Fate of Mountain Waves in the Stratosphere: A Spectral Approach

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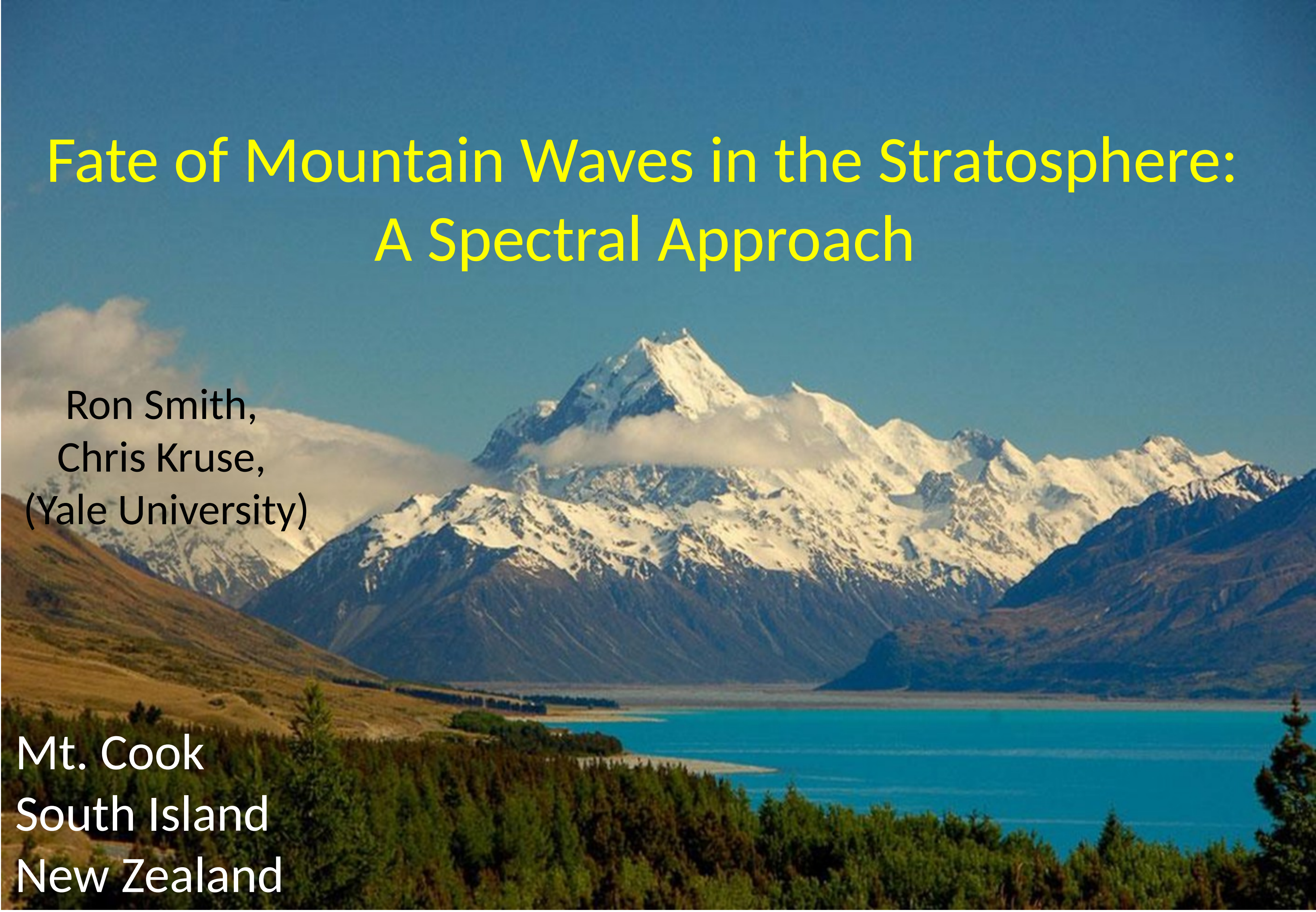


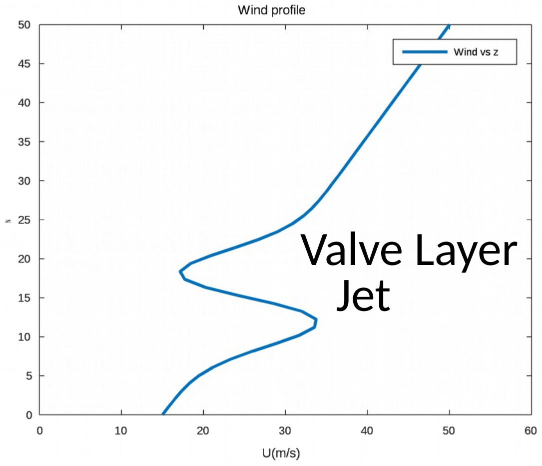
Mt. Cook
South Island
New Zealand

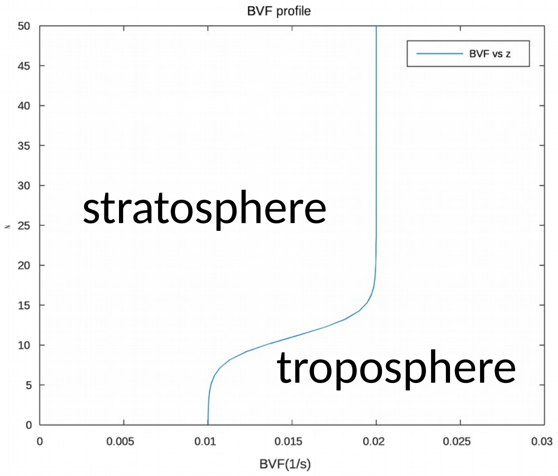
Support from NSF-AGS-1338655

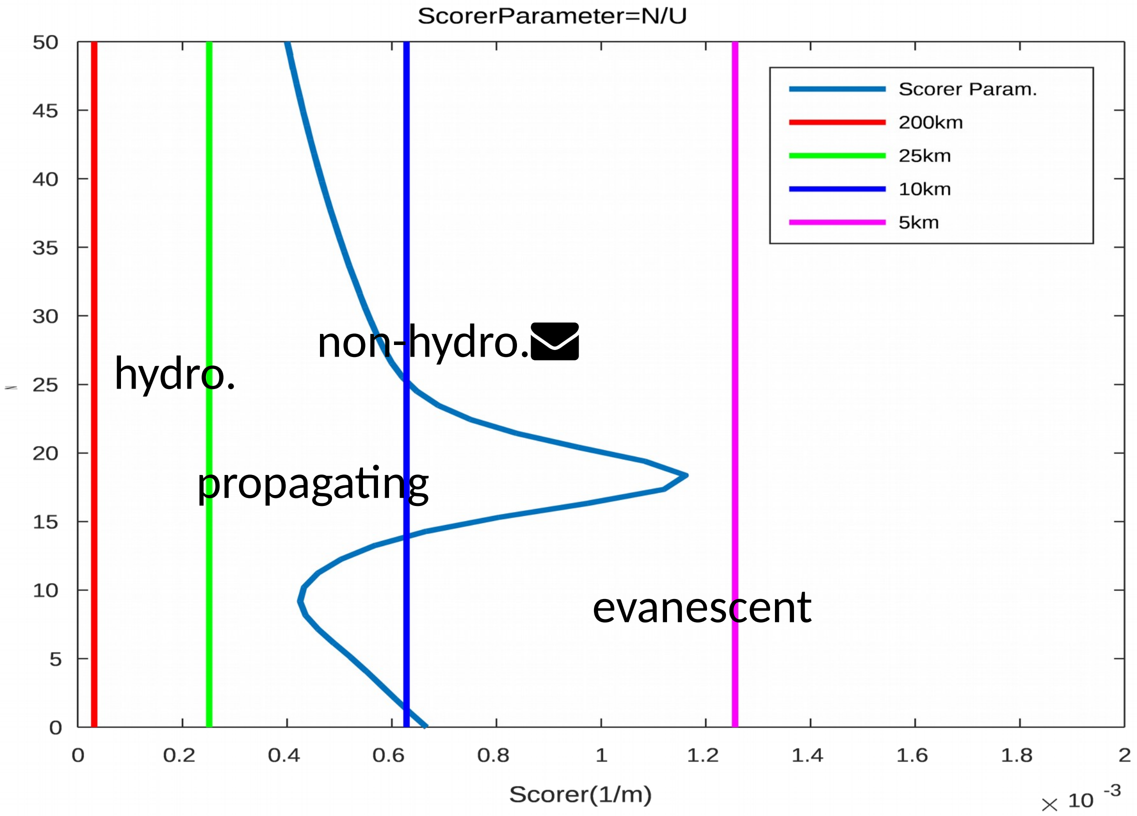
Outline

- Theoretical expectations
  - Scorer profiles
  - Broad spectra terrain
  - Wave measurements
  - Non-hydrostatic effects
- DEEPWAVE analysis
  - Aircraft data
  - WRF model results
  - Fate of waves in the stratosphere

Generic Deepwave Sounding







Gravity Wave Spectral Variances

- Mountain shape or streamline displacement $\eta(x)$
- Fourier Transform $\hat{\eta}(k) = \int_{-\infty}^{\infty} \eta(x) \exp(-ikx)\, dx$
- $Var(\eta) = \int_{-\infty}^{\infty} \eta^2(x) dx = \left(\frac{1}{2\pi}\right) \int_{-\infty}^{\infty} \hat{\eta}(k)\hat{\eta}(k)^* dk$

From **hydrostatic** mountain wave theory

weights

1. $Var(w) = \int_{-\infty}^{\infty} w^2(x) dx = \left(\frac{U^2}{2\pi}\right) \int_{-\infty}^{\infty} k^2\, \hat{\eta}(k)\hat{\eta}(k)^* dk$
2. $Cov(u, w) = \int_{-\infty}^{\infty} u(x) w(x) dx = -\left(\frac{NU}{2\pi}\right) \int_{-\infty}^{\infty} |k|\, \hat{\eta}(k)\hat{\eta}(k)^* dk$
3. $Var(u) = \int_{-\infty}^{\infty} u^2(x) dx = \left(\frac{N^2}{2\pi}\right) \int_{-\infty}^{\infty} \hat{\eta}(k)\hat{\eta}(k)^* dk$

Note: P-power and T-power are similar to u-power (3)

Weights are unimportant if spectrum is narrow, but important if the spectrum is broad .

Mountain shape or streamline displacement

Fourier Transform

From **hydrostatic** mountain wave theory

1.

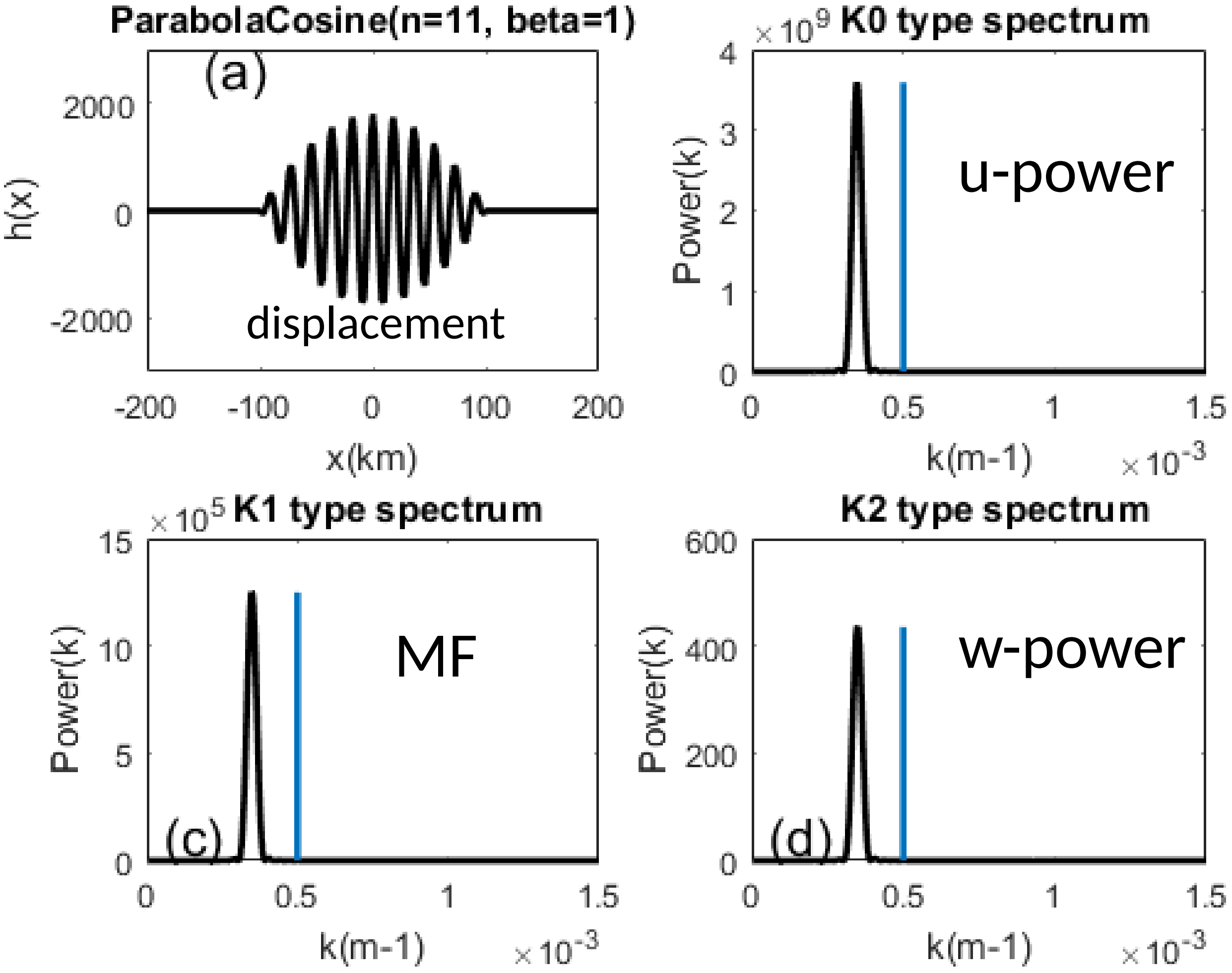
2.

