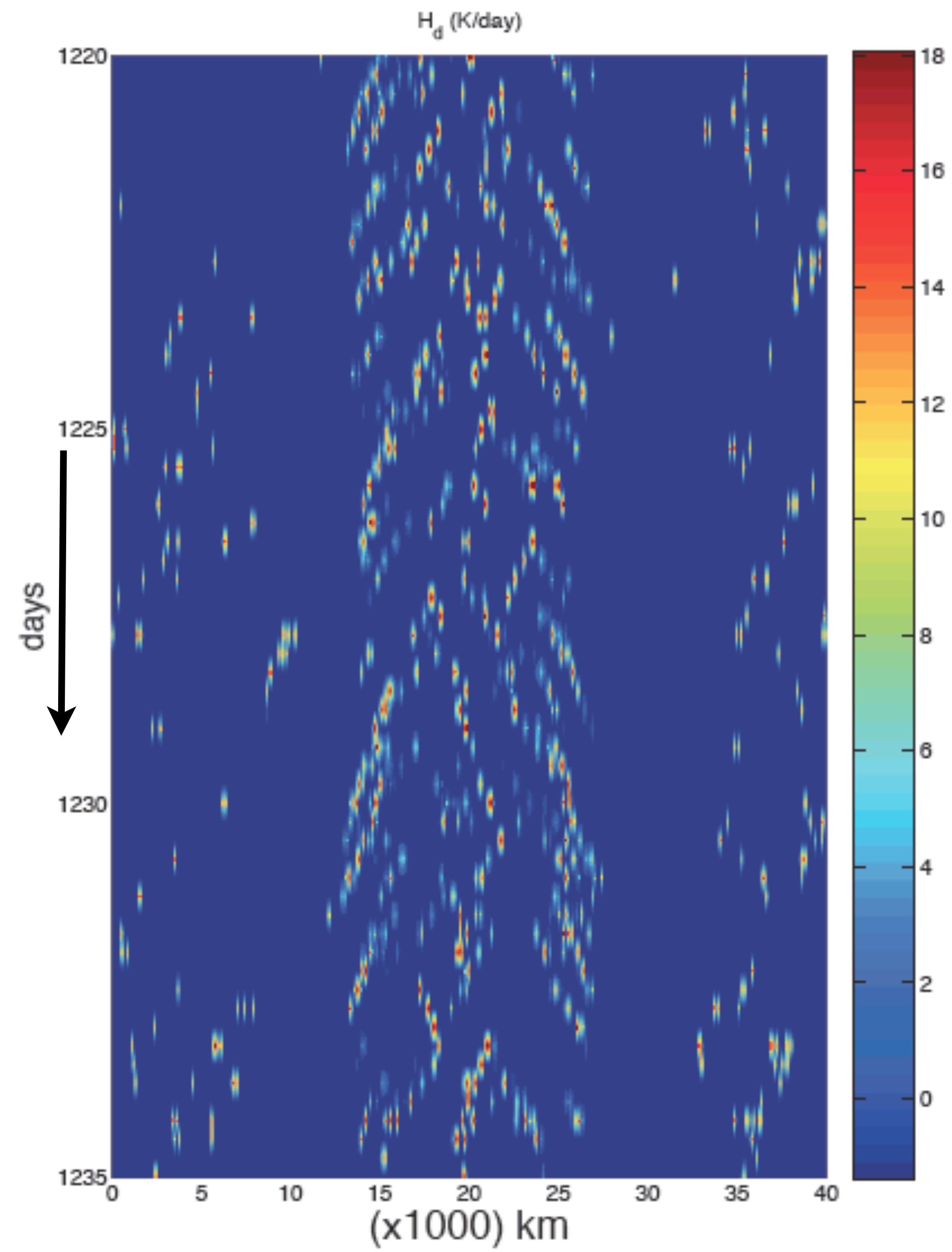
$$\partial_t H_s = \frac{1}{\tau_s} (\alpha_s \sigma_s H_d / \bar{\sigma}_d - H_s)$$

