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Fractal-based lightning channel length estimation from convex-hull flash areas for DC3 Lightning Mapping Array Data

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Motivation

Use VHF Lightning Mapping Array data to estimate NOx per flash and per unit channel length, including the vertical distribution of channel length. What's the best way to find channel length from VHF sources?

Challenges

- Channel is fractal. Infinite length?
- LMA detection variability with range, individual network noise floor, number of stations, etc.

Strategy

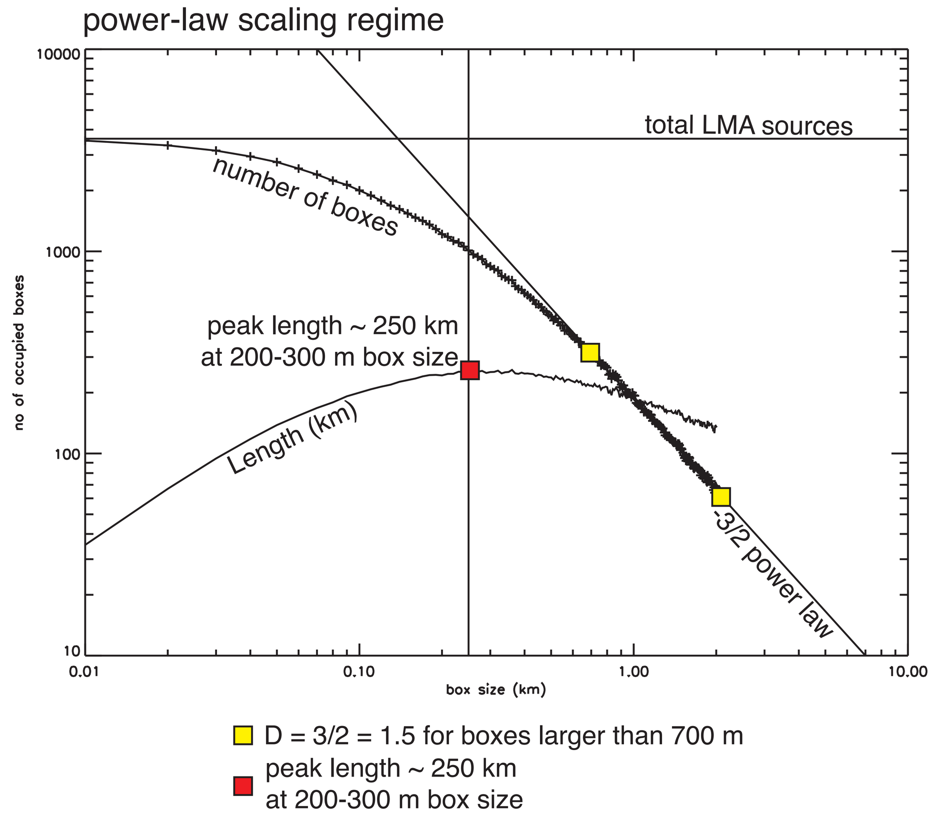
Investigate geometric properties of lightning flashes to inform length estimates, while also seeking to discover fundamental characteristics of the channel geometry.

In the adjacent poster (Thomas et al.), a comparision of three methods of calculating flash length are presented:
- connect-the-dots
- box-covering
- fractal estimate

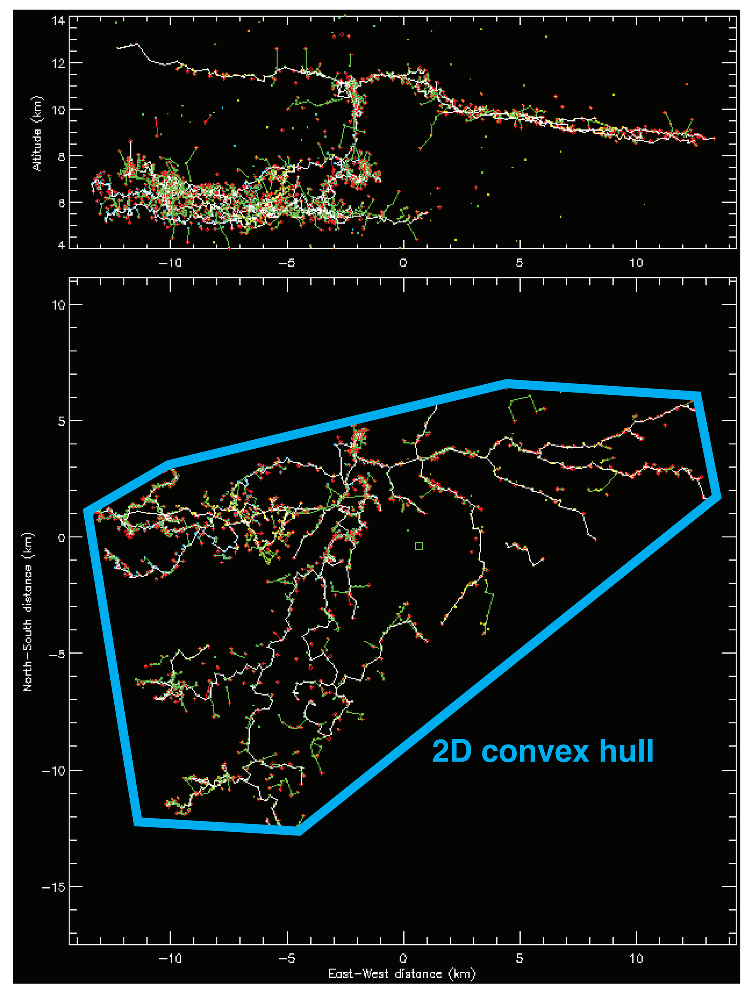
This paper presents the rationale for the fractal method, which is closely related to the box-covering method.