Note: P-power and T-power are similar to u-power (3)

Weights are unimportant if spectrum is narrow, but important if the spectrum is broad

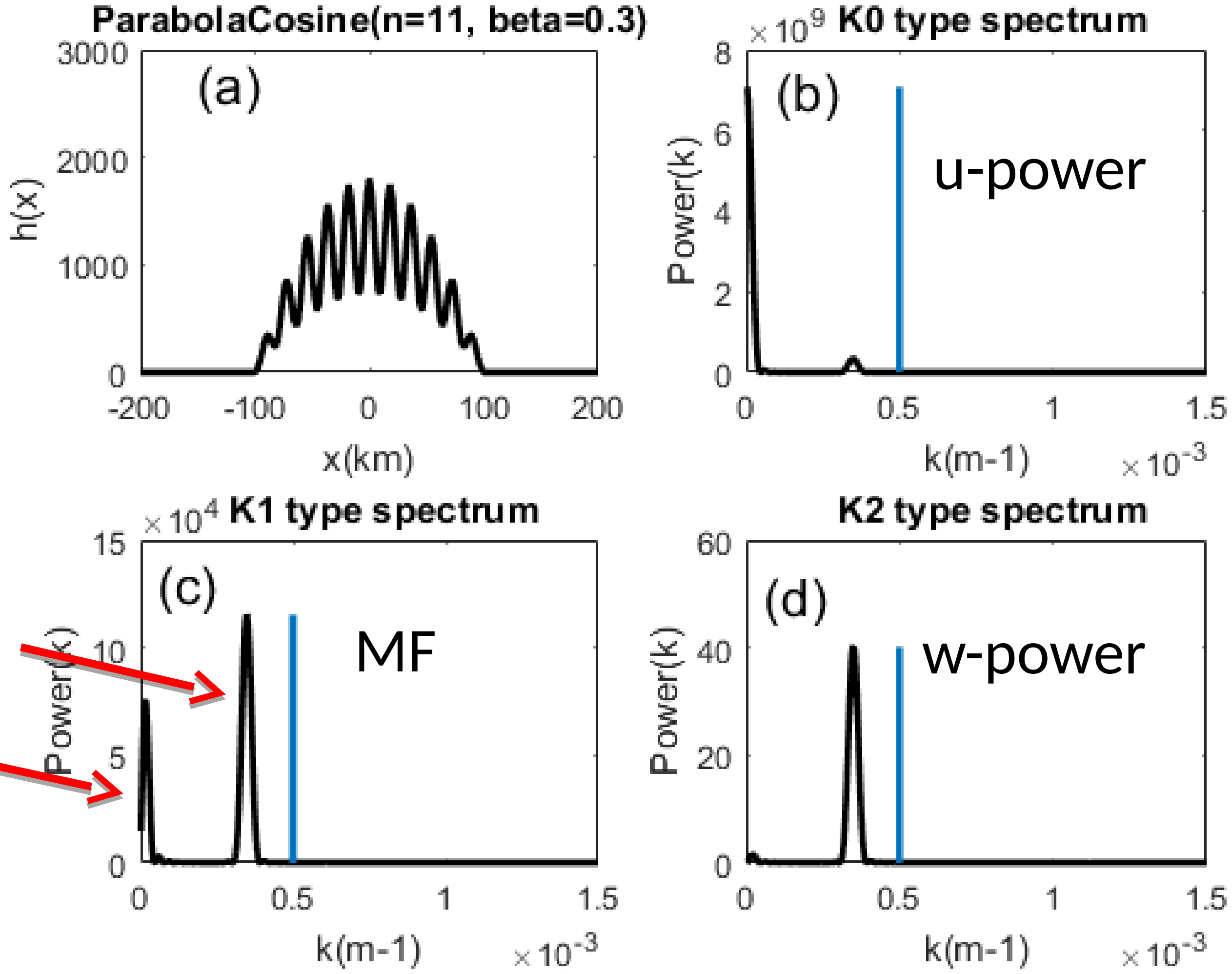
Monochromatic Wave

(blue line: typical buoyancy cut-off)



Ideal Rough hill

(blue line: typical buoyancy cut-off)

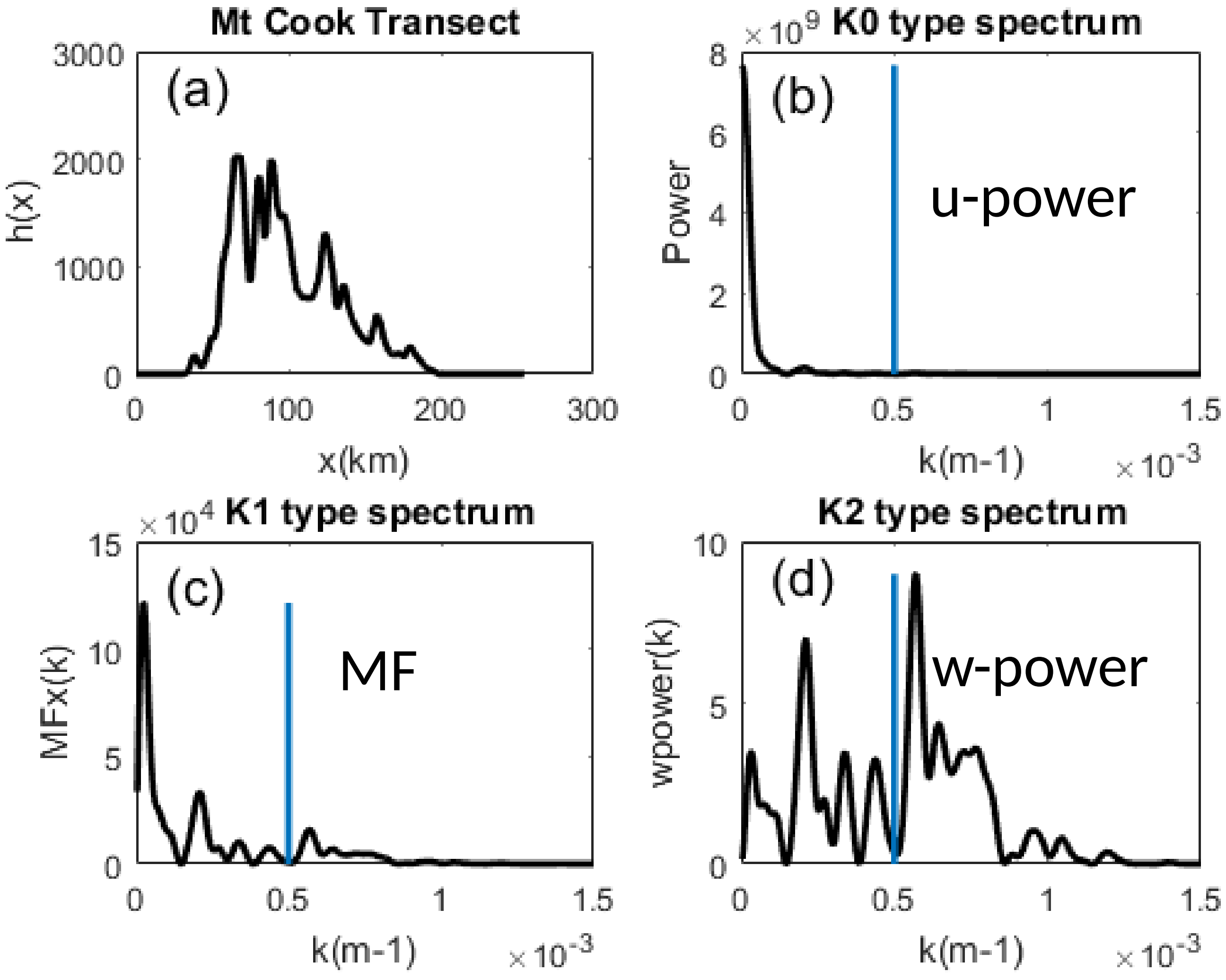


Roughness Mode

Volume Mode

New Zealand transect, Mt. Cook

(blue line: typical buoyancy cut-off)



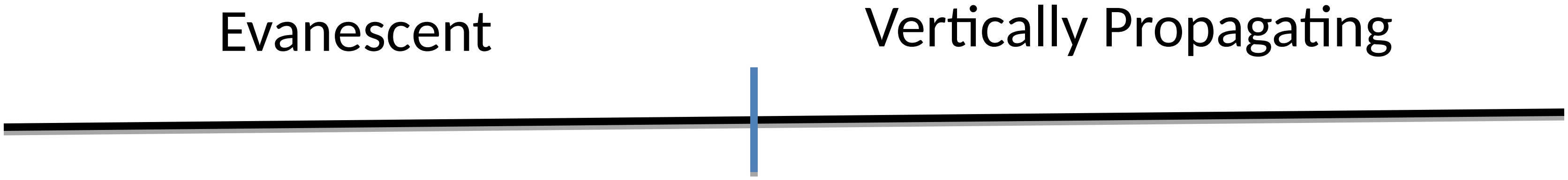
For terrain with volume and roughness
(hydrostatic results)

- Variance spectra are broad and varied
- Volume mode dominates the u-power
- Roughness mode dominates the w-power
- Both modes contribute to MF
  - Volume mode: large u’ and small w’
  - Roughness mode: small u’ and large w’

Measuring GWs and MFs

- T’ measurements mostly see the Volume mode
  - Passive IR (e.g. AIRS)
  - Rayleigh LIDAR
- U’ measurements mostly see the Volume mode
  - Ascending balloons
  - VHF Doppler radar
  - Constant pressure balloons
- W’ cannot be inferred from T’ and U’ as polarization relations do not apply to broad spectra. Can’t get MF.
- Aircraft can directly observe U’ and W’

Non-hydrostatic mountain waves near the buoyancy cut-off



Evanescent

Vertically Propagating

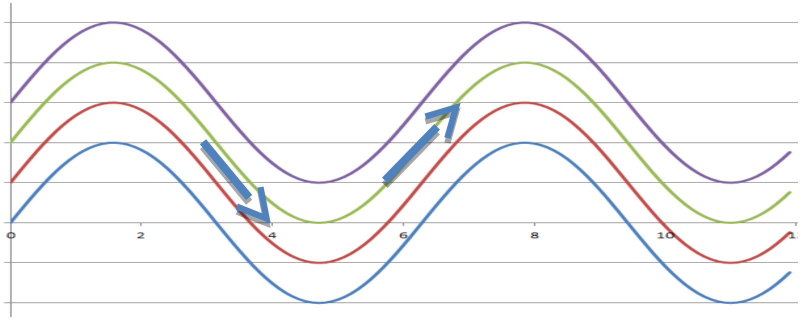
Wavelength →

$$k = N/U$$

$$\lambda = 2\pi U/N \sim 10\text{km}$$

$$m = \left(\frac{N}{U}\right)[1-(\frac{kU}{N})^2]^{1/2} \rightarrow 0$$

$$\hat{u} = \left(\frac{m}{k}\right)\hat{w} \rightarrow 0$$



m=0

constant streamline spacing

u’=0

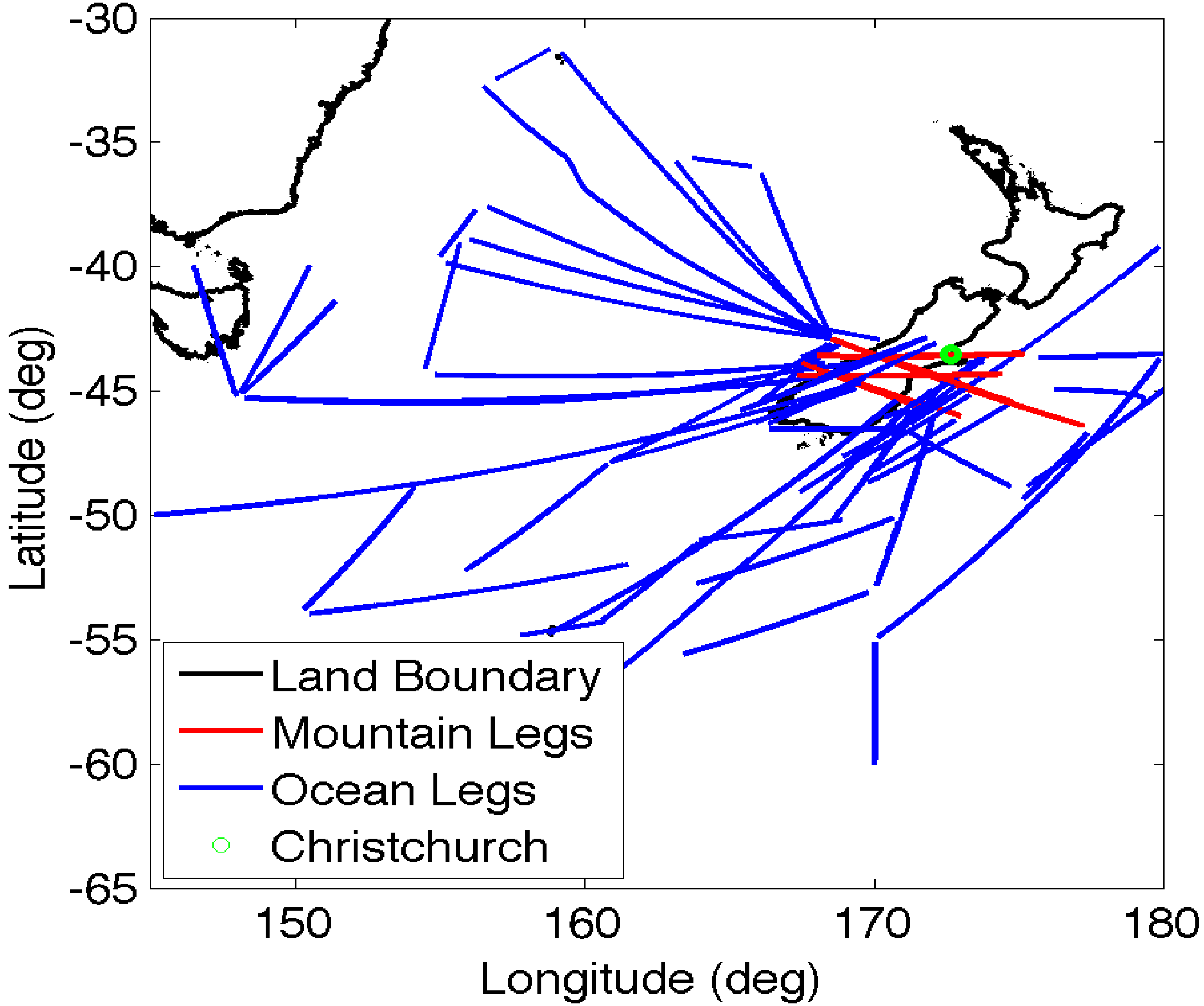
Short waves in the stratosphere

- Wavelengths near 10km may be found in the stratosphere over rough terrain due to their slow decay or leakage.
- They will have strong w-power but little u-power or MF
- Wavelengths near 5km may be generated by rough terrain but will not reach the stratosphere due to evanescence.