Warm Pool Simulation using the stochastic MC Model

CRM (Grabowski et al. 2000)



Stochastic MC



Congestus Moistening

- Believed to be main driver of deepening of convection: transition from shallow to deep convection
- Occurs via two main mechanisms
  - ✦ Large-scale low-level moisture convergence due to congestus heating of second baroclinic mode. Instability of convectively coupled waves in multicloud model disappears when low-level moisture convergence is ignored (K. and Majda, 2006) .
  - ✦ Detrainment of non-precipitating congestus clouds of up to 2g/kg/day occurs prior the deepening of convection in a small domain CRM simulation (Waite and K. 2010).

Main Goal of Talk

1. The effect of adding effect of congestus detrainment
2. Systematic Link and comparison between stochastic and deterministic multicloud models

Detrainment of congestus clouds

- Introduce evaporation due to detrainment of congestus

$$E_c = \frac{\sqrt{2}}{\pi} \frac{H_c}{Q_{R,1}^0} (\theta_{eb} - \theta_{el}); \quad \theta_{el} = 2q + \frac{2\sqrt{2}}{\pi}(\theta_1 + 2\theta_2)$$

- The new moisture budget equations ...

$$\frac{\partial q}{\partial t} + ... = -\frac{2\sqrt{2}}{\pi} P + (D + E_c)/H_T$$

$$\partial_t \theta_{eb} = \frac{1}{h_b}(E - E_c - D)$$

The Deterministic Mean Field Limit

- **Mean field equations of cloud area fraction**

$$\dot{\sigma}_c = (1 - \sigma_c\sigma_d - \sigma_s)R_{01} - \sigma_c(R_{10} + R_{12})$$

$$\dot{\sigma}_d = (1 - \sigma_c\sigma_d - \sigma_s)R_{02} + \sigma_c R_{12} - \sigma_d(R_{20} + R_{23})$$