The LNOM algorithm (Koshak, Peterson) also uses box-covering ideas. It uses b_s=100 m and incorporates claims about effeciveness of various NOx-producing processes. See posters by Peterson and Carey this session.

Results from Thomas et al. (adjacent poster) for the flash shown above. There is a power law relationship between box size and the number of boxes. A maximum length estimate is achieved for a box size smaller than that implied by the power-law scaling regime



Fractal length from flash area and volume



Connect-the-dots estimate: 260 km for the main white channel, another 250 km from green segments (Thomas et al. result)

A power-law relationship between count and box size indicates the dimension D of an object, including objects with fractal dimension (Theiler, 1990, J. Optical Soc. Am.)

Use D to estimate flash length from flash area
Use box-counting principles
- Flash-spanning A_h is largest box
  - Area of the convex hull is the minimum convex measure
- Channel step length b_s is smallest sensible box
  - Lower bound to box size prevents length from growning infinitely, i.e., only part of the flash obeys a scaling law.

Fractal length estimate L is:

$$L = N_s b_s = b_s \left(\frac{\sqrt{A_h}}{b_s} \right)^D = \frac{(\sqrt{A_h})^D}{(b_s)^{D-1}}$$

where N_s is number of boxes of size b_s

The method above is easily extended into three dimensions by using the cube root of the volume of 3D convex hull.

Vertical distribution of length

To determine the channel length distibution with height, partition the whole-flash total length estimate as follows:
- The convex hull volume V_h is calculated from an underlying Delaunay triangulation
- Use the Delaunay sub-volumes V_i (tetrahedra) that give the natural-neighbor connectivity between VHF sources
- The local length estimate is a volume-weighted partitioning of the global flash length estimate

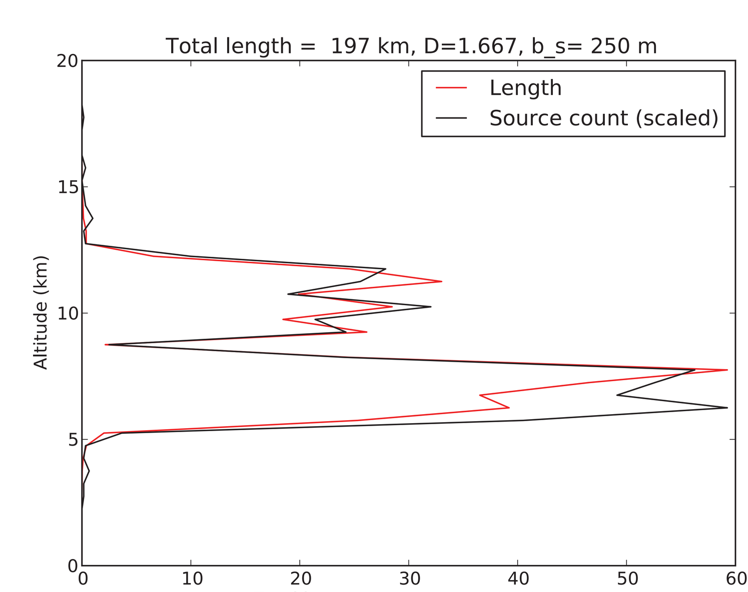
$$S_i = \sqrt[3]{V_i}$$ Simple length from sub-volume

$$P_i = \frac{(\sqrt[3]{V_i})^D}{(b_s)^{D-1}}$$ Predicted fractal length from sub-volume