DEEPWAVE GV legs over New Zealand

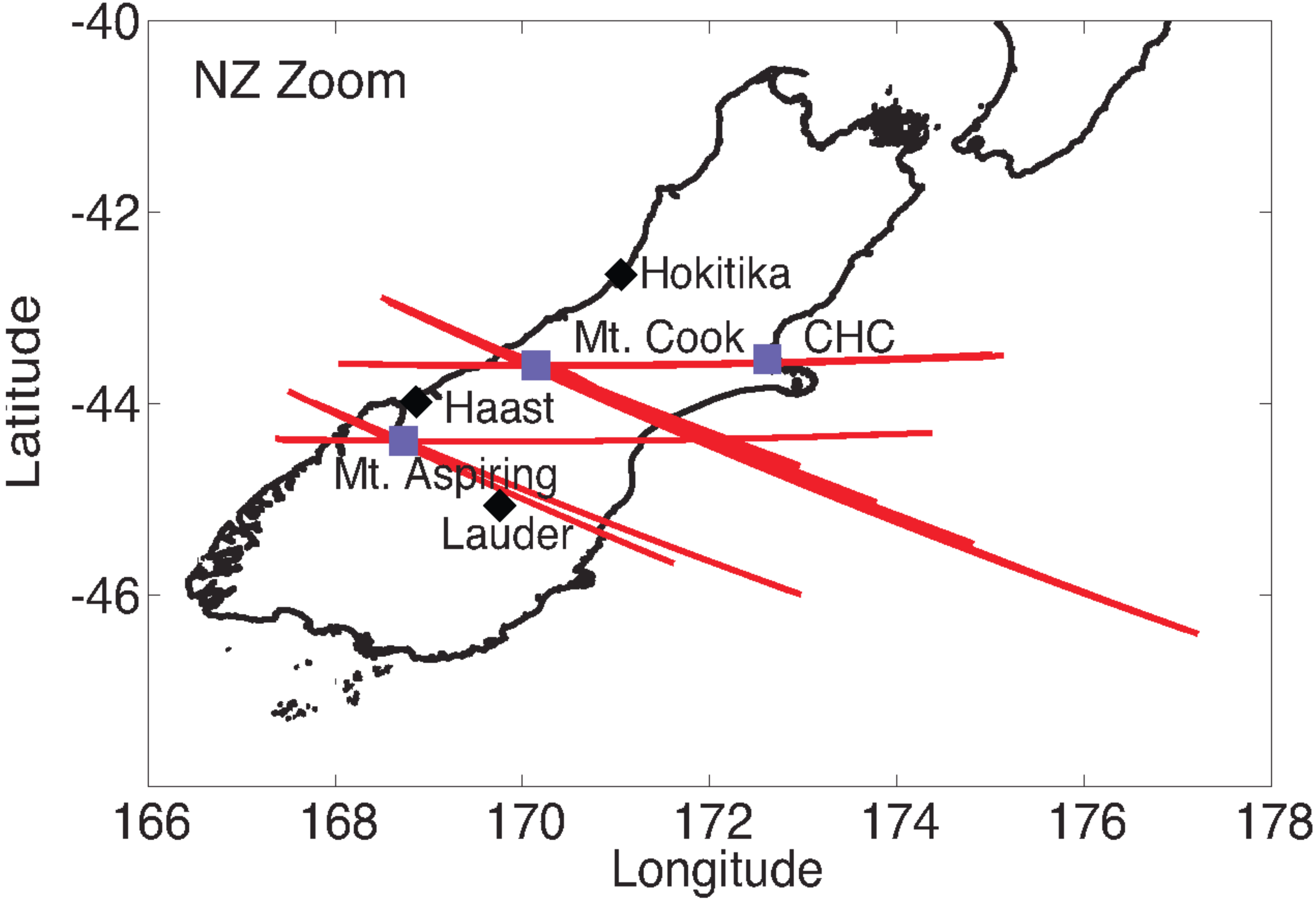
- Observing period: SH Winter: June/July 2014
- Total (26 flights, 180 hours)
- Over New Zealand (97 legs; 49.1 hours)
- Over Ocean (157 legs; 84.3 hours)
- Altitude: mostly 12.1km
- Typical leg length: 350km
- **Variables measured: u,v,w,p,T**
- Publications
  - Smith et al. 2016, *J. Atmos. Sci.*
  - Smith and Kruse, 2017, *J. Atmos. Sci.*

NGV Legs: mostly at z=12.1km



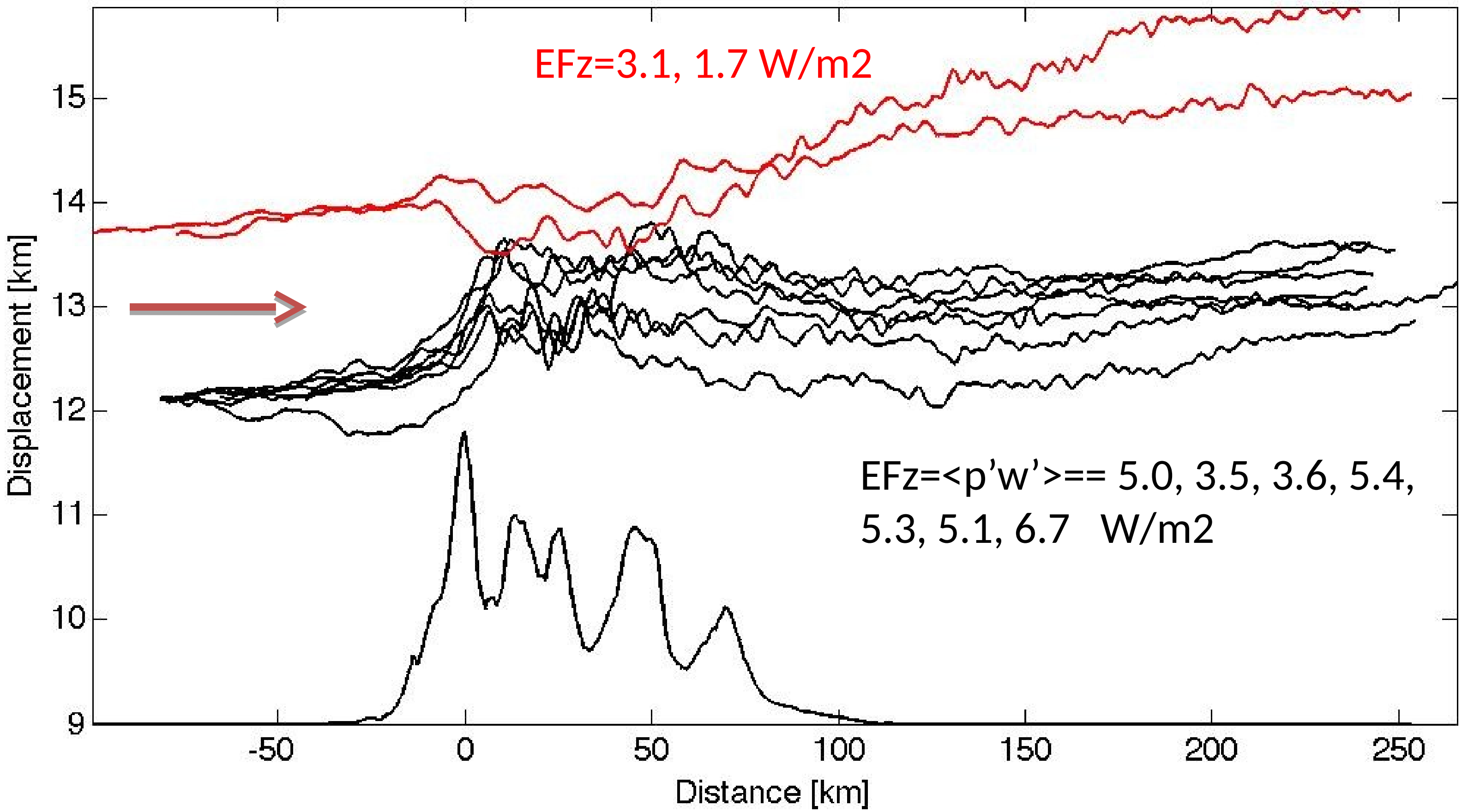
NGV Legs over New Zealand

(mostly at z=12.1km)



Mt
Cook

RF05: 9 Legs
Vertical displacement



EFz=3.1, 1.7 W/m2

EFz=<p'w'>== 5.0, 3.5, 3.6, 5.4, 5.3, 5.1, 6.7 W/m2

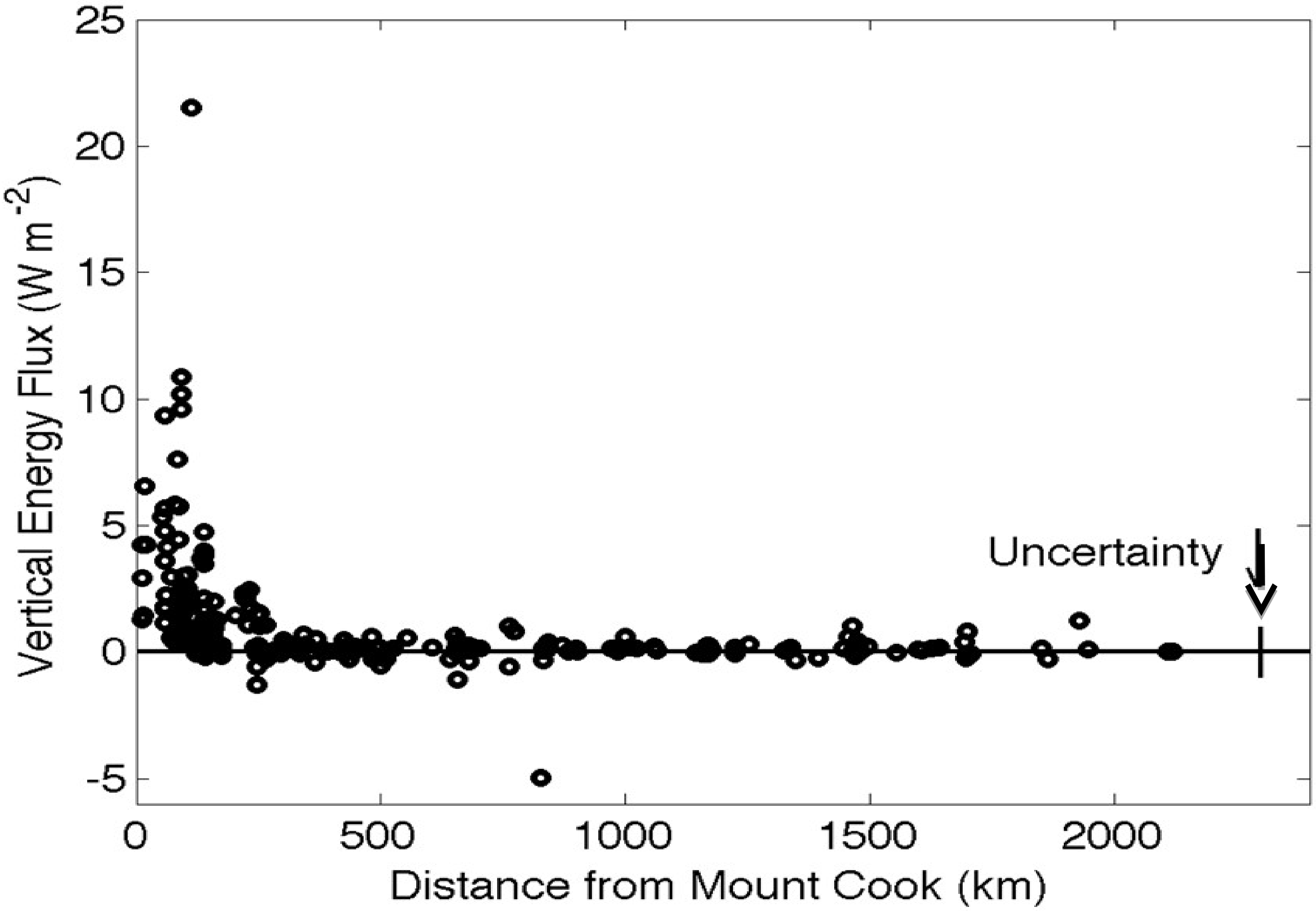
Mountain to scale but offset vertically
Vertical exaggeration = 25:1