$$\dot{\sigma}_s = \sigma_d R_{23} - \sigma_s R_{30}$$

- **Analogy with (original) Deterministic MC**

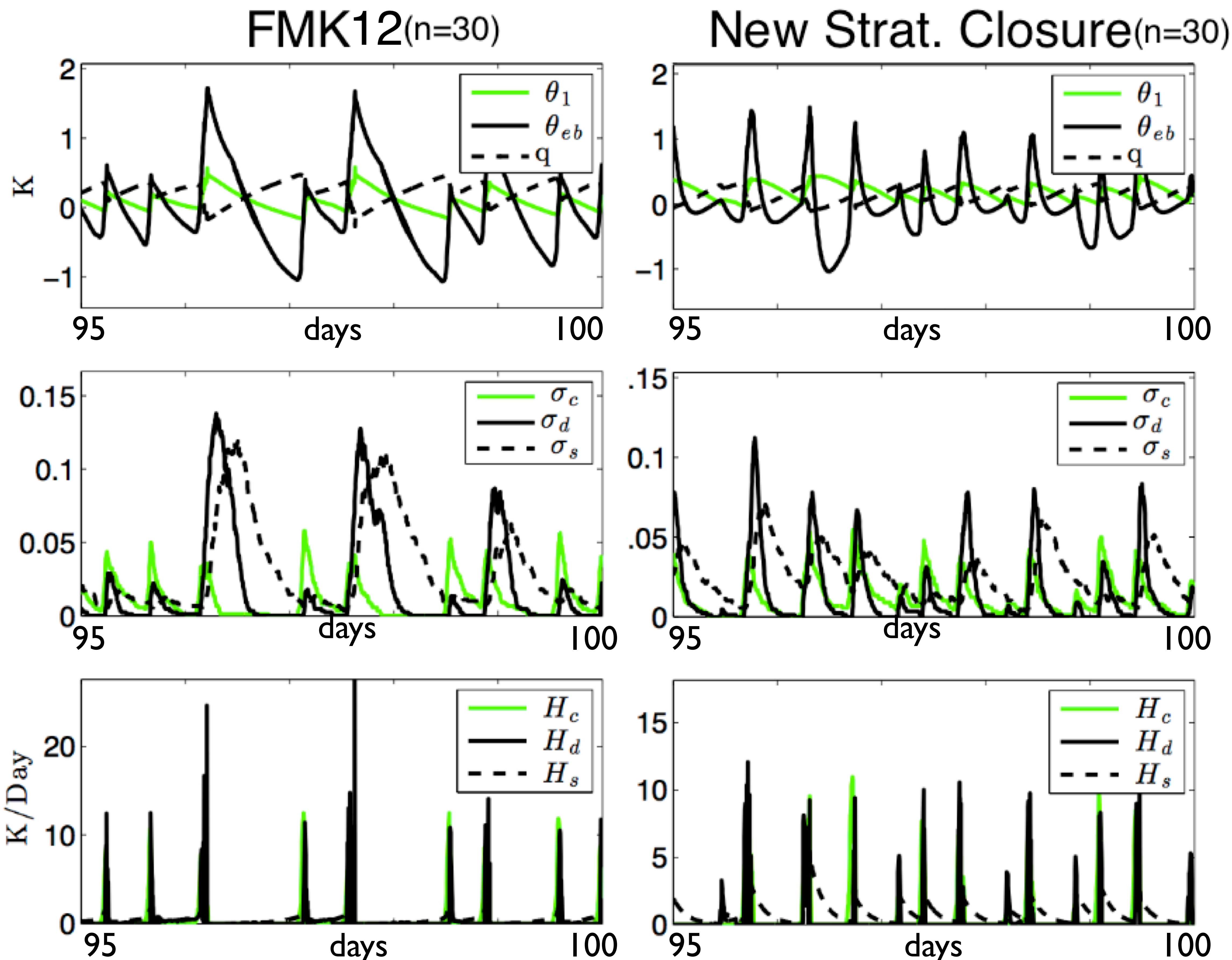
$$\partial_t H_c \approx \frac{1}{\tau_c^{MFL}}(\alpha_c^{MFL}\Gamma(D)\sqrt{CAPE_l^+} - H_c), \quad \tau_c^{MFL} = \frac{\tau_{12}}{\Gamma(\bar{C})}$$

$$H_d \approx \left[\frac{\tau_{23}\tau_{20}\Gamma(\bar{C})}{\tau_{02}(\tau_{20} + \tau_{23}(1 - \Gamma(\bar{C}))}\right]\Big(1 - \Gamma(D)\Big)\times$$