Hull volume acts as large box size

$$L_i = k \frac{S_i}{P_i} L = \sqrt[3]{\frac{V_h^D}{V_i^{D-1}}}$$

Sub-volume V_i acts as local b_s



Estimating the total length using this method, we find that the vertical length partitioning is nearly identical to the VHF source histogram

Application of this method and the addition of IC/CG discrimination is planned as part of a delivery of flash-level analysis products

Fractal method: results

Try various b_s and use values for D from literature

D = 4/3 (photos of lightning, Tsonis 1996; Niemeyer et al. 1984)
D =1.5 (Sañudo et al. 1995; Thomas et al., adjacent)
D = 5/3 (correlation dimension from LMA data, Allen et al. 2011)
D =1.6-1.75 (Garik et al. 1987, diffusion-limited aggregation)

The tables below illustrate that the power-law relationship has the expected high sensitivity to D, b_s

Box sizes on the order of 100-300 m match best with box-cover and connect-the-dots methods, adding confidence to the importance of those length scales

2D Hull Area

D | b_s	10 m	100	250	300	500
1.5	720 km	228	144	132	101
5/3=1.67	2500	538	292	259	184
1.7	3200	639	336	296	207

3D Hull Volume

D | b_s	10 m	100	250	300	500
1.5	560 km	177	112	102	79
5/3=1.67	1890	407	221	195	139
1.7	2410	481	253	223	156

Summary

Two-step approach accounts for the intrinsic geometry of a lightning flash
- ***Global*** per-flash length estimation from D, V_h, b_s
  - Mostly independent of detection efficiency (only needs accurate volume)
  - Appropriate for whole-storm estimates
- ***Local*** flash length weighted by local natural-neighbor volume
  - Appropriate for vertical or horizontal distributions of flash length

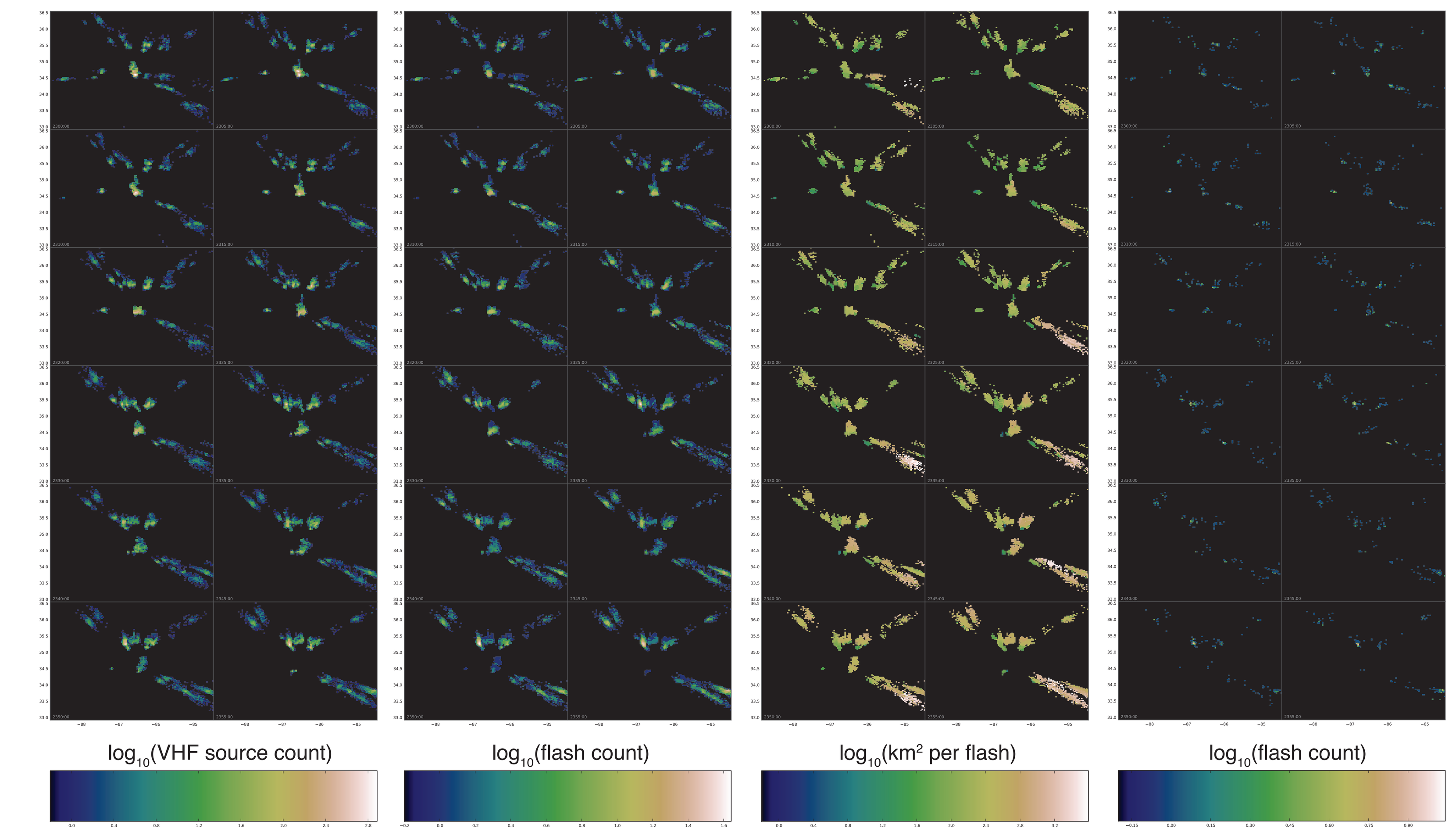
With tuning, various flash length estimation methods give compatible estimates
- Errors are highly sensitive to changes in global parameters, as expected for a fractal object
- Requires input from lightning physics and chemistry specialists to inform physically-based choices for D, V_h, b_s.