Flight level Flux calculations

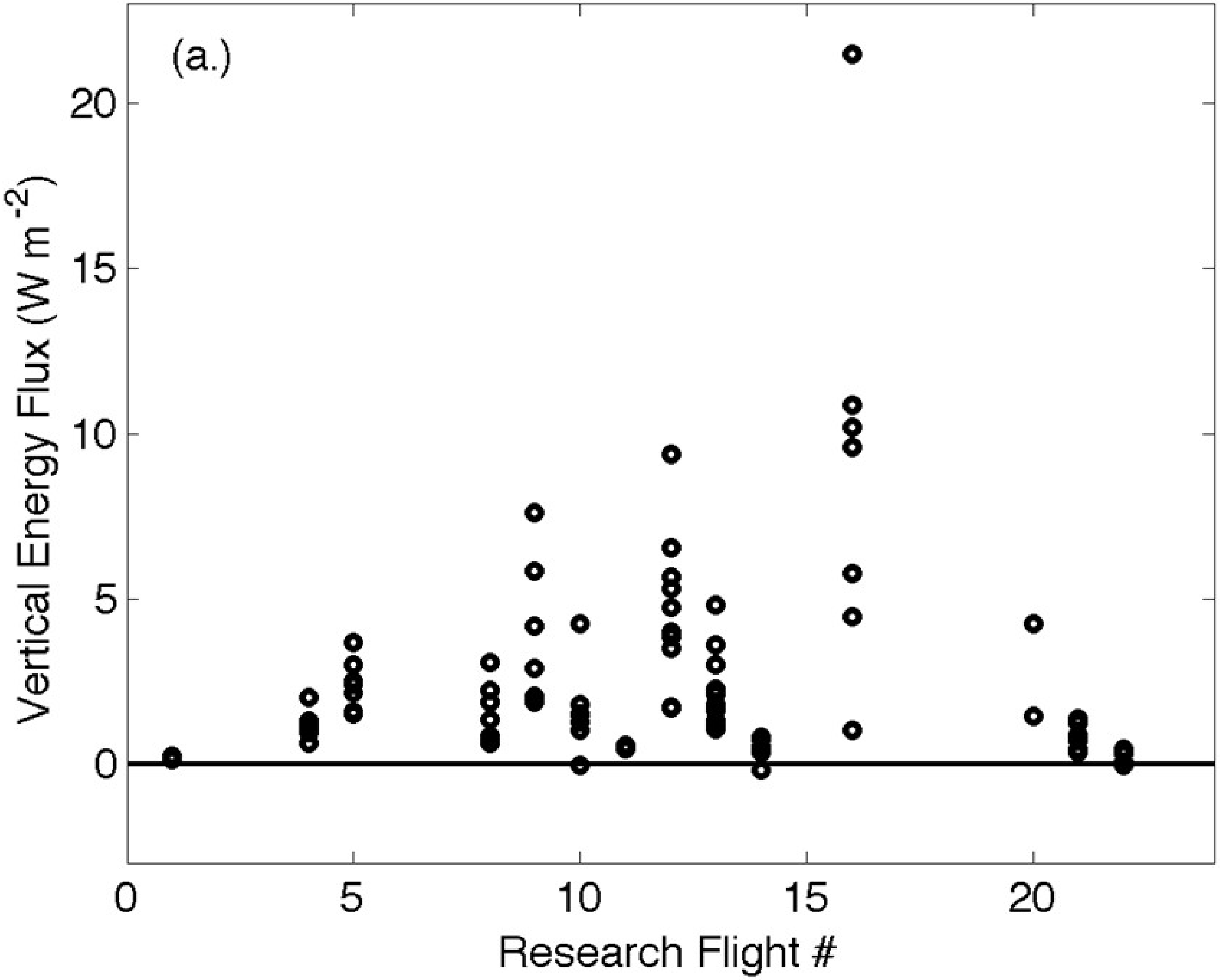
- The fluxes are computed from
  - $MFx = \bar{\rho} < u'w' >$
  - $MFy = \bar{\rho} < v'w' >$
  - $EFz =< p'w'>$
  - $EFx =< p'u' >$
  - $EFy =< p'v' >$
  - EFzM = -(U*MFx+V*MFy) (Eliassen-Palm, 1960)
  - $NER = (u'^{2} + v'^{2})/(U^2 + V^2)$

- The fluxes are computed from
- EFzM = -(U*MFx+V*MFy) (Eliassen-Palm, 1960)

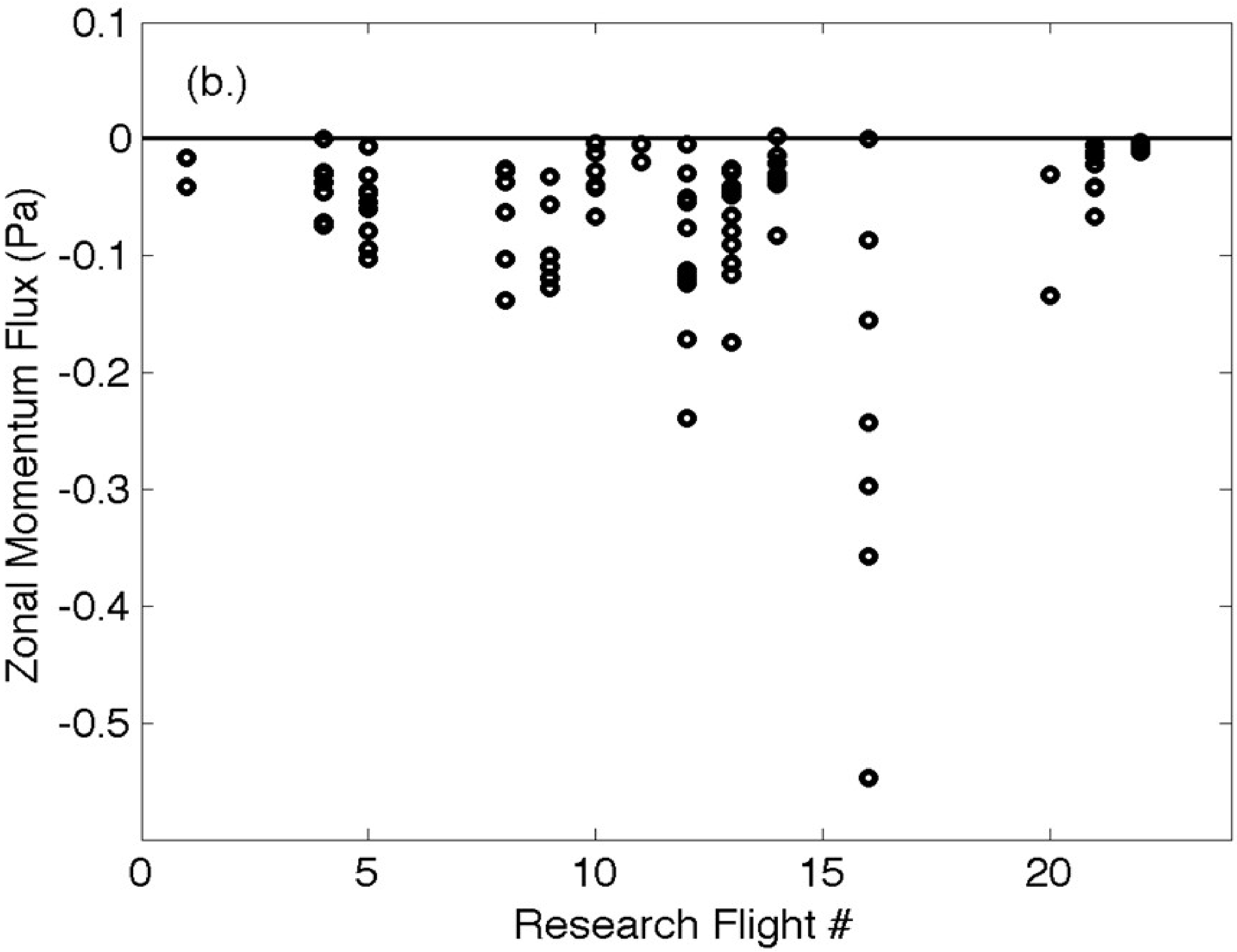
Vertical Energy Flux:
all Deepwave GV flights



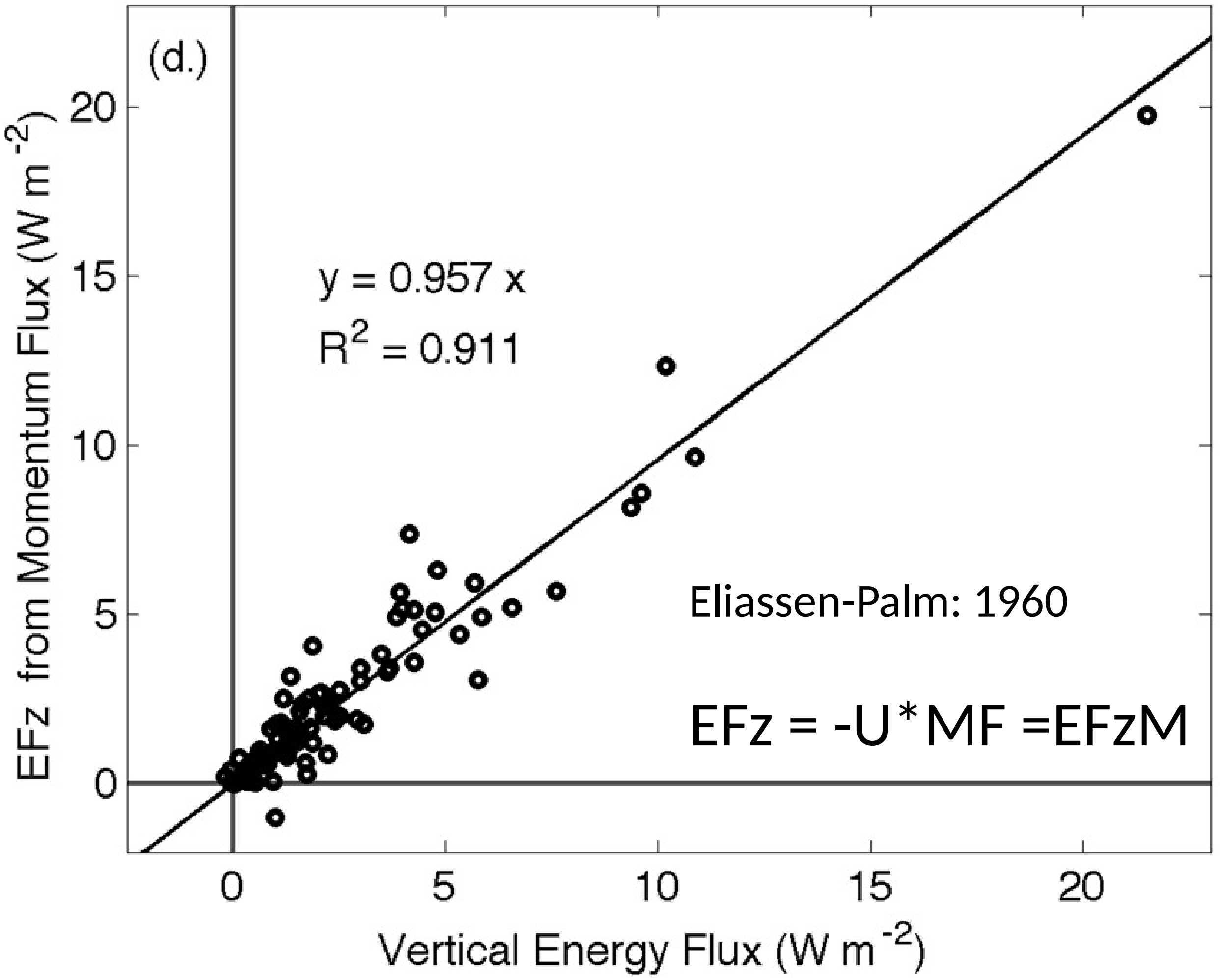
Vertical Energy flux for 14 NZ flights



Zonal Momentum Flux for 14 NZ flights



Comparing Energy and Momentum Fluxes
EFz versus EFzM

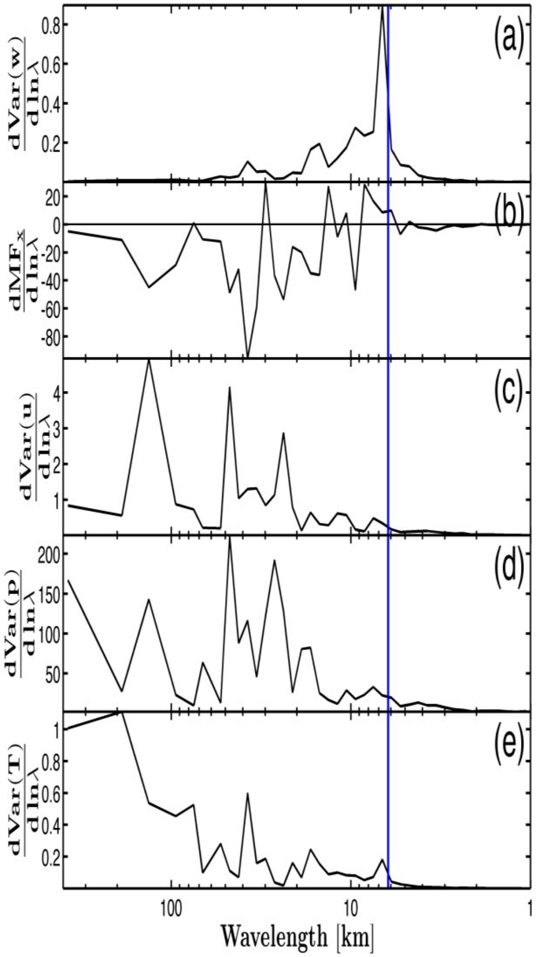


Eliassen-Palm: 1960

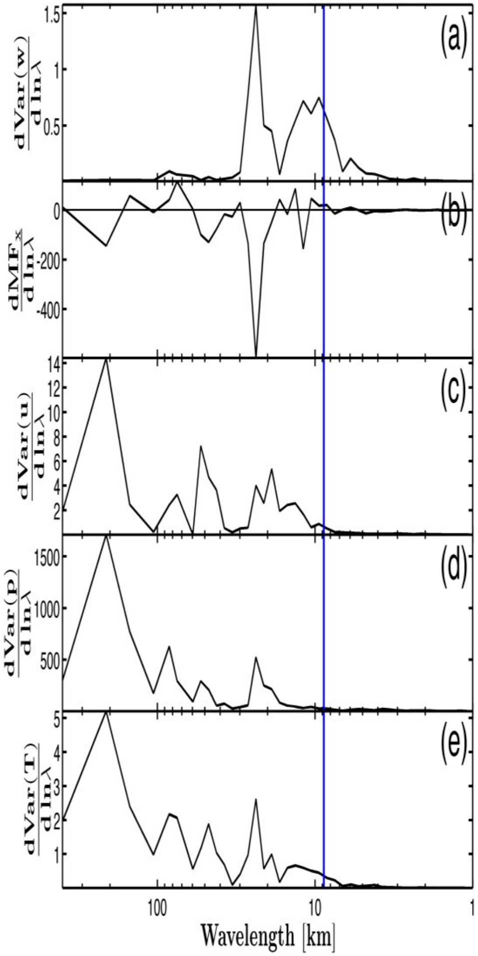
EFz = -U*MF =EFzM

Implies u’ and p’ are proportional

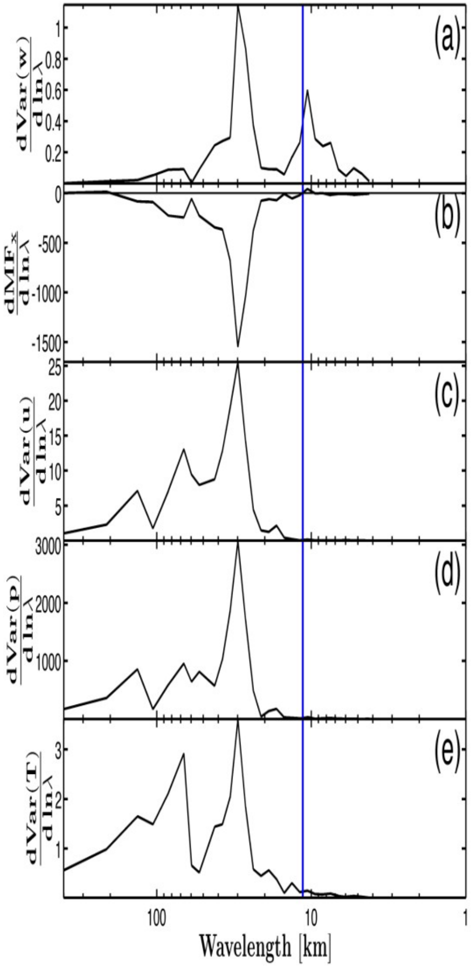
GV Aircraft Mountain Wave Spectra



RF05



RF09



RF16

w-power

MFx

u-power

P-power

T-power

Blue line is the buoyancy cut-off

DEEPWAVE cross-terrain flight statistics

DEEPWAVE cross-terrain flight statistics

Volume Mode: $\lambda > 60km$, Roughness Mode: $\lambda < 60km$

Volume Mode: ;Roughness Mode:

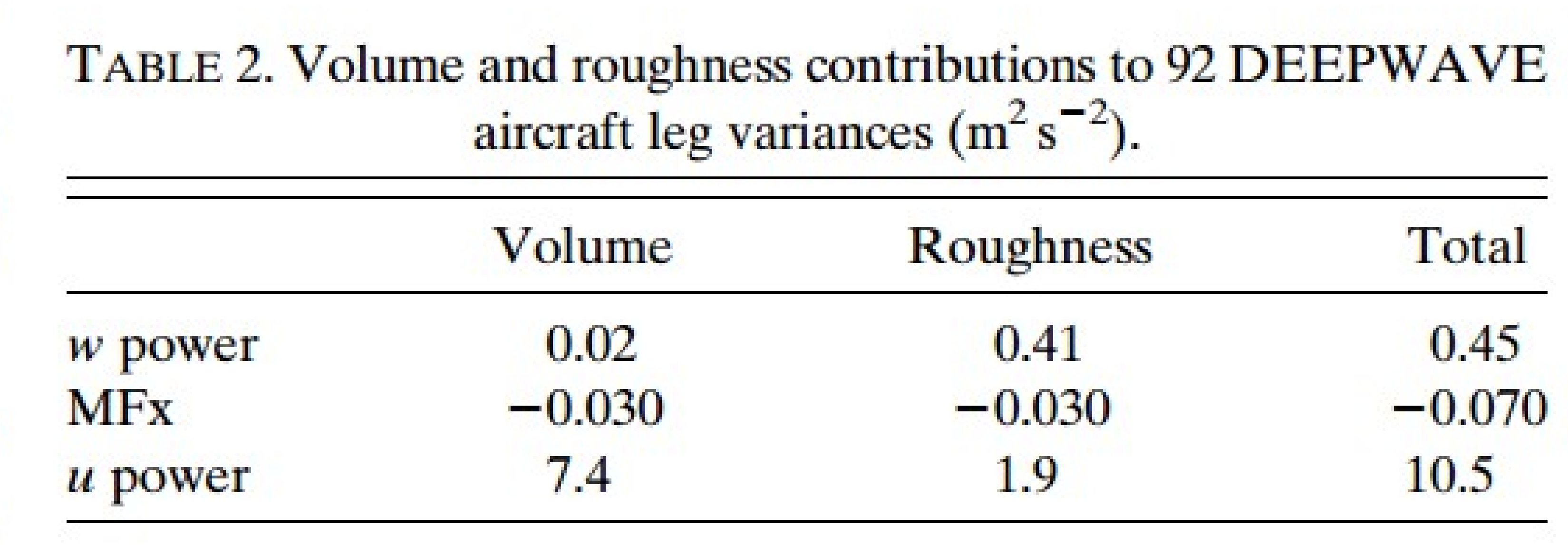
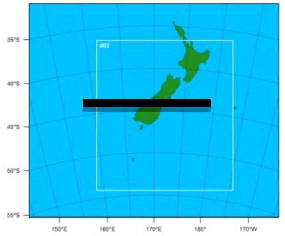


TABLE 2. Volume and roughness contributions to 92 DEEPWAVE aircraft leg variances ($m^2 s^{-2}$).

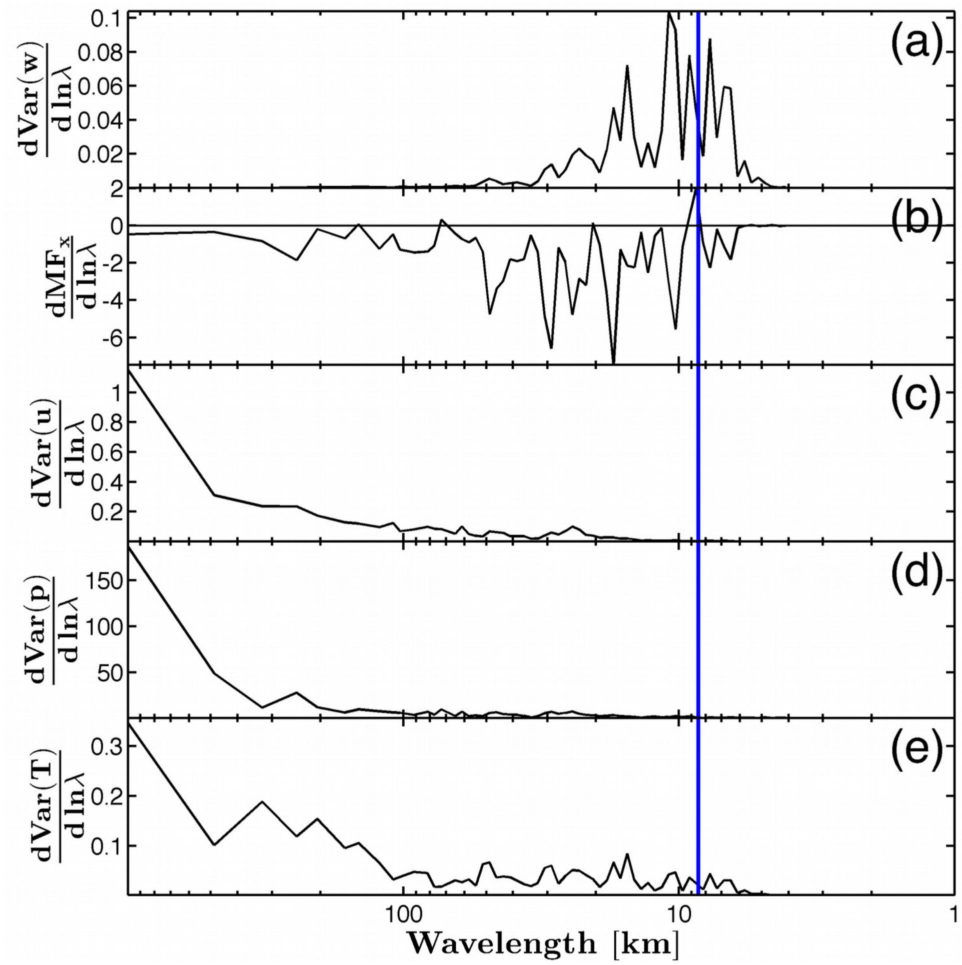
	Volume	Roughness	Total
w power	0.02	0.41	0.45
MFx	−0.030	−0.030	−0.070
u power	7.4	1.9	10.5

Smith and Kruse, 2017

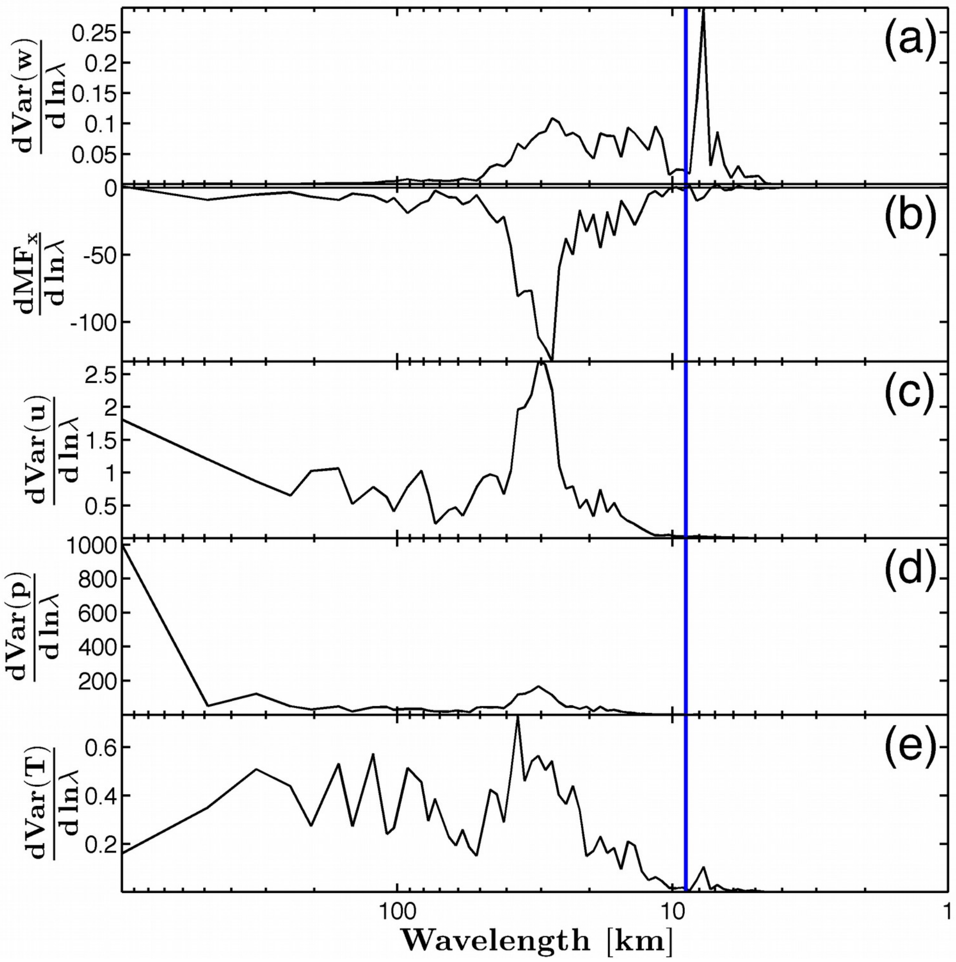
WRF Spectra for RF days during Deepwave



RF04 15UTC 14 June



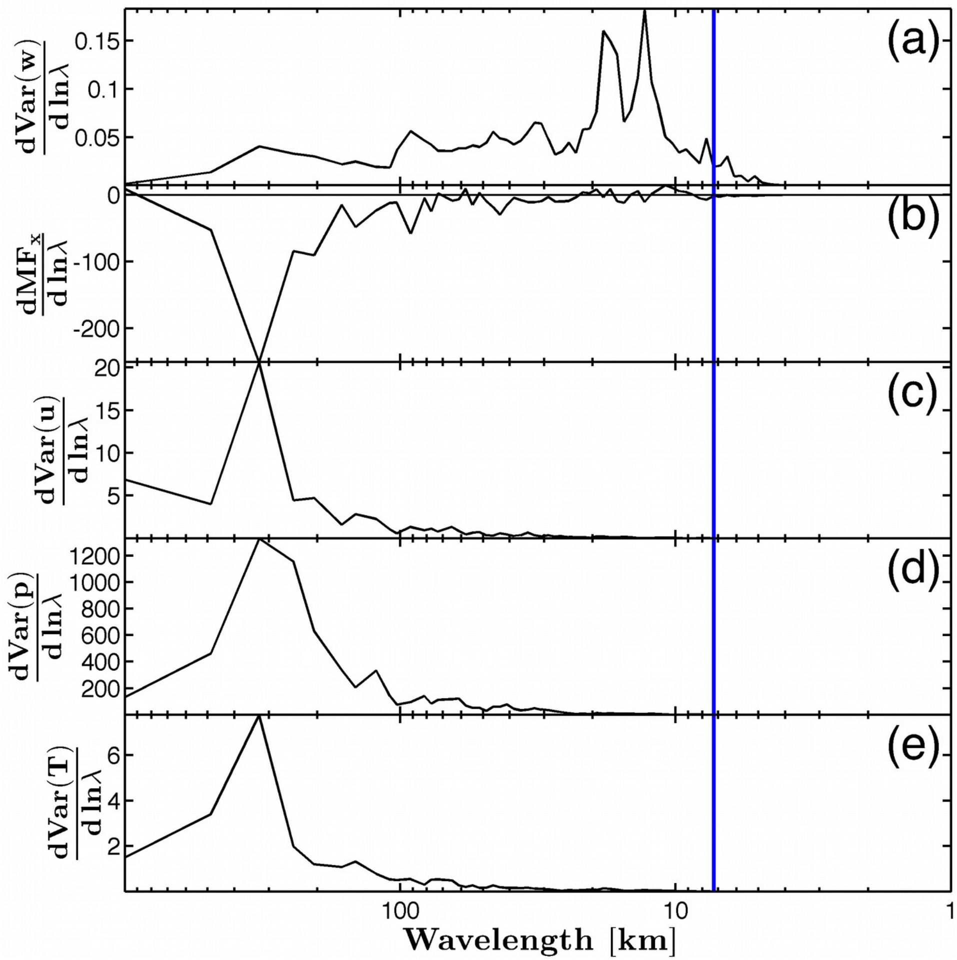
RF08 00UTC 19 June



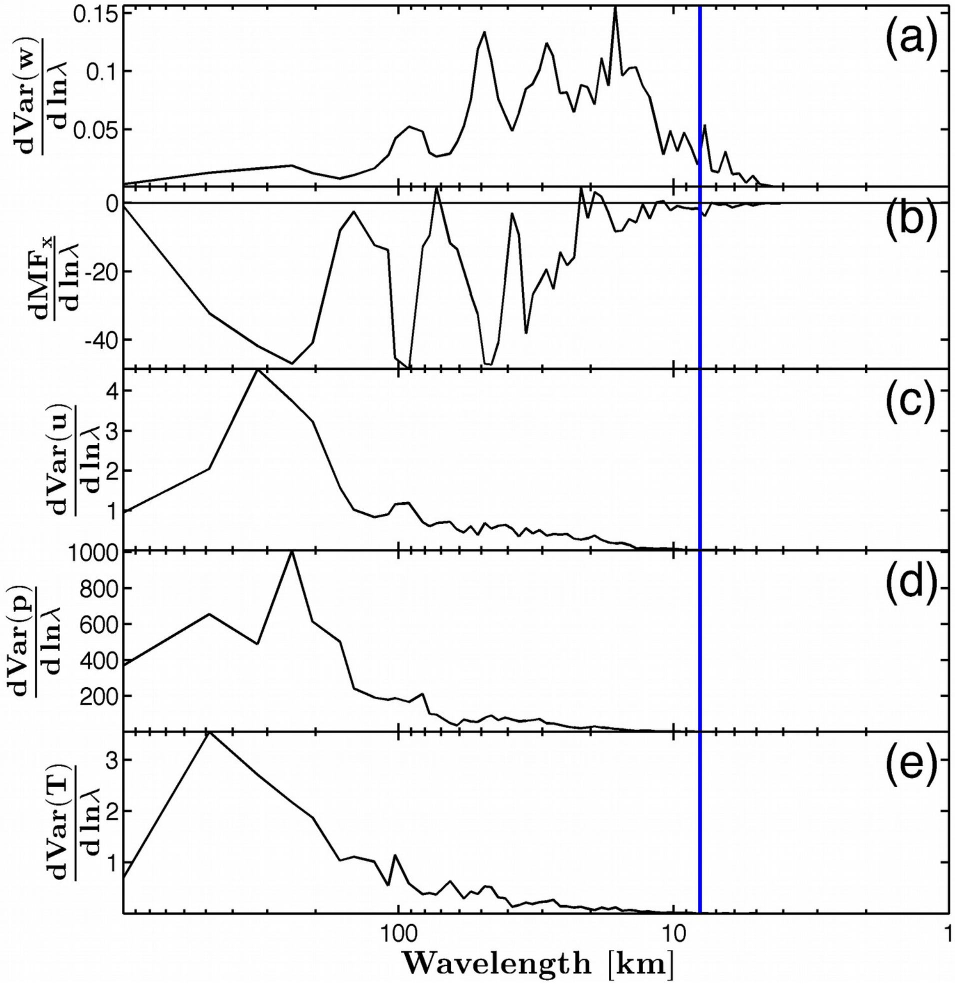
12Hour averages

WRF Spectra for RF days

RF09 15UTC on 24 June

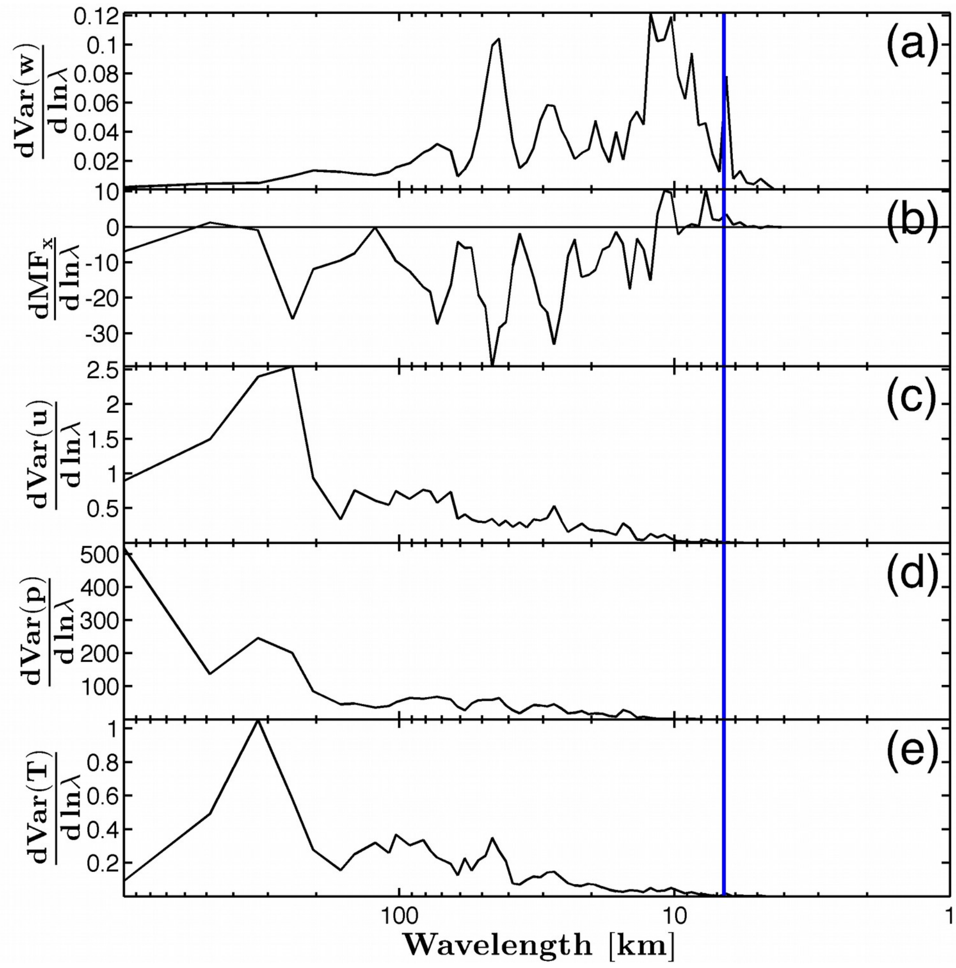


RF09 03 UTC on 25 June

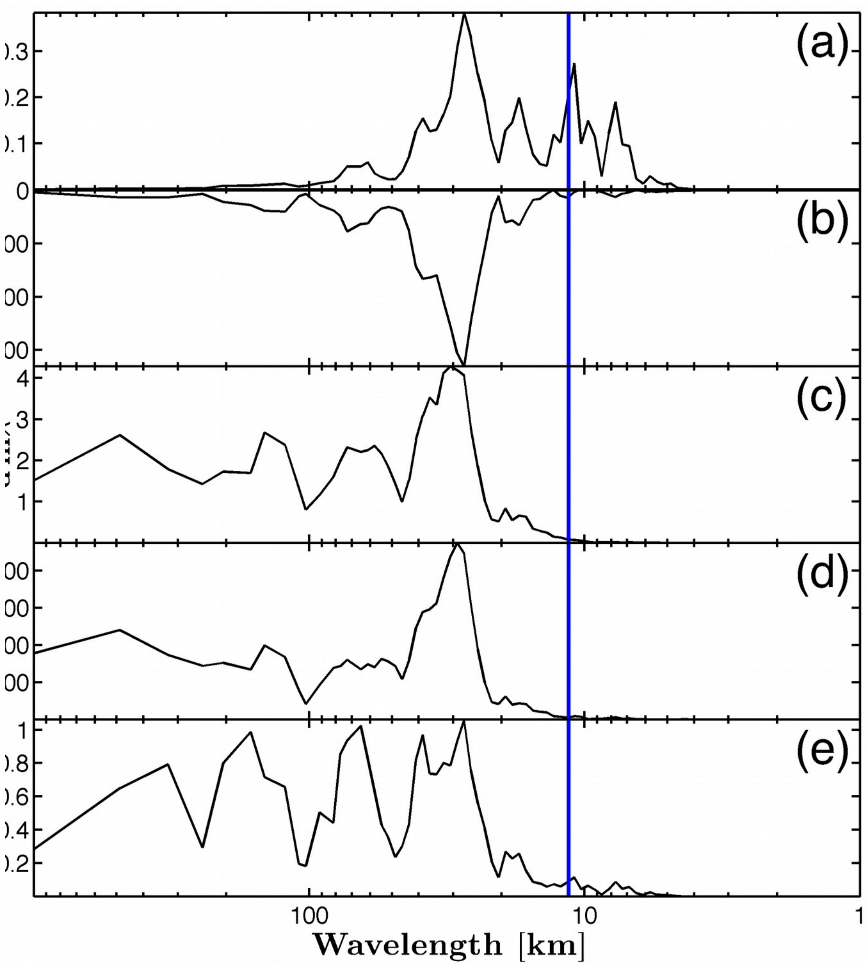


WRF Spectra for RF days

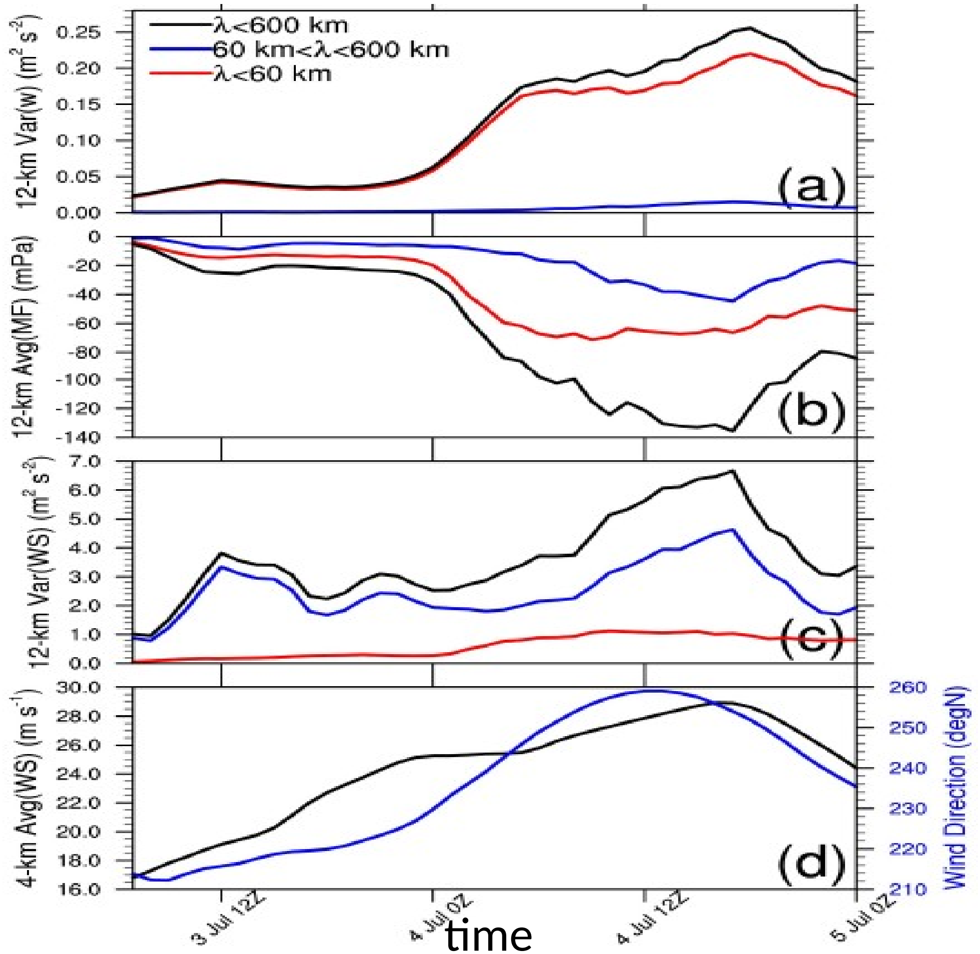
RF12 09 UTC 30 June



RF16 07 UTC 4 July



2km WRF Simulation: 3-day wave event; z=12km



time

w-power

MFx

u-power

Wind Speed
& Direction

Red = Roughness Mode $\lambda < 60km$

Red = Roughness Mode

Blue = Volume Mode $\lambda > 60km$

Blue = Volume Mode

Black = Total

Black = Total

DEEPWAVE Mountain Waves Spectral Categories

- **Roughness Mode I:** $\lambda = 8\ to\ 15km$
  - Near the buoyancy cut-off, Non-hydrostatic
  - Large w-power, Very little u-power or MF
- **Roughness Mode II:** $\lambda = 15\ to\ 60km$
  - Nearly hydrostatic
  - Large w-power, Small but significant u-power
  - Largest Momentum Flux
- **Volume Mode:** $\lambda = 60\ to\ 300km$
  - Hydrostatic
  - Large u-power, P-power and T-power
  - Small but significant w-power
  - Significant Momentum Flux