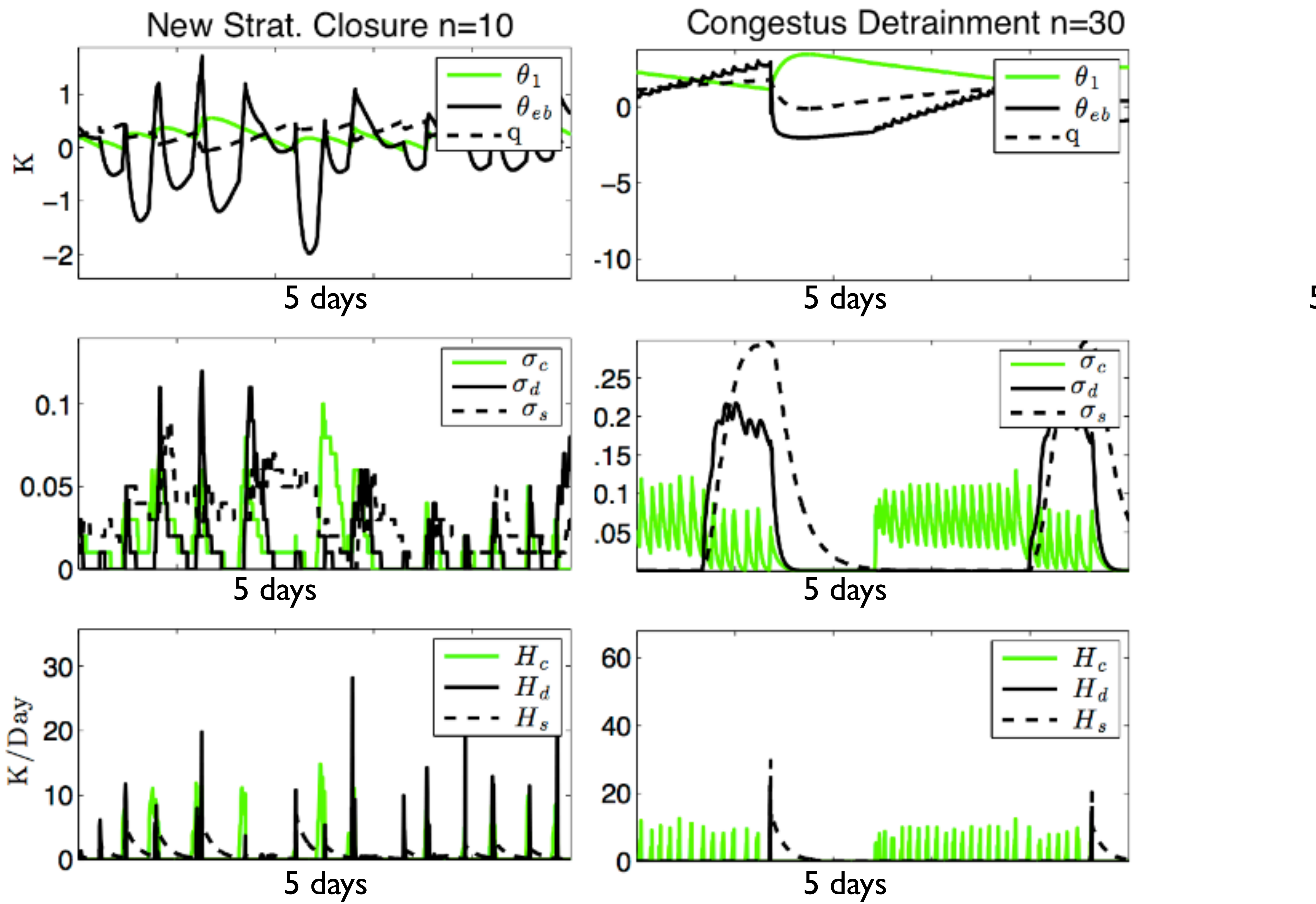
$$\left[\bar{Q} + \frac{1}{\tau_{conv}^0\bar{\sigma_d}}(a_1\theta_{eb} + a_2 q - a_0(\theta_1 + \gamma_2\theta_2))\right]^+$$

- $\Gamma(D)$ is an increasing function of $\theta_{eb} - \theta_{em}$ like Λ but highly non-linear