Future work

Incorporate polarity-specific D
Positive end more brush-like -> larger D?
Account for step length variation with altitude
200 m at 10 km, smaller at lower altitude
H. Edens, Ph.D. Thesis, NMT 2011.
See also Winn et al. (2011) and Kitagawa & Brook (1960)

Does the 3D spatial geometry and additional information in the time coordinate relate to the (re)distribution of electrostatic potential in some way? Can the flash area product (top right) be exploited in this role?

DC3 gridded flash products



Source density
Simple count of VHF sources in the grid cell

Flash extent density
Count of how many flashes passed through the grid cell. Acts as a column-local flash rate

Average flash area
Sum of the area of all flashes passing through the grid cell divided by flash extent density

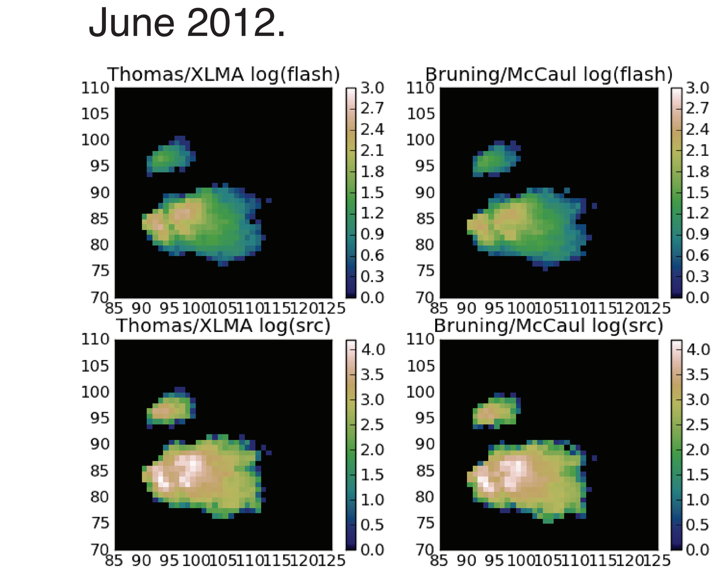
Flash initiation density
Count of how many flashes began in a given grid cell. Sum of all grid values equals domain-wide flash rate

Example of preliminary data above is for the hour beginning 2300 UTC on 21 May 2012 in North Alabama, and has also been produced for West Texas using this method.

Grids are defined with constant lat-lon increments equal to 3km at the network center location. Grids are produced for each five minute window in the hour. Flash sorting is the McCaul et al. (2009) method, with 0.15 s and 3 km VHF source spacing thresholds and 10 point minimum per valid flash. VHF source criteria are chi-squared less than 5.0 and at least six contributing stations.

Code (Python) to produce these grids may be found in a public repository at http://bitbucket.org/deeplycloudy/lmatools

Below, comparison of TTU gridding with an independent flash sorting and gridding implementation in IDL by Thomas. Data from Colorado on 22 June 2012.



Preliminary storm-total length estimate

We have conducted a preliminary fractal length estimate using flashes identified in the gridding process for the time period shown above. For the cell near the network center, a preliminary fractal total length of 16249 km can be obtained using D = 1.5, b_s = 130 m.

With a significant caveat about the robustness of the flash-matching process between datasets, the LNOM-estimated total length for the same flashes was estimated at 16152 km. (Preliminary data courtesy Koshak and Peterson.)

Other D, b_s combinations are possible, but this preliminary analysis shows the possibility of correspondence between the LNOM and fractal methodologies.