- **Roughness Mode I:**
  - Near the buoyancy cut-off, Non-hydrostatic
  - Large w-power, Very little u-power or MF
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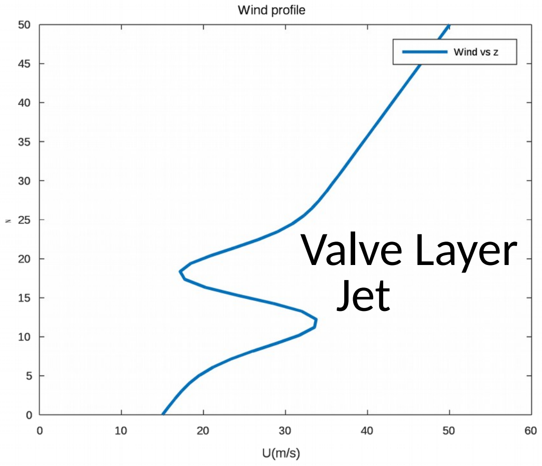
  - Large u-power, P-power and T-power
  - Small but significant w-power
  - Significant Momentum Flux

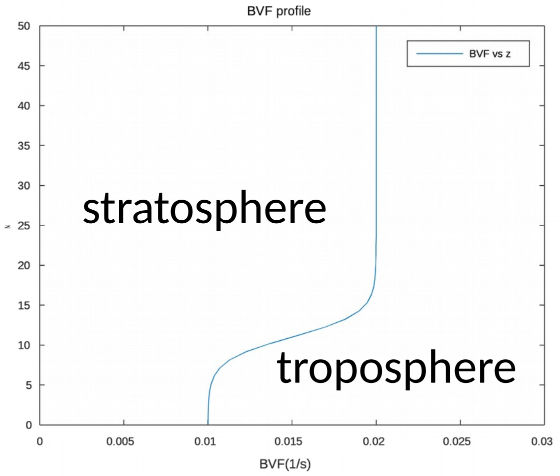
Fate of Mountain Waves in the Stratosphere: A spectral approach

- Questions
  - How do the broad spectrum waves, observed by aircraft at z=12m, propagate and break down in the stratosphere?
- Method
  - Split spectrum into long and short propagating waves with equal MFs (L>60km and L<60km)
  - Project upwards using the hydrostatic assumption and constant MF
  - Use a generic Deepwave sounding

Generic Deepwave Sounding

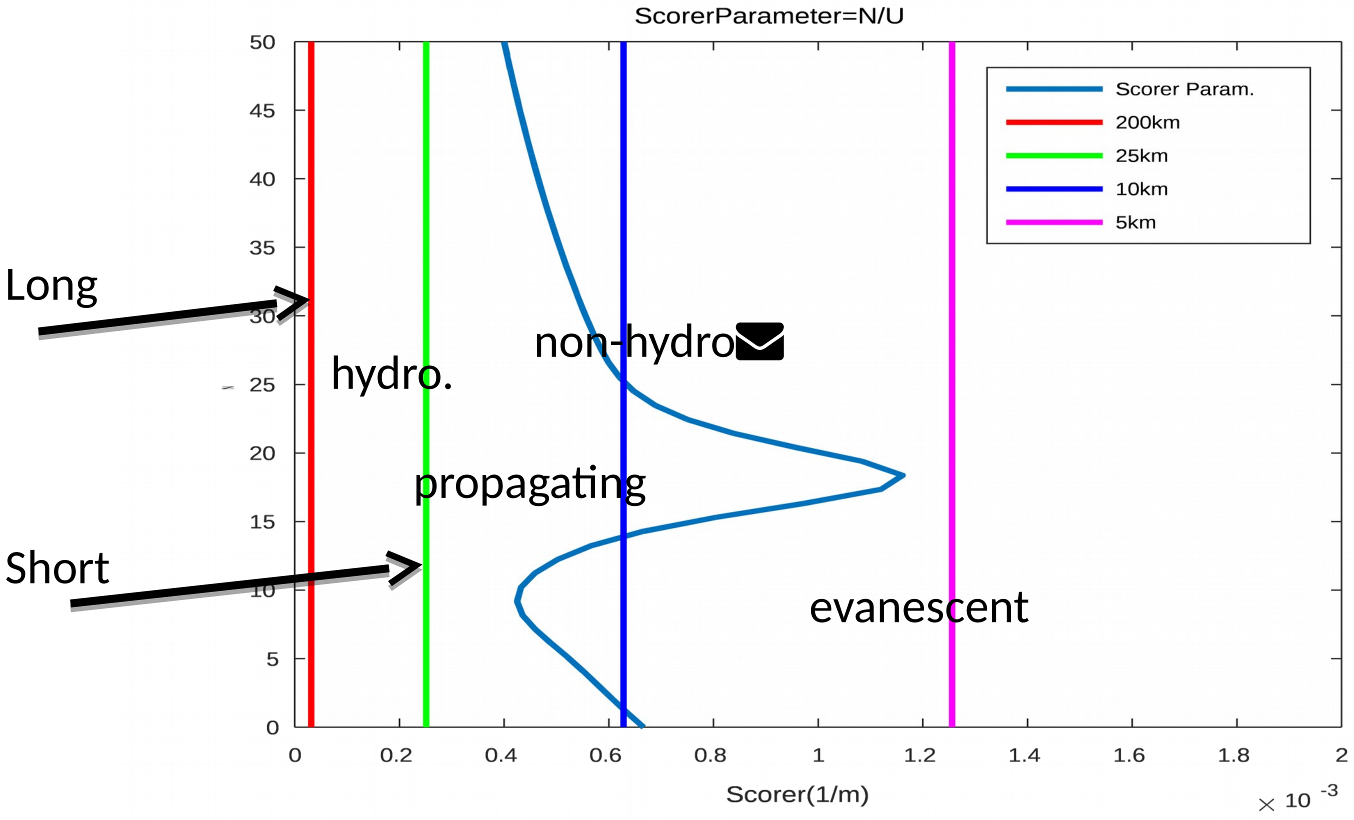


Valve Layer Jet



stratosphere

troposphere



Long

Short

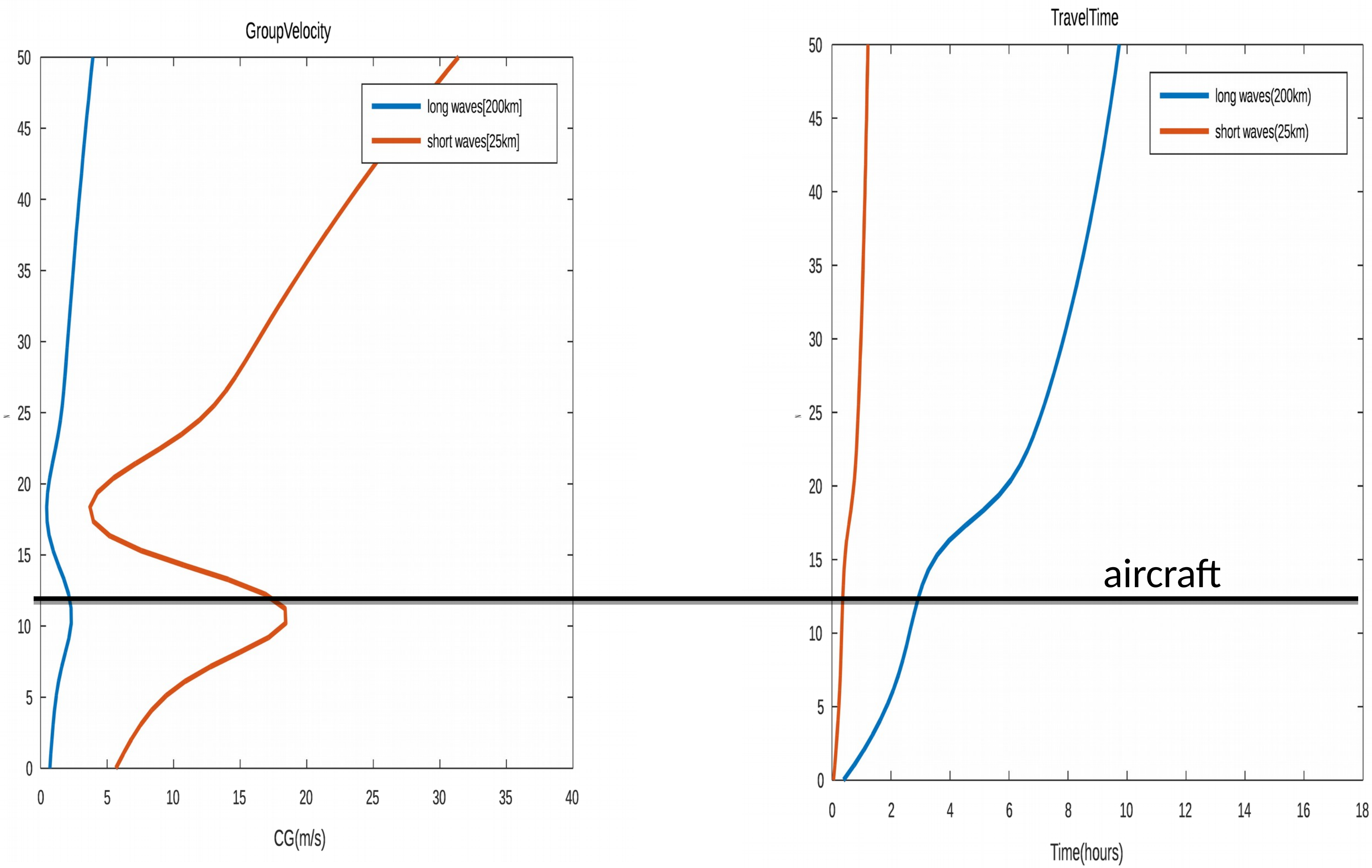
hydro.

non-hydro✉

propagating

evanescent

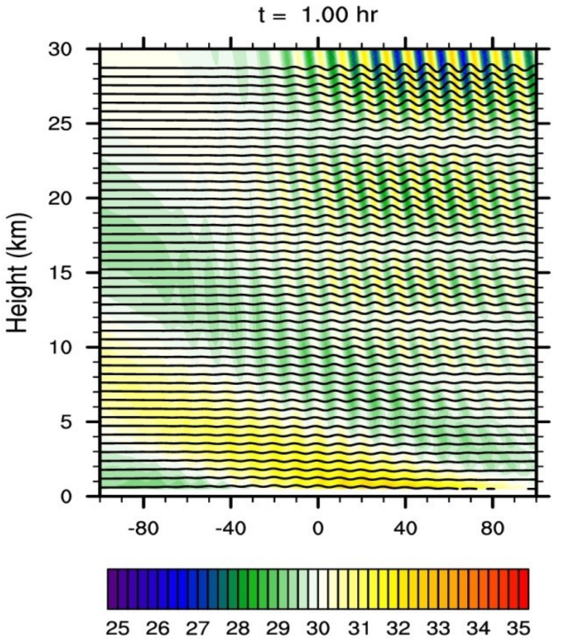
Group Velocity and Travel Time



$$Cg = kU^2/N$$

$$Time(z) = \int_0^z \frac{dz}{Cg}$$

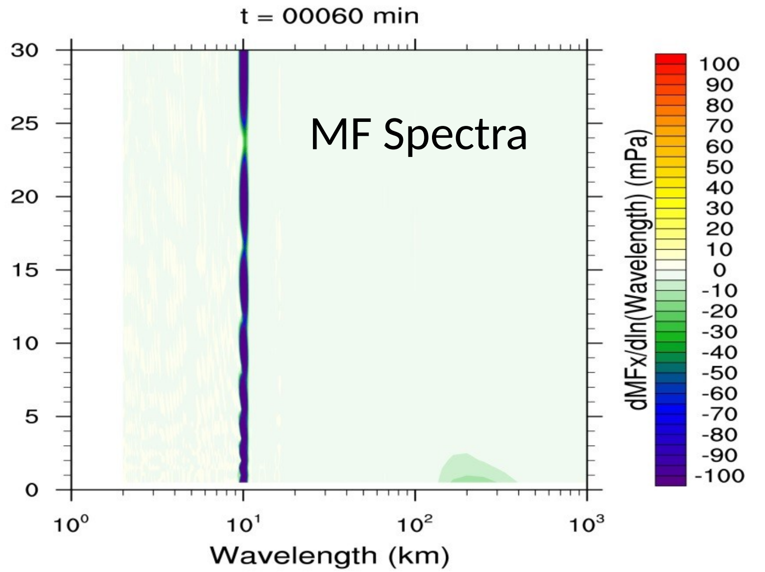
Idealized WRF simulated 12 hour wave event with rough terrain



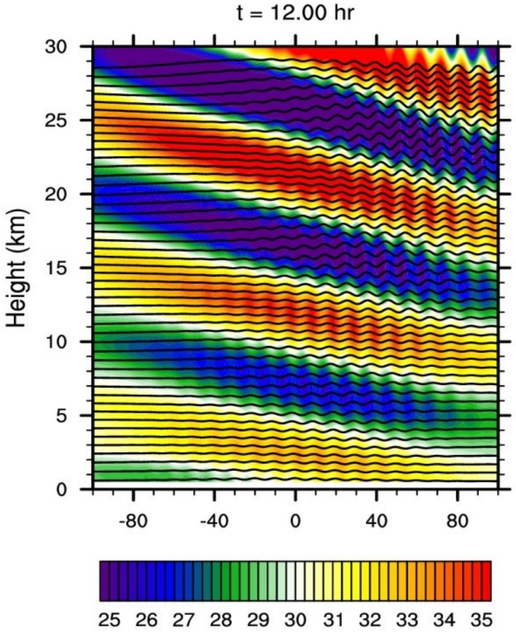
T=1 hour

Roughness waves reach 30km.
Cgz=9m/s
Small u-power

Volume mode waves reach 10km.
Cgz=1.4m/s

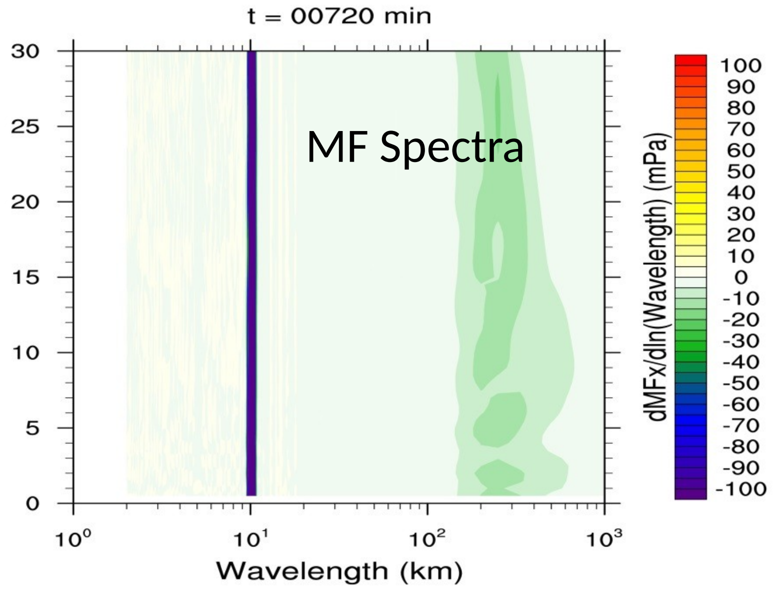


MF Spectra



T=12 hours

Volume mode waves reach 30km.
Large u-power
Susceptible to breaking



MF Spectra

Ideal Rough Hill

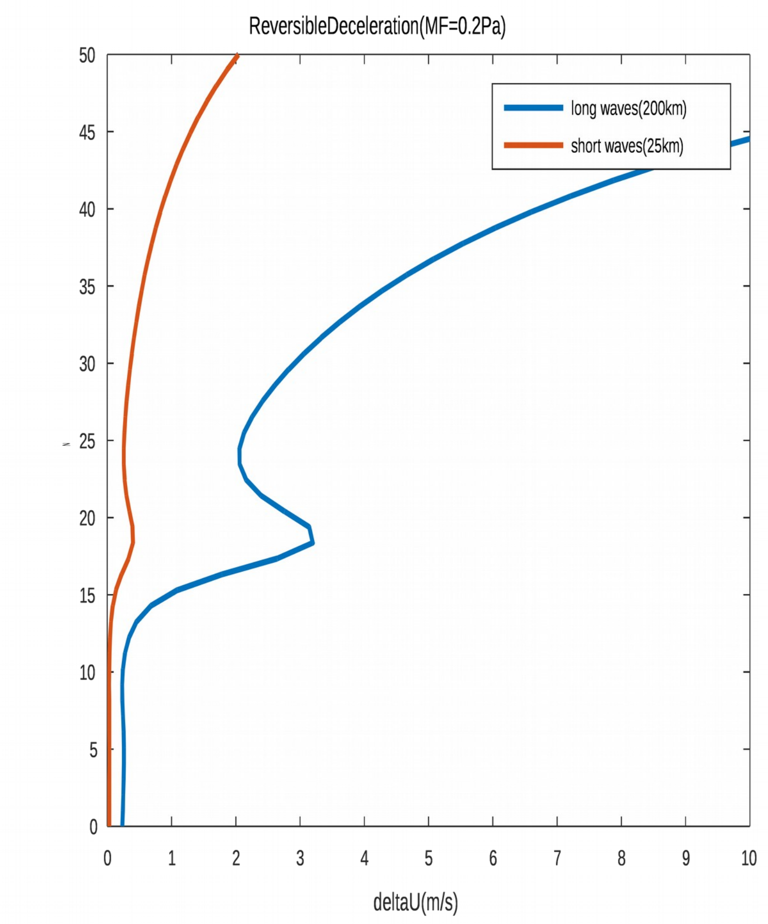
Ideal Rough Hill

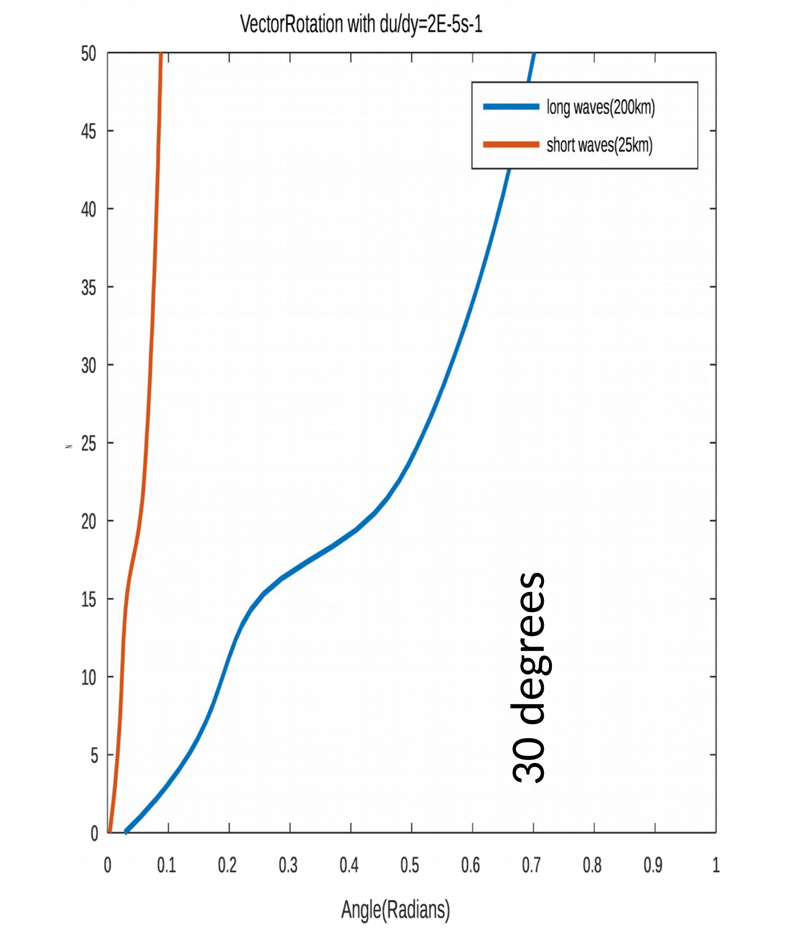
n=20, λ=10km , λ_c=9.4km

n=20, =10km , =9.4km

No shear

Reversible deceleration and Wavenumber Vector Rotation



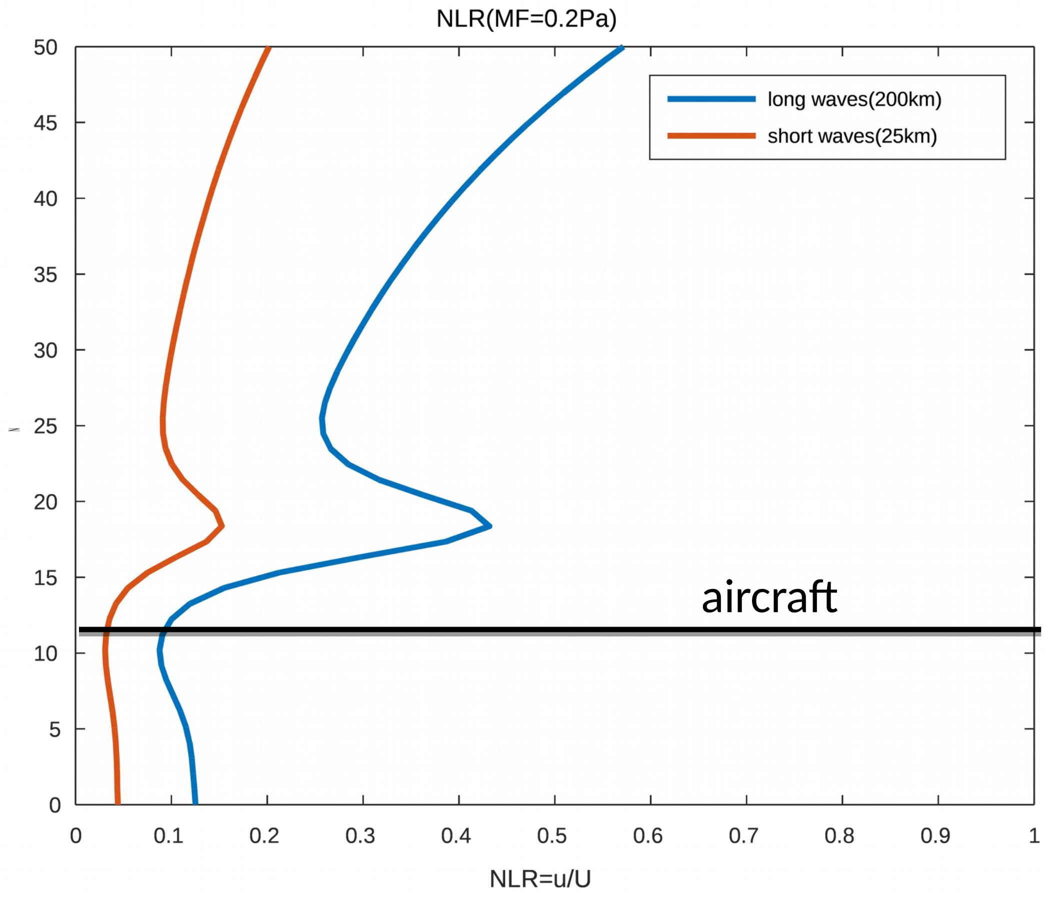


$$\Delta U = MF/(\rho Cg)$$

$$Rotation\ Angle = \int_0^z (du/dy)\ \frac{dz}{Cg}$$

See next talk by Chris.

Non-linearity Ratio and Wave Breaking



$$NLR = \left(\frac{u}{U}\right) = \sqrt{\frac{N[constant\ MF]}{\rho U^3 k}}$$

Properties of Long and Short Hydrostatic Mountain Waves
(i.e. 200 and 25km)

Properties of the volume mode

1. Wavelengths from 60 to 300km
2. Carries half of the MF
3. Large u-power, small w-power
4. Detectable with T' and U'
5. Likely to break in the valve layer
6. Slow group velocity
   a. Large time delay
   b. Large refraction
   c. Large reversible deceleration
7. Explicitly resolved in new GCMs

Properties of the Roughness Mode

1. Wavelengths from 15 to 60km
2. Carries half of the MF
3. Small u-power, large w-power
4. Not seen with T' and U' instruments
5. Resistant to breaking until
   a. High altitude
   b. Impact of the volume mode; flow stagnation or turbulence
6. Fast group velocity
   a. Little time delay
   b. Little refraction
   c. Little reversible deceleration
7. Must be parametrized; easier to do

Implications for GWD schemes

- If the longer waves can now be explicitly resolved in global models, only the shorter waves need to be parametrized.
- The short waves have:
  - A narrower spectra
  - Faster group velocity
  - Little time delay
  - Little refraction
  - Little reversible deceleration
  - They are invisible to T’ and U’ sensors
- This may not be the right time to “improve” GWD schemes to include delay, refraction and reversible deceleration.
- Challenge: How to predict and verify short wave generation and dissipation?

The End

Typical assumptions in GWD parametrization

1. Instantaneous propagation
2. Neglect Stokes drift (irreversible deceleration)
3. Vertical propagation (no refraction)
4. WKB; no reflection
5. Monochromatic
6. Saturation hypothesis
7. No Gray region (no waves resolved, all parametrized)
8. Launching amplitude related to terrain roughness