Single Column Simulations



Congestus detrainment



Deterministic Mean Field Limit

Deterministic Nonlinearity

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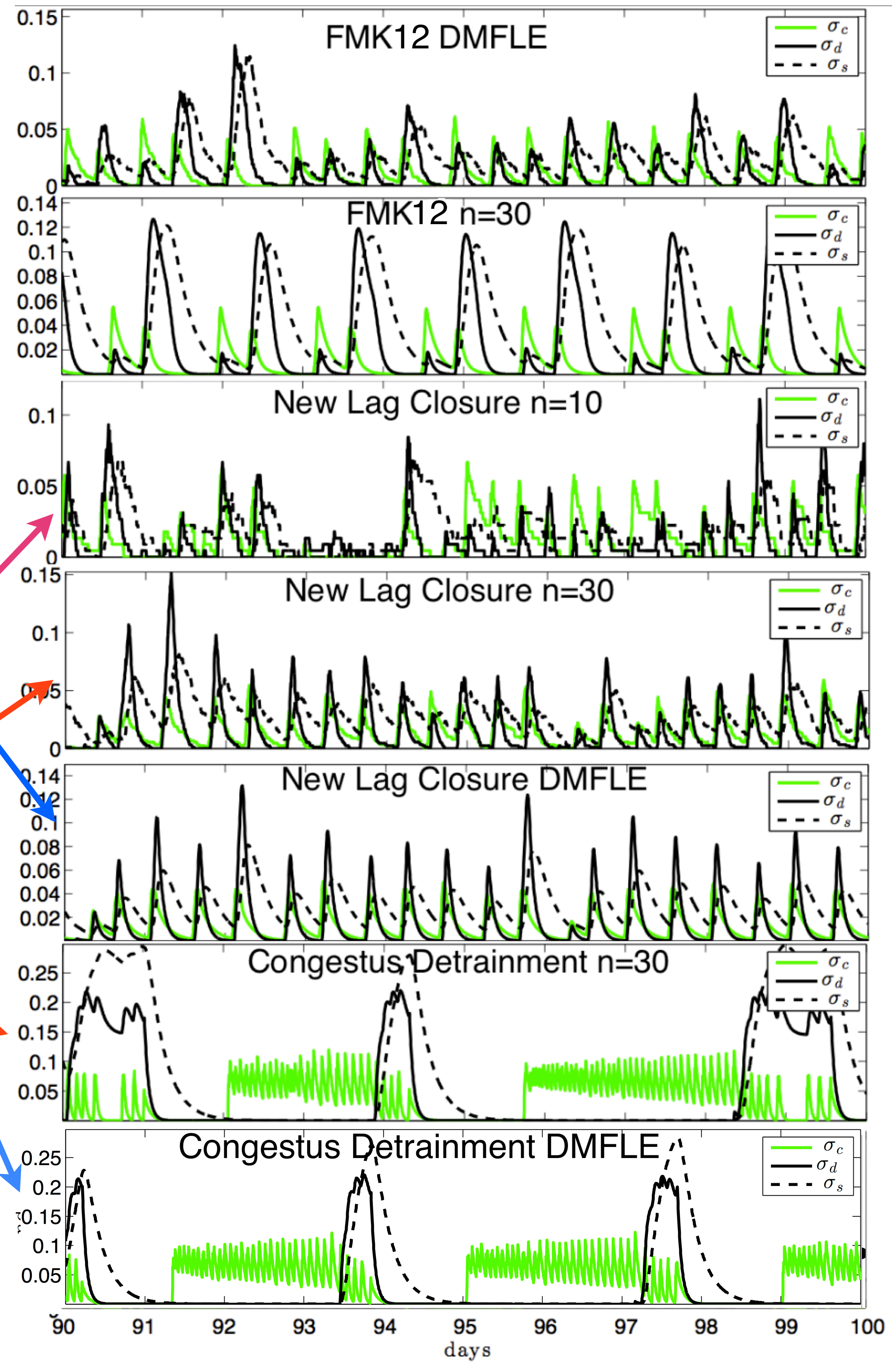
Stochasticity



Conclusions

- Multicloud model captures key features of organized tropical convection including MJO
- Here we showed how to build physically constrained stochastic model to account for sub-grid scale variability
- Non-trivial effect of congestus detrainment
- Bridging deterministic mean-field limit elucidates the effects of nonlinearity v.s. stochasticity
- Local-interaction effects: Thursday, 9:30 AM - 9:45 AM NG41E-03
- Assessment of SMC against observations: K. Peters' Talk, Friday, 12:05 PM - 12:20 PM A52C